Banking Crises and the Lender of Last Resort: How crucial is the role of information?

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February 27, 2006

Abstract

This article develops a model of bank runs and crises and analyses how the presence of a lender of last resort (LOLR) affects the solvency of the banking system. We obtain a one to one mapping from the depositors' equilibrium strategy to an optimal contract prevailing in the economy. The study finds that the difference between a perfectly informed and an imperfectly informed LOLR can be crucial. Our results indicate that a perfectly informed LOLR is a Pareto improvement. However, if the supervisory process of the LOLR is subject to noise, then the gains from ex post efficiency may be outweighed by ex ante inefficiency induced by moral hazard which is conducive to lower lending rates in the economy.

1 Introduction

Banks are an integral part of the economy as they provide an important channel through which funds are transferred from investors to the entrepreneurial sector. However, history has shown that banks are subject to runs and panics. A bank run occurs when depositors fearing that the bank will be unable to fulfill its obligations, attempt to withdraw their funds immediately. If a bank run is severe enough, then even healthy banks can ultimately become insolvent or even bankrupt. Such banking crises can seriously disrupt economic activity.1

*I would specially like to thank Amil Dasgupta, Atish Rex Ghosh, Itay Goldstein, Charles Goodhart, Alexandros Mourmouras, Spyros Pagratis, Jean-Charles Rochet and Hyun Song Shin for useful suggestions and conversations. I would also like to thank Ronald Anderson, Abhijeet Banerjee, Sudipto Bhattacharyya, Douglas Gale, and Timothy Lane for helpful comments. Comments from the participants of LSE/FMG doctoral seminar, IMF/PDR seminar, MIT finance lunch seminar, University of Illinois Urbana-Champaign economics seminar, University of Florida finance seminar and Duke finance seminar are also appreciated. Any errors remain my responsibility. Correspondence to be addressed to Hassan Naqvi, NUS Business School, National University of Singapore, 1 Business Link, Singapore 117592, or email: naqvi@nus.edu.sg

1Bernanke (1983) claims that a substantial part of the decline in real output during the Great Depression was a consequence of the breakdown of economic institutions and the subsequent collapse of credit rather than the decline in the quantity of money.
Because of the central position of financial intermediaries in the economy, the adverse impact of banking crises on economic activity cannot be overemphasised. Since banks hold only a fraction of their deposits as reserves, they are vulnerable to liquidity shocks which might hit the economy as such shocks might induce panic and may affect the behaviour of the depositors. The role of the central bank as a lender of last resort was thus a natural response to the fractional reserve system. Some economists claim that the LOLR is not necessary in a well developed financial system as the interbank market can provide liquidity to solvent banks facing liquidity problems. However, as argued by Goodhart and Huang (2003), the interbank market cannot provide liquidity in two instances. First, the interbank market might not suffice in case of a market failure, for instance, when a large amount, which is too much for a single bank, is needed to bail out a solvent institution. Second, the market mechanism cannot provide insurance against liquidity shocks which affect the whole economy.

Since Diamond and Dybvig (1983) there has been a growing interest in models of bank runs. However the problem with Diamond-Dybvig type models is that runs take place because of self-fulfilling equilibria subsequent to liquidity shocks experienced by depositors and hence are random events. The Diamond Dybvig model exhibits multiple equilibria and the good or the bad equilibrium might prevail irrespective of the underlying fundamentals. In practice, however, bank runs take place when the depositors doubt the solvency of the bank given their beliefs regarding the underlying fundamentals. Thus the bad equilibrium is more likely to prevail if fundamentals are weak and vice versa. Evidence by Gorton (1988) supports this view. He finds that during the US National Banking Era (1865-1914), panics were triggered when the leading indicator of recession reached a threshold level. His results therefore reject the sunspot theories of panics.

Our approach is based on the ‘global games’ methodology first introduced by Carlsson and van Damme (1993) and later modified by Morris and Shin (1998). As discussed in more detail later in this paper, it is not straightforward to apply this approach to banking crises because it is based on the assumption that an agents incentive to take a particular action increases as more and more agents take that action. In general, however, bank run models do not satisfy this assumption of ‘full strategic complementarities’ because if the bank is already bankrupt then an agents payoff from withdrawing decreases when more and more depositors run. Nevertheless, Dasgupta (2004) and Goldstein and Pauzner (2005) get round this problem and show that a unique equilibrium can still be obtained in bank run models. The advantage of using global games analysis is that it enables us to link the probability of crises to the real economy.

Our paper also provides a methodological contribution to the global games literature. We show that for any equilibrium strategy of depositors, there exists

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2See, for example, Goodfriend and King (1988).
3For example, on November 21st 1985, the Bank of New York required a bail-out because of a computer bug in its T-Bills clearing system which denied any incoming payments. The Fed then had to provide an emergency loan of $22.6 billion which was too much for a single bank and because of coordination problems could not be provided by the market as a whole.
a corresponding optimal lending rate in the economy. Thus by using global games we are not only able to identify a unique equilibrium in the depositors’ strategy but are also able to pin down and study the unique optimal contract.

As mentioned by Goodhart and Huang (2003), there have been few formal models analysing the role of the LOLR. Goodhart and Huang study the trade off faced by the LOLR between contagion and moral hazard effects. They show that even in the presence of moral hazard, providing LOLR facilities is justified given the cost of contagion. Freixas (1999) considers the optimal bail out policy of the LOLR. However, Freixas restricts attention to the bail out (or liquidation) of insolvent banks.\(^4\) He justifies the ‘too big to fail’ argument by assuming that the cost of bank liquidation increases with size and hence concludes that it might be rational to bail out an insolvent bank. In contrast, we focus on the bail out of solvent but illiquid banks, in the presence of both perfect and imperfect information.

In a related study Rochet and Vives (2002) analyse the role of the LOLR in the presence of coordination failure among depositors. They study a LOLR whose objective is to bail out solvent banks facing liquidity problems and they show that for an intermediate range of fundamentals, there may exist solvent but illiquid banks. These features of their model are similar to ours. Rochet and Vives show that the LOLR is a Pareto improvement as it can avoid the cost of inefficient liquidation. They thus conclude that Bagehot was right after all in claiming that there exists a role for the LOLR in lending to illiquid solvent financial institutions. However, what makes their paper fundamentally different from ours is that they ignore the moral hazard aspect of the LOLR. This is because throughout their analysis they assume that the LOLR has perfect information about the bank’s fundamentals. We show in our work that moral hazard sets in once a small amount of noise is introduced in the supervisory process of the LOLR. This can dramatically change the results obtained by Rochet and Vives as in the presence of incomplete information, a bank realises that it might be bailed out even if it is insolvent, and consequently ex ante incentives are affected. It is important to study the imperfect information scenario as it is realistic given the difficulty often faced by policymakers in distinguishing between solvency and liquidity problems.

Corsetti, Guimaraes and Roubini (2004) and Morris and Shin (2003) also study a model whereby the presence of a LOLR induces moral hazard. The objective of their work is to study how the presence of a LOLR affects the adjustment policies of the borrower. Both these studies focus on the LOLR’s ‘indirect catalytic effect’ whereby a bail-out by the LOLR reduces the willingness of the investors to withdraw early. On the other hand, our study focuses mainly on the ‘direct catalytic effect’ whereby a bail-out by the LOLR avoids premature liquidation. Corsetti et al show that an improvement in the precision of LOLR’s information is beneficial since the indirect catalytic effect becomes stronger. However, in our model, the increase in LOLR’s precision is beneficial since it

\(^4\)Freixas assumes that solvent banks will be bailed out by the interbank market. However as discussed before this need not be the case.
reduces the probability that the LOLR will inadvertently bail out an insolvent bank and hence reduces the extent of the moral hazard problem.

There are three main objectives of our model. First, we intend to show clearly how shocks are transmitted within sectors via the banking system. This can be done by endogenising the entrepreneurial sector in a bank runs model. Most of the existing literature on banking crises takes the asset side of the bank activities as given and assumes that the bank’s returns are determined by an exogenously given production function. However, a general equilibrium setting gives a clear picture of where the bank’s return comes from and it is then possible to see clearly how a liquidity shock is transmitted from the entrepreneurial sector to the depositors via the banking system, and conversely how the depositors’ equilibrium behaviour affects the behaviour of the entrepreneurs. More importantly, such a setting enables us to characterise the optimal contract between the banks and the entrepreneurs, and thus the lending rate is determined endogenously in the model. To the best of our knowledge, our model is the first one which analyses if and how the presence of the LOLR has any effect on the lending rate and hence on entrepreneurial investment.

Second, an important objective of the model is to study how the presence of the LOLR affects the solvency of the banking system. Many economists have argued that the presence of the LOLR is conducive to moral hazard. However, these arguments have tended to be informal and have thus failed to show under what circumstances the presence of the LOLR will have an adverse effect on the solvency of the banking system. Because of this, precise policy recommendations have been difficult to justify. We clearly show when and how the presence of the LOLR will cause a moral hazard problem and how this can be mitigated.

Lastly, but not least important, we analyse implications for the transparency of the banking system. Since rational agents base their decisions on all available information, it is crucial to study how more or less transparency of the banking system has an effect on the evolution of crises. We thus carry out a comprehensive study of how a banking crisis occurs in the presence of both perfect and imperfect information, with and without the presence of the LOLR.

The rest of the paper is organised as follows. Section 2 introduces the basic setup and the main players in the model. Section 3 considers the second best contract which will prevail in the absence of any bank runs. Section 4 analyses the solvency of banks when it is subject to runs. Section 5 studies the equilibrium in the presence of perfect information with and without a LOLR. Section 6 introduces asymmetric information between banks and the depositors and studies the equilibrium behaviour of the depositors in this imperfect information setting. Section 7 analyses how the presence of an imperfectly informed LOLR affects the economy. Section 8 provides a discussion of the model and finally section 9 gives a summary of the main results.

See, for example, Calomiris (1998) and Krugman (1998).
2 The basic setup and the players

Consider an economy with three periods, $t = 0, 1, 2$. There exists a single divisible consumption good in each period. There are three types of agents in the economy: depositors, financial intermediaries or banks, and entrepreneurs. Later on we will introduce a fourth agent, the central bank or the LOLR. The model can also be applied to an international setting, in which case the depositors can be interpreted as international investors, and the central bank can be thought of as the international lender of last resort, like the IMF.

We study an economy with competitive credit markets, i.e. there are more agents who wish to invest in the risky assets than there are investment opportunities available. Thus the number of depositors is large relative to the available entrepreneurial projects. All agents are risk neutral. We next give a description of the three agents in the economy and then briefly explain the nature of the macroeconomic shock that hits our economy.

2.1 Entrepreneurs

The economy is populated with a total of $T$ entrepreneurs each of which has access to a perfectly divisible risky technology. The entrepreneurs have zero wealth and hence require funding for their projects. The risky technology converts 1 unit of the consumption good at $t = 0$ to $X$ units at $t = 2$ with probability $\pi$ and 0 units with probability $1 - \pi$. The probability $\pi$ is realized at $t = 1$ and hence in the interim period the entrepreneurs know whether or not their projects have succeeded.

All entrepreneurs are heterogeneous and have differing skill levels. Let $p_j$ denote the skill of entrepreneur $j$ where the entrepreneurial skill, $p$, is uniformly distributed on $[0, 1]$. Alternatively, $p_j$ can be thought of as a measure of the quality of the project which the entrepreneur has access to. Furthermore, $p_j$, is private information and is observed neither by the intermediary nor the depositors. Naturally, the entrepreneurial probability of success, $\pi$, is some function of $p$. More precisely, for a project to be successful, it must be ‘good’ which is with probability $p$ and it must survive the macroeconomic shock which hits the economy and finally it must not be liquidated by the bank. Thus an entrepreneur’s probability of success, $\pi$, does not only depend on the quality of his project but is also affected by common risk factors. We will evaluate this probability in later sections.

The reservation utility of the entrepreneurs is $b$ units of the consumption good. $b$ can be interpreted as the wage income of the entrepreneurs if they decide not to take up their projects. Hence it represents the value of the entrepreneurs’ outside option.

2.2 Depositors

There are $D$ depositors each of which is endowed with 1 unit of the consumption good for investment purposes. As in Diamond and Dybvig (1983) there are two
types of depositors: patient depositors who prefer to consume at \( t = 2 \) and impatient depositors who can only consume at \( t = 1 \). A proportion \( \theta \) of the investors are impatient. At \( t = 0 \) the depositors are not aware of their types and this information is revealed to them at \( t = 1 \). The depositors’ type is iid and is their private information.

All depositors have access to a risk free storage technology such that 1 unit of the good at \( t = 0 \) becomes \( 1 + r \) units at \( t = 2 \). However if an investor experiences a liquidity shock and withdraws early then his return will be \( 1 + r_1 \), where \( r_1 < r \). Thus the opportunity cost of funds between \( t = 0 \) and \( t = 2 \) is given by \( \tilde{r} \), where \( \tilde{r} = \theta (1 + r_1) + (1 - \theta) (1 + r) \).

2.3 Banks

The banks just act as intermediaries between the depositors and the entrepreneurs and hence channel funds from the investors to the entrepreneurial sector. The banks exist in this model primarily because of two reasons. First, the banks can perfectly and costlessly monitor whether the entrepreneurs’ projects succeeded or not. There is therefore no moral hazard problem between the entrepreneurs and the banks. Second, as in Diamond (1984), banks can rely on the strong law of large numbers (SLLN) and hence diversify out any idiosyncratic risk. This allows us to focus on systemic risk.

The banking sector is perfectly competitive. Since banks make zero profits, they offer the same contract to entrepreneurs as the one that would be offered by a single bank maximising the welfare of the agents in the economy. It would thus be simple to think of the homogenous group of banks as one single bank.

The interim deposit contract specifies that if an investor withdraws early, then the bank will pay him \( 1 + r_1 \) units. This is just for simplicity and all that we need is that the rate of return on the interim deposit contract be less than or equal to \( \tilde{r} \), the opportunity cost of funds. In the final period, \( t = 2 \), the bank’s returns are equally divided among the depositors who did not withdraw early.

The bank can allocate its endowments to three possible alternatives. It can invest its endowments in the risky projects of the entrepreneurs; it can invest the funds in the riskless storage technology; and finally it can retain a fraction of its endowments as reserves to meet the demand of the early withdrawers. ‘Reserves’ can be interpreted as a short term storage technology such that 1 unit retained as reserves at \( t = 0 \) gives 1 unit at \( t = 1 \). Thus, reserves have a zero net rate of return. Let \( \Omega \) denote the reserve level of the bank and let \( I \) represent the investment portfolio of the bank comprising of investments in risky projects and the riskless storage technology. Finally, let \( \omega_p \) denote the fraction of investment funds, \( I \), invested in the entrepreneurial projects. Note that it should be the case that \( \Omega + I \leq D \).

Given competitive credit markets, we assume that the bank always has enough funds to finance all of the entrepreneurs willing to initiate their risky projects. Clearly, holding reserves is costly, and the bank would like to hold as low a level of reserves as possible. This is because, the bank faces a positive
opportunity cost to holding reserves as any consumption good not retained as reserves can be stored in the risk free storage technology. Thus the opportunity cost of retaining one unit of the endowment in reserves is \((1 + r)\), which is the return from the riskless storage technology.

The bank can also liquidate its investments in the interim period to service withdrawals. However, premature liquidation by the bank is costly. The rate of return on premature liquidation is given by \(R_l\), where \(0 < R_l < 1\). \(R_l > 0\) since the average value of the bank’s portfolio is positive. However, \(R_l < 1\) as otherwise it would never be in the interest of any bank to hold positive reserves. The restriction is only sensible and implies that liquidation is costly enough to induce banks to hold some reserves.

It would be simple to think of the bank’s investment portfolio as a mutual fund (comprising of both investment in the risky project and investment in the riskless asset). Thus when the bank’s reserves are insufficient to meet withdrawals, the bank sells a portion of its fund in the market. For the sake of tractability and simplicity, we therefore do not either assume that the bank liquidates its risky investments first or its riskless investment. The bank just liquidates a certain proportion of its mutual fund to meet the demands of its depositors. This is without loss of generality and simplifies our exposition.

One explanation of why premature liquidation is costly is that the secondary market is characterised by a problem of adverse selection as in Flannery (1996) and Rochet and Vives (2002). The investments of the bank consist of a continuum of assets and agents in the secondary market are infinitesimal and cannot observe the type of asset being sold to them given asymmetric information. Each agent fears that he might end up with the worst quality asset and hence because of adverse selection, the bank’s assets are sold at a discount to their face value.\(^6\)

### 2.4 Macroeconomic shock

In the interim period, \(t = 1\), a macroeconomic shock, \(\phi\), hits the entrepreneurial sector and subsequently adversely affects the return of the bank. We model the macroeconomic shock as a multiplicative shock that affects the proportion of ‘good’ projects. Thus when the shock hits the economy the proportion of ‘good’ projects are scaled by \(\phi\), where \(\phi \sim U[\phi, 1]\). Hence for an entrepreneur’s project to succeed it must not only be ‘good’ which is with probability \(p\), but it must also survive the shock, the expected value of which is given by \(\phi = \frac{1 + \phi}{1 + \phi}\).

Let \(f(\phi)\) be the density function of \(\phi\) and \(F(\phi)\) represent the corresponding cumulative distribution of the shock.

\(^6\)On the contrary, note that if the depositors had invested their endowments in the riskless storage technology instead of the bank, then all impatient depositors would have liquidated their investments at a return of \(1 + r_1\). This is because the problem of asymmetric information does not arise when the impatient investors in isolation liquidate their investments since they hold only one type of asset.
The information structure of the model is such that the banks perfectly observe the realisation of the shock. The investors, however, may perfectly or imperfectly observe the realisation of $\tilde{\phi}$. (We analyse these two cases in later sections.) The ex ante distribution of the shock is public information and is thus known by all the agents. The shock, $\phi$, can also be interpreted as a measure of the fundamentals of the banks. Thus, banks are aware of their fundamentals but the investors or the regulator may or may not have perfect information regarding the bank fundamentals. As we will see this will have interesting implications for the behaviour of the depositors and the entrepreneurs in the economy. Finally, note that since the shock is systemic in nature, it cannot be insured by the interbank market.

3 The optimal contract with no bank runs

We now derive the optimal contract with no bank runs taking the interim deposit contract as given. The 'no runs' optimal contract will provide a useful benchmark against which we can compare the changes in the economy when it is subjected to bank runs.

As a first step note that not all entrepreneurs will be willing to undertake their projects. An entrepreneur will invest if and only if the expected benefit of doing so is greater than the opportunity cost as measured by the value of the outside option, $b$. Therefore only the entrepreneurs who are good enough in terms of having a high likelihood of succeeding will undertake their risky projects. The measure of entrepreneurial skill is $p_j$ and thus if $p_j$ is high enough, entrepreneur $j$ will take up his project. Let $p^*$ be the reservation skill level, i.e. entrepreneurs with $p_j \geq p^*$ undertake their projects whilst the others consume their outside option. Given $p \sim U [0, 1]$, $p^*$ is the fraction of projects that are rejected and $1 - p^*$ is the fraction that are accepted. Thus, the total number of active entrepreneurs who accept their projects is $N = T (1 - p^*)$, while the rest of the entrepreneurs consume their outside option. Then the average skill level of the active entrepreneurs is given by $\tilde{p}$, where $\tilde{p}$ is defined as follows:

$$\tilde{p} = E (p|p \geq p^*) = \frac{1 + p^*}{2}.$$

Given the strong law of large numbers, the actual proportion of ‘good’ projects (i.e. projects that will succeed unless they are hit by the shock or are liquidated) out of the total projects of the active entrepreneurs, will be nonstochastic and will be given by $\tilde{p}$. However, the proportion of projects which are good and survive the shock will be stochastic and will be given by $\tilde{p} \tilde{\phi}$. Hence, the lower the value of $\tilde{\phi}$, the larger will be the number of projects that will fail.

We can now deduce the bank’s rate of return at $t = 2$. Let $\bar{R}(\phi)$ denote the bank’s rate of return. Suppose that $(1 + \rho)$ is the lending rate charged by the banks to the entrepreneurs. Then the bank’s rate of return from its investment portfolio is a weighted average of the return from the entrepreneurial projects.
and the return from the storage technology. Hence the rate of return on the proportion of the bank’s portfolio which it does not liquidate is given by

\[ \tilde{R} = \omega p \left[ (1 + \rho) \tilde{\phi} \right] + (1 - \omega p) (1 + r) . \]  

(1)

If the bank liquidates a proportion \( \xi \) of its investment portfolio then its net rate of return will be given by \( R(1 - \xi) \).

Having derived an expression for the bank’s rate of return we can now characterise the optimal contract with no bank runs. Suppose only the truly impatient depositors withdraw their funds in the interim period. Let \( \hat{\pi} \) denote the average success probability of entrepreneurs. Then the optimal contract solves the following problem:7

\[
\max_{\Omega, 1+\rho} \left( 1 - p^* \right) \left[ \hat{\pi} (X - (1 + \rho)) \right] + p^* b 
\]

subject to

\[
\theta (1 + r_1) + (1 - \theta) \int_{\phi}^{1} \frac{R(\phi) I + [\Omega - \theta D (1 + r_1)]}{(1 - \theta) D} f(\phi) d\phi \geq \tilde{r} 
\]

(3)

\[ I + \Omega \equiv D \]

(4)

\[ \Omega \geq \theta D (1 + r_1) \]

(5)

\[ p^* = \arg\max \left( 1 - p^* \right) \left[ \hat{\pi} (X - (1 + \rho)) \right] + p^* b. \]

(6)

Expression (2) is the entrepreneurs’ expected utility which is a weighted average of the expected net profit from a typical project and the value of the outside option. With no runs, the bank would never have to prematurely liquidate its assets and thus the average success probability of entrepreneurs will be given by \( \hat{\pi}_{sb} = \hat{\pi} \), where the subscript \( sb \) implies that this is the average success probability in the second best economy characterised by no bank runs.

Constraint (3) is the investor rationality constraint of the depositors. At time \( t = 0 \), the depositors do not know whether they are patient or impatient, and thus they will invest if and only if their expected ex ante return is at least equal to the opportunity cost of their funds. A typical investor realises that with probability \( \theta \) he will withdraw in which case his payoff will be \( 1 + r_1 \). With probability \( 1 - \theta \) the investor will not withdraw and his payoff will be the return on the bank’s investment portfolio, \( R(\phi) I \), plus any left over reserves from the interim period, \( \Omega - \theta D (1 + r_1) \), divided over the total number of patient depositors.

7The optimal contract here bears some resemblance to the optimal financial contract studied by Bernanke and Gertler (1990) when borrower type is unobservable. However, Bernanke and Gertler consider a two period model and hence they do not model reserves.
depositors. Thus, if constraint (3) is satisfied then all investors will deposit their money in the bank since their expected payoff from investing in the bank will be higher than or equal to their reservation utility.

Constraint (4) is actually a balance sheet identity which states that the total assets of the bank be equal to its total liabilities. Given that the investor rationality constraint is satisfied, this constraint will hold with equality and hence the sum of the bank’s investment plus reserves will equal the total deposits. Hence it can be interpreted as the budget constraint of the bank.

Constraint (5) is the reserve constraint of the bank which states that reserves be at least sufficient to satisfy the impatient depositors. If the reserve level were less than \( \theta D (1 + r_1) \), then the bank would have had to resort to premature liquidation which as discussed earlier is inefficient and results in a deadweight loss.

Finally constraint (6) is the incentive compatibility condition of the entrepreneurs which states that the reservation skill level, \( p^* \), chosen by the entrepreneur is such that it maximises his expected utility subject to the terms of the financial contract. Taking the derivative of the objective function (2) with respect to \( p^* \) yields the following first order condition:

\[
\left( p^* \hat{\phi} \right) [X - (1 + \rho)] = b. \tag{7}
\]

According to equation (7), \( p^* \) is the threshold skill level such that entrepreneurs are just indifferent between proceeding with their risky project and consuming their outside option.

We assume that \( X - (1 + \rho) - b > 0 \) so that the projects are feasible in the sense that if the project is successful than the return exceeds the interest payment to the bank and the reservation utility of the entrepreneurs. This also ensures that the second order condition \( X > (1 + \rho) \) is satisfied.

Thus, given competitive credit markets, the optimal contract maximises the expected profits of the entrepreneurs subject to the investor rationality constraint of the depositors, the budget constraint of the bank, the reserve constraint, and the incentive compatibility condition of the entrepreneurs. The solution to the ‘no runs’ optimal contract is provided in the Appendix.

In particular it should be noted that the second best optimal reserve level of the bank will be such that it is just sufficient to satisfy the impatient depositors, i.e. \( \Omega_{sb} = \theta D (1 + r_1) \). The intuition behind this result is that the bank holds as low a level of reserves as is possible since reserves have a positive opportunity cost.

Furthermore, for every depositor who withdraws in the interim period, the bank has to hold \( (1 + r_1) \) units as reserves instead of one unit, given that the

\[8\text{Krasa and Villamil (1992) also consider competitive credit markets and the optimal contract in their setup maximises the entrepreneurs' profits subject to investor rationality constraints. However, they do not model the entrepreneurs and hence they do not have an incentive compatibility condition. Further, they have a two period model. However in a three period model, we also need a reserve constraint as investors can withdraw in the interim period.}\]
reserves have a zero net rate of return. This extra cost of \( r_1 \) units per depositor who withdraws in the interim period, ultimately has to be recovered from the entrepreneurial projects. Thus if \( \theta D \) depositors are withdrawing then the extra reserve cost per entrepreneur will be \( \theta D r_1/N \). This explains why the (second best) lending rate charged by the bank to each entrepreneur, as given in equation (19) is equal to the risk free rate scaled by average risk and then adjusted to retrieve the extra reserve cost incurred by the bank to service withdrawals in the interim period.

4 Bank runs and solvency

Now suppose that patient depositors may also withdraw their funds in the interim period. Let \( n \) be the proportion of depositors who withdraw early at \( t = 1 \). Furthermore, let \( \xi \) denote the fraction of investment portfolio \( I \) which the bank liquidates if its reserves are insufficient to service early withdrawals. Finally, let \( \Omega_2 \) denote the level of reserves, if any, the bank has at \( t = 2 \). More formally, \( \Omega_2 = \max [\Omega - nD(1 + r_1), 0] \). We now consider the following possible cases which the bank may encounter.

**Case 1** We assume the existence of an upper dominance region such that the dominant strategy of a patient investor is not to run irrespective of the strategy followed by other investors. More formally, there exists a range of fundamentals \( \phi \in [\phi_U, 1] \) for \( \phi_U < 1 \), such that the payoff from waiting exceeds \( 1 + r_1 \), regardless of the number of people who run. In other words we are assuming that if systemic risk is very limited, then the payoff from waiting always exceeds that from running.

**Case 2** The bank is insolvent if the patient depositors’ utility from waiting is lower than that from running, even if only the impatient depositors withdraw early. Hence if \( n = \theta \), then there will be no liquidation at \( t = 1 \), and the bank will be insolvent if and only if

\[
\frac{\tilde{R}I + \Omega_2}{(1 - \theta)D} < 1 + r_1 \quad \text{or} \quad \tilde{R}I + \Omega_2 < (1 - \theta)D(1 + r_1).
\]

The insolvency point, \( \phi_L \), is such that it solves the following:

\[
R(\phi_L)I + \Omega_2 = (1 - \theta)D(1 + r_1)
\]

where \( R(\phi_L) = \omega_p \left( 1 + \rho \phi_L \right) + (1 - \omega_p) (1 + r) \) given equation (1). We refer to the range \([\phi, \phi_L]\) as the lower dominance region since the bank is insolvent in this range of parameters and consequently the dominant strategy of a patient investor is to run irrespective of the strategy followed by other investors.
Case 3 A bank failure occurs at $t = 2$ when the payoff to the patient depositors who did not run is even less than the payoff to the depositors who withdrew early. If $nD(1 + r_1) > \Omega$ then the bank will need to liquidate some of its investments. The maximum amount that the bank can receive from liquidation is $R_l I$. Hence if $\Omega < nD(1 + r_1) \leq \Omega + R_l I$ then there will be partial liquidation at $t = 1$. Failure will now occur at $t = 2$ if and only if

$$\tilde{R}(1 - \xi) I < (1 - n) D (1 + r_1)$$

where $\xi = \frac{\max[nD(1 + r_1) - \Omega, 0]}{R_l I}$. Analogous to equation (8) the failure point, $\phi_f$, is such that the above inequality just binds. Notice that $\phi_f \geq \phi_L$ since $n \geq \theta$. Thus there is a possibility that a solvent bank might fail if the proportion of depositors who withdraw early is large enough.

Case 4 Finally if $nD(1 + r_1) > \Omega + R_l I$, then the bank will be closed in the interim period $t = 1$. Thus the bank will be bankrupt whenever $n > n_B$, where $n_B = \frac{\Omega + R_l I}{D(1 + r_1)}$. Since $I = D - \Omega$, the bankruptcy point can be rewritten as follows:

$$n_B = \frac{\Omega (1 - R_l) + R_l D}{D (1 + r_1)}.$$  \hfill (9)

It is clear from equation (9) that the bankruptcy threshold is increasing in the level of reserves. Hence, the higher the reserve level, the lower will be the probability of bankruptcy.

To summarise the discussion so far, the bank always fails if $\phi < \phi_L$; the bank never fails if $\phi \geq \phi_U$; and in the range $\phi \in [\phi_L, \phi_U)$, failure depends on the proportion of depositors who withdraw their funds in the interim period $t = 1$. Thus for intermediate fundamentals there is a possibility that banks might be solvent but illiquid.

Assume that the bank’s returns and any assets are equally divided among investors. Then the payoffs to the depositors are summarised in the following matrix.

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<tr>
<th></th>
<th>$n \leq n_B$</th>
<th>$n &gt; n_B$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Run</td>
<td>$1 + r_1$</td>
<td>$\frac{\Omega + R_l I}{nD}$</td>
</tr>
<tr>
<td>Wait</td>
<td>$\frac{\tilde{R}(1 - \xi) D + \Omega}{(1 - n) D}$</td>
<td>0</td>
</tr>
</tbody>
</table>

Note that if the bank is not bankrupt, then the payoff from waiting will be higher than the payoff from running if the bank does not fail. Conversely, the payoff from running will be higher than the payoff from waiting in the event of bank failure. If the bank is bankrupt, then the proceeds from bankruptcy will be equally divided over the number of people who withdrew early, whilst the depositors who wait till $t = 2$ will get zero. It is clear from (10) that the depositors are better off running if the bank fails and by waiting if the bank does not fail.
5  Agents with perfect information

5.1  Multiple equilibria with no LOLR

Let us now analyse the depositors’ equilibrium strategy when the agents can perfectly observe the realised value of $\phi$ at $t = 1$. It is clear from (10) that if the realised value of $\phi$ was common knowledge at $t = 1$, then all investors will not run if $\phi \geq \phi_U$ as in this region the bank never fails. Conversely, if $\phi < \phi_L$, then the dominant strategy of the patient investors will be to run irrespective of the decision of the other depositors. However in the intermediate region $\phi_L < \phi \leq \phi_U$, there exist a multiplicity of equilibria a la Diamond and Dybvig (1983) and the payoff to each investor will now depend on the proportion of patient depositors who withdraw. If a patient depositor expects all other patient depositors not to withdraw then he will also not withdraw his funds and hence an equilibrium will exist where all patient depositors do not withdraw. On the other hand, if a patient depositor expects all the other depositors to run, then an equilibrium will exist where everyone runs.

The Pareto efficient equilibrium is for all (patient) investors to refrain from withdrawing as long as the bank is solvent. This is because as long as the bank is solvent, the payoff from waiting is higher than the payoff from running if the investors can coordinate on their strategies. Thus as long as $\phi > \phi_L$, the (patient) depositors should not run. However, because of coordination failure the dispersed depositors might not be able to achieve the Pareto efficient outcome.

One problem with assuming perfect information about fundamentals is that we cannot characterise and study the optimal contract offered by the bank to the entrepreneurs. In fact for each possible equilibrium there will exist a different optimal contract. If the bank picks the ‘runs equilibrium’, then that would entail a 100% reserve level strategy by the bank since it expects all of its investors to run early. Consequently, there will be no investment and all agents will store their endowments in the riskless storage technology. On the other hand, if the bank picks the ‘no runs equilibrium’, then it will only hold minimal reserves to insure against the possibility of fundamentals being in the lower dominance region. Hence, there will be investment in the entrepreneurial projects. In fact, this ‘good equilibrium’ will correspond to the one that will be obtained in the presence of the LOLR. (We study this below). Finally, in the presence of a mixed strategy equilibrium, the reserve level will be less than 100% of deposits but higher than the one corresponding to the ‘no runs equilibrium’. Thus there exists an optimal contract for any given equilibrium strategy of the depositors. Since we are unable to attach probabilities to the different equilibria, we cannot write down a unique investor rationality constraint and hence we cannot study a unique contract offered by banks to entrepreneurs. We can restate this phenomenon in the following remark.

**Remark 1** There exists a one to one mapping from the depositors’ equilibrium strategy to the optimal contract offered by the banks to the entrepreneurs.
Hence it is clear that in the presence of multiple equilibria policy analysis becomes very difficult since a policymaker is unable to attach probabilities to the different outcomes.

5.2 Unique equilibrium with LOLR

Now suppose that there exists a LOLR who also has complete information regarding the fundamentals. The LOLR announces that it will bail out all solvent banks facing liquidity problems. More precisely the LOLR operates as follows: if a solvent bank is unable to service withdrawals in the interim period, then the LOLR is prepared to lend resources at a zero interest rate, and cover the bank’s shortfall. However, the LOLR facility only covers withdrawals of agents with no liquidity needs and hence a bank is required to hold resources of at least \( \theta D (1 + r_1) \) to satisfy the impatient depositors. Hence, if a solvent bank is illiquid then the LOLR will lend an amount equal to \( n D (1 + r_1) - \Omega \), where \( \Omega \geq \theta D (1 + r_1) \). Clearly the advantage of having a LOLR is that the cost of premature liquidation is avoided.

It follows that in the presence of a perfectly informed LOLR who is willing to bail out all solvent banks, the dominant strategy of the patient depositors is not to run as long as the bank is solvent. This is because in the presence of the LOLR, the investors’ payoffs are independent of the proportion of agents who run, as any inefficient liquidation is avoided. We will thus no longer have a situation where the bank is solvent but illiquid. Hence we now have a unique equilibrium unlike the multiple equilibria result which we got in the absence of the LOLR. Moreover, this unique equilibrium corresponds to the ‘good equilibrium’ of the previous game in the absence of the LOLR, where no patient depositors run as long as the bank is solvent. Therefore, it is clear that in a perfectly transparent economy, the presence of a LOLR is welfare improving since the unique equilibrium with the LOLR, is the one that corresponds to the ‘good equilibrium’ in the case where there is no LOLR.

Given a unique equilibrium we can now also characterise a unique optimal contract offered by banks to the entrepreneurs. The investor rationality constraint will now be given by

\[
\int_{\phi_L}^{1} \frac{\Omega + B I}{D} f(\phi) \, d\phi + 1 \left[ \theta (1 + r_1) + (1 - \theta) \left( \frac{R(\phi) I + [\Omega - \theta D (1 + r_1)]}{(1 - \theta) D} \right) \right] f(\phi) \, d\phi \geq \bar{r}. \quad (11)
\]

The above constraint says that if the bank is insolvent, which is with probability \( F(\phi_L) \), then everyone runs and the reserves and the proceeds from liquidation are divided among depositors. Alternatively if the bank is insolvent, which is with probability \( 1 - F(\phi_L) \), then the impatient depositors get \( 1 + r_1 \), while the
patient depositors receive the bank’s returns plus any left-over reserves divided among the patient agents. Overall the lending rate will be such that the expected return to the agents is at least equal to their reservation utility. Thus the optimal contract will maximise (2) subject to constraints (4), (5), (6) and (11).

From a straightforward comparison of constraint (11) with the second best constraint (3) it is clear that now the bank explicitly prepares for the possibility of fundamentals falling in the lower dominance (insolvency) region by endogenising this possibility in the investor rationality constraint. This is because the bank realises that in case of insolvency it will be bankrupt as everyone runs and it will not be bailed out. This in turn affects the expected return of the depositors and thus a rational bank needs to accommodate for this possibility in the optimal contract. Since an increase in the lending rate increases the expected return of the depositors, a competitive bank will increase the lending rate just up to the point where the above constraint holds with equality.\(^9\) Hence compared to the second best economy, the lending rate will now be higher because of the possibility of fundamentals being in the lower dominance region as in this region everyone runs as they are aware that the bank will not be bailed out.

Let \(\Omega^*_1\) denote the optimal reserve level in the presence of LOLR with perfect information. Although a closed form solution for \(\Omega^*_1\) does not exist, it is nevertheless easy to see that \(\Omega^*_1 > \Omega^*_{sb}\). This is because in an economy with no runs only impatient agents withdraw early regardless of fundamentals. However, in a perfectly transparent economy with runs and a LOLR, everyone withdraws if fundamentals are in the lower dominance region, whilst it is only in the region \(\phi \in [\phi_L, 1]\) that only the impatient agents withdraw early. Thus a rational bank will be prepared for the possibility of insolvency and hence will hold higher reserves vis-a-vis the case where there are no runs.

Finally, it should be noted that the entrepreneurial probability of success will also be affected by the presence of a LOLR. Let \(\hat{\pi}_1\) denote the average success probability in a runs economy with LOLR and perfect information. Then \(\hat{\pi}_1 = \bar{p}(1 - F(\phi_L))\). This is because for a project to be successful it has to be good \(\text{and}\) the bank needs to be solvent. Thus the probability of insolvency has a direct impact on the entrepreneurs’ probability of success.

### 6 Agents with imperfect information

Now suppose that depositors do not perfectly observe the shock but get precise albeit imperfect information regarding the fundamentals of the bank. In the interim period investor \(i\) receives the realisation of the private signal

\[
s_i = \phi + \epsilon_i
\]  
\(^{12}\)

\(^9\)Since the expected profit of entrepreneurs is decreasing in the lending rate, a competitive bank always sets the lending rate to ensure that the investor rationality constraint just holds with equality. Further increases in the lending rate would drive a bank out of the market as other banks will attract all entrepreneurs.
where the noise term $\epsilon_i$ is independent across depositors and is uniformly distributed over the interval $[-\epsilon, \epsilon]$. Since the noise term is iid, thus the signals conditional on the fundamentals are also independently and uniformly distributed across the depositors.

Rational agents use their noisy signals primarily in two ways. First, after observing the signals, the depositors update their beliefs of the shock $\phi$ and thus each investor updates the prior distribution of $\phi$ with the posterior distribution. Second, the signals allow the agents to infer the beliefs of the other agents. Thus in this environment, given the private signal, a depositor forms beliefs not only about the underlying fundamentals but also about the beliefs of other players and other players’ beliefs about other player’s beliefs and so on. This is because the investors realise that their payoffs depend not only on the economic fundamentals but also on the proportion of people who run.

Agents will now condition their actions on their private signals and will run if the expected conditional payoff from running exceeds the expected conditional payoff from waiting and vice versa. As discussed by Morris and Shin (2000), the equilibrium strategy of an investor will be such that it maximises his expected utility conditional on his private information and the strategies followed by the other agents. We thus need to solve for the Bayesian Nash equilibrium of the imperfect information game.

More specifically, we need to solve for the equilibrium of a global game, where a global game was first defined by Carlsson and van Damme (1993) as a game of incomplete information where the actual payoff structure is determined by a random draw from a given distribution and where each player receives a noisy signal of the realisation. Carlsson and van Damme (1993) and Morris and Shin (1998) showed that if a binary action global game satisfied full strategic complementarities, i.e. an agent’s incentive to take a particular action increases when more and more agents take that action, then there would exist a unique equilibrium such that all agents will take a particular action if their signal is below a threshold signal and vice versa.

However, a general feature of bank run models is that they do not satisfy the property of full strategic complementarities. As is clear from the payoff matrix in (10), once the bank is already bankrupt, the payoff from running decreases as the number of agents who are running increases. It is therefore not straightforward to show that a unique equilibrium exists in models of banking crises. Corsetti, Guimaraes and Roubini (2004) and Rochet and Vives (2002) get round this problem by assuming that the decision to withdraw in the interim period is delegated to fund managers who face reputation costs. The fund managers prefer not to withdraw early but their reputation suffers if they do not withdraw when the bank fails. With this assumption the payoffs to the fund managers satisfy full strategic complementarities and therefore the standard argument to show the uniqueness result can be used. Nevertheless we follow the technical

\[\text{Carlsson and van Damme (1993) showed this result for a two player binary action game. Morris and Shin (1998) extended their result to the case where there are a continuum of agents. See Morris and Shin (2003) for a comprehensive review of the literature on global games.}\]
approach adopted by Dasgupta (2004) and Goldstein and Pauzner (2005) to show that a unique equilibrium will exist even if the payoffs do not satisfy full strategic complementarities.

**Proposition 1** There exists a unique threshold, \( s^* \), such that patient agents who receive a signal below \( s^* \) will run and withdraw their funds at \( t = 1 \), while patient agents who receive a signal above \( s^* \) do not run and wait till \( t = 2 \).

**Proof.** Omitted.\(^{11}\)

The cutoff signal \( s^* \) is such that a patient agent who receives the signal \( s^* \) will be indifferent between withdrawing early at \( t = 1 \) or waiting till \( t = 2 \). In other words, the unique \( s^* \) is implicitly defined in a manner such that it solves the following:

\[
\int_{n(\phi,s^*)=\theta}^{n_B} \left( \frac{R(\phi)(1 - \xi) I + \Omega_2}{(1 - n) D} - (1 + r_1) \right) dG(\phi, s^*) = \int_{n(\phi,s^*)=n_B}^{1} \left( \frac{\Omega + R I nD}{nD} \right) dG(\phi, s^*)
\]

where \( n(\phi,s^*) \) is the proportion of depositors who run for any given \( \phi \) and \( s^* \) and \( G(\phi, s^*) \) is the cdf of \( n(\phi, s^*) \). Given that a proportion \( \theta \) of the depositors are impatient and always withdraw early, \( n(\phi, s^*) \) is given by

\[
n(\phi, s^*) = \begin{cases} 
1 & \text{if } \phi \leq s^* - \epsilon \\
\theta + (1 - \theta) \left( \frac{1}{2} + \frac{s^* - \phi}{2\epsilon} \right) & \text{if } \phi \in (s^* - \epsilon, s^* + \epsilon) \\
\theta & \text{if } \phi \geq s^* + \epsilon
\end{cases}
\]

Equation (13) says that at the threshold signal \( s^* \) the expected payoff from waiting exactly equals the expected payoff from running. The uniqueness result enables us to find ex ante the expected proportion of depositors who will run. Given the distribution of \( n(\phi, s^*) \) in (14), the expected proportion of agents who will withdraw early can thus be calculated.

In section 5 we established that there exists a one to one mapping from a depositors’ equilibrium strategy to an optimal contract. Since we have a unique equilibrium in the presence of imperfect information it follows that there must also exist a unique optimal contract in the economy. Hence we have the following corollary to Proposition 1:

**Corollary 1** There exists a unique optimal contract in the presence of imperfect information.

The investor rationality constraint will now be given by

\(^{11}\)Most of the proof of Proposition 1 is along the lines of Goldstein and Pauzner (2005) and has therefore been omitted. Please refer to their paper for details regarding the existence of a unique equilibrium in bank run models.
\[
\int_{s^* - \phi}^{e} \left[ \int_{\phi_B}^{1 - \phi} \frac{1}{n} \frac{(\phi - \phi_B)}{D} \frac{(1 + r_1) f(\phi) f(\epsilon)}{f(\epsilon)} d\epsilon d\phi + \int_{\phi_B}^{\phi} \frac{1}{n} \frac{(\phi - \phi_B)}{D} \frac{(1 + r_1) f(\phi) f(\epsilon)}{f(\epsilon)} d\epsilon d\phi \right] \geq -r. \quad (15)
\]

where \( \phi_B \) is the fundamental corresponding to the bankruptcy threshold \( n_B \).

The above constraint says that if an investor withdraws early, which is the case when \( s_i < s^* \), then he will either receive \( 1 + r_1 \) or the proceeds from liquidation depending on whether or not the bank is bankrupt. Furthermore, if \( s_i \geq s^* \), the investor will wait till \( t = 2 \), in which case he will get the return from the bank’s investments which were not liquidated in the interim period plus any leftover reserves divided among the agents who did not withdraw early. Constraint (15) says that the overall expected return of the investors should at least be equal to their reservation utility. Thus the optimal contract in the presence of bank runs and asymmetric information maximises entrepreneurial utility as given by (2), subject to the budget constraint (4), the reserve constraint (5), the incentive compatibility condition (6) and the investor rationality constraint (15).

### 6.1 Optimal reserves

Let \( \Omega^*_2 \) denote the optimal reserve level of the bank in the presence of asymmetric information. The optimal reserve level chosen by the bank will be such that it maximises the entrepreneurs’ expected utility, as stated in the objective function (2). Even though a closed form solution does not exist for \( \Omega^*_2 \), but nevertheless we are able to do some comparative statics with respect to the reserve level.

As a first step, note that \( \partial I / \partial \Omega = -1 \), i.e. every unit of endowment retained as reserves reduces investment by one unit. Thus, the opportunity cost of holding reserves is \( 1 + r \) per unit, since the same unit could have been invested in the riskless storage technology. As the opportunity cost of holding reserves increases, the lending rate rises since the reserve costs are eventually borne by the entrepreneurs. This in turn reduces entrepreneurial utility.

On the other hand, the advantage of holding higher reserves is that the probability of bankruptcy decreases. From equation (9), it is clear that \( \partial n_B / \partial \Omega = \frac{1 - R_l}{D(1 + r_1)} > 0 \). Thus, as the level of reserves rises, the bankruptcy threshold increases, which in turn implies that bankruptcy becomes less likely. Hence, the probability of bankruptcy, \( 1 - G(n_B) \), declines as the reserve level rises.

---

12 More formally, \( \phi_B \) is such that it solves: \( u(\phi_B, s^*) D(1 + r_1) = \Omega + R_l I \). Thus \( \phi_B \) is the threshold fundamental below which the bank is bankrupt.

13 Note that the probability that an investor withdraws early is given by \( \text{Pr}(s_i < s^*) = \text{Pr}(\epsilon_i < s^* - \phi) \) which is the area under the probability distribution of \( \epsilon \) ranging from \(-\epsilon\) to \( s^* - \phi \). Conversely, the probability that an investor waits is given by the area under the distribution of \( \epsilon \) ranging from \( s^* - \phi \) to \( \epsilon \).
A lower probability of bankruptcy induces competitive banks to reduce lending rates, subsequently increasing entrepreneurial utility.

Furthermore, a higher reserve level may reduce liquidations which in turn may increase the entrepreneurial probability of success. Recall that for a project to survive till $t = 2$, it has to be good, it has to survive the macroeconomic shock and it has to avoid liquidation. Thus the entrepreneurs’ average probability of success in the presence of bank runs and imperfect information is given by

$$\hat{\pi}_2 = \hat{p} E[\phi (1 - \xi)] = \hat{p} \left[ \hat{\phi} - E(\phi\xi) \right].$$

Since the proportion of investments liquidated, $\xi$, has a direct effect on the entrepreneurs’ probability of success, it is clear that the reserve level chosen by the bank will also have an impact on the probability of entrepreneurial success.

The optimal reserve level, $\Omega^*_2$, chosen by the bank will be such that it balances the above trade-offs. More precisely, at the optimal level, $\Omega^*_2$, the marginal benefits of holding higher reserves will be just equal to the marginal costs of holding higher reserves and hence the bank will have no further incentive to increase reserves. At this optimal level, the entrepreneurs’ expected utility will be at a maximum given the constraints.

### 6.2 Perfectly informed LOLR

Now suppose that depositors imperfectly observe the bank fundamentals but there exists a LOLR who has complete information regarding the macroeconomic shock. As before, the LOLR commits to bail out all solvent banks facing liquidity problems. Thus as long as the bank is solvent the cost of panic runs will be avoided and there will be no premature liquidations. We will no longer have a situation where a bank is solvent but illiquid. Hence depositors will always be better off from waiting as long as $\phi > \phi_L$ regardless of the number of people who run. Conversely, depositors will always be worse off from waiting if the bank is insolvent. Thus a depositor’s optimal strategy will now be independent of the strategy of other players and will only depend on fundamentals.

Depositors will realise that their expected payoff from waiting rather than running will be higher if and only if $\phi \geq \phi_L$ and thus they will wait as long as $E[\phi|s_i] \geq \phi_L$ and will run if the converse is true. Given that $\phi|s_i \sim U[s_i - \epsilon, s_i + \epsilon]$, it follows that $E[\phi|s_i] = s_i$ and hence all depositors will wait as long as $s_i \geq \phi_L$ and will run otherwise. We will thus have a unique equilibrium around the switching strategy $\phi_L$. Now all depositors will run if $\phi < \phi_L - \epsilon$; only the impatient depositors will withdraw if $\phi \geq \phi_L + \epsilon$; and in the range $\phi \in [\phi_L - \epsilon, \phi_L + \epsilon]$, there will exist a unique equilibrium where some depositors will withdraw while others will not depending on their posterior beliefs regarding bank’s solvency.

It can be shown that in an environment with imperfectly informed agents, the presence of a LOLR with complete information is always a Pareto improvement. We prove the following Proposition in the appendix.

\footnotetext[14]{Note that $\phi$ and $\xi$ are not independent. This is because liquidations, $\xi$, will be higher the worse the shock i.e. the lower the realised value of $\phi$.}
Proposition 2 The presence of a perfectly informed LOLR is a Pareto improvement.

Proof. See Appendix. ■

The intuition behind the above Proposition is that the presence of a perfectly informed LOLR avoids the cost of premature liquidation as long as the bank is solvent. Hence with the presence of the LOLR the cost of panic runs is avoided. Entrepreneurs are better off while no one is worse off and thus a perfectly informed LOLR is a welfare improvement. In the next section we check the robustness of this result when we drop the assumption that the LOLR is perfectly informed.

7 Imperfectly informed LOLR

We now consider the case where the LOLR also has incomplete information and only observes a noisy signal of the bank fundamentals.\textsuperscript{15,16} In the interim period the LOLR receives a signal, $s_l$, of the bank fundamentals where

\[ s_l = \phi + \epsilon_l. \tag{16} \]

The noise term $\epsilon_l$ is uniformly distributed over $[-\epsilon_L, \epsilon_L]$. We assume that the distribution of $\epsilon_l$ is known by the bank. Further, the precision of the LOLR’s signal is higher than that of the investors’ signals, i.e. $\epsilon_L < \epsilon$. This is realistic since the LOLR generally has more information about the banks than the individual investors.

The objective of the LOLR is to bail out all solvent banks facing short term liquidity problems. But if the information set of the LOLR is noisy then it might not always be easy to distinguish the solvent but illiquid banks. Indeed an imperfectly informed LOLR might not bail out a solvent bank and conversely a noisy LOLR might bail out an insolvent institution. Thus there is a possibility that an imperfectly informed LOLR may make Type I or Type II errors. We refer to the situation of not bailing out a solvent bank as a Type I error and the situation where the LOLR bails out an insolvent bank as a Type II error.

Let $s^*_l$ define the bail-out strategy of the LOLR such that it bails out the bank if and only if $s_l \geq s^*_l$ and does not bail out if $s_l < s^*_l$. Our results do not hinge on any particular form of the LOLR’s objective function. All that we need is that $s^*_l$ be such that there exists a positive probability that the LOLR may make Type I and Type II errors. This is plausible and is likely to be the case in practice if the LOLR is imperfectly informed.

\textsuperscript{15} For the purpose of brevity, we will often refer to an imperfectly informed LOLR as a ‘noisy’ LOLR.

\textsuperscript{16} The LOLR will be imperfectly informed because it may have incomplete information regarding the exact withdrawals and exact reserve level of a bank or alternatively because it may not be aware of the precision of the depositors’ signals or finally it could be a combination of these effects. This would imply that the LOLR will not be perfectly able to infer $\phi$ given the noise in its information.
We assume that it is common knowledge that the LOLR follows a bail-out strategy around $s^*_l$. Then the investor rationality constraint of the depositors in the presence of a noisy LOLR is given by expression (24) in the Appendix. The constraint says that in the presence of a noisy LOLR which may or may not bail the bank out, the expected payoff to the depositors should at least be equal to $\bar r$, given that the agents follow a switching strategy around a threshold signal. Thus the optimal contract in the presence of a noisy LOLR maximises entrepreneurial utility as given by (2), subject to the budget constraint (4), the reserve constraint (5), the incentive compatibility condition (6) and the investor rationality constraint (24).

It is noteworthy that the introduction of a noisy LOLR also affects the average entrepreneurial probability of success, $\hat \pi$. Thus we have seen that the entrepreneurial probability of success is not only affected by regulation but also by the information structure of the players. Such an insight is only possible in a model that endogenises the asset side of the bank’s balance sheet.

We next establish the following proposition.

**Proposition 3** (i) In the imperfect information setting where the bank fundamentals are not perfectly observable to the investors, the level of bank reserves in the presence of a (noisy) LOLR will be lower relative to the case where the LOLR does not exist. (ii) The lending rate, $(1 + \rho)$, decreases in the presence of a (noisy) LOLR when bank fundamentals are not common knowledge.

**Proof.** See Appendix.

The reason why the level of reserves falls in the presence of a noisy LOLR is that now the bank will only hold reserves to insure against the possibility that it might not be bailed out rather than holding reserves to insure against bank runs in general. The presence of a LOLR reduces the possibility of bankruptcy because of a potential bail-out, and hence for any given reserve level, the probability of bankruptcy will be lower in the presence of a LOLR relative to the case where the LOLR does not exist.

Recall that with imperfect information the bank holds more reserves, relative to the second best level, as some patient depositors may also withdraw in the interim period. Thus the fall in the level of reserves in the presence of a LOLR is an improvement as it brings us closer to the second best reserve level.

Furthermore, for any given lending rate the bank faces a lower expected cost of premature liquidation in the presence of a LOLR because of the possibility of a bail-out. Hence given a competitive banking system the bank will lower the lending rate and pass all the benefits on to the entrepreneurs. Moreover, the bank will realise that there exists a probability that it might be bailed out even when it is insolvent in the presence of an imperfectly informed LOLR. This asymmetry of information will act as a subsidy that will relax the bank’s zero

\[ \hat \pi = \Pr (s_i < s^*) \bar p E [\phi (1 - \xi) | s_i < s^*] + \Pr (s_i \geq s^*) \tilde p. \]  
Thus, for a project to succeed it has to be good and it has to avoid liquidation whereby the probability of liquidation is affected by the probability of a bail-out.
profit condition. A competitive bank will internalise the implicit subsidy, $S$, provided by the LOLR in its zero profit condition which will further decrease the lending rate.

The source of the implicit subsidy provided by the LOLR is the bank’s inability to repay the LOLR loan in the event of insolvency. The bank repays the LOLR loan as long as it is solvent. However, in case of insolvency, the bank defaults on the loan as all returns are paid off to depositors. Hence the LOLR suffers a loss if it inadvertently bails out an insolvent institution.\(^{18}\)

It is noteworthy that the higher the probability of a Type II error, the higher will be the value of the implicit subsidy provided by the LOLR and consequently the lower will be the lending rate. On the other hand, we show in the proof to Proposition 4 that the higher the probability of a Type I error, the higher will be the level of bank reserves. The intuition is that an increase in the Probability of Type I error implies that the likelihood of a solvent bank not being bailed out increases and consequently a rational bank realises this and takes this into consideration when setting its reserve level.

### 7.1 Transparency and insolvency

We next examine the important question of how the presence of an imperfectly informed LOLR affects the solvency of the banking system. We show that in the presence of a noisy LOLR, banks are more likely to be adversely affected by the macroeconomic shock as long as the precision of the LOLR’s signal is sufficiently low. This result is stated in the following proposition.

**Proposition 4** The probability of insolvency of the banking system increases in the presence of an imperfectly informed LOLR as long as the noise in the LOLR’s information set is sufficiently high.

**Proof.** See Appendix. ■

The intuition as to why the probability of insolvency of a bank increases in the presence of a sufficiently noisy LOLR is that the bank realises that there is a high likelihood that the LOLR will bail it out even when it is actually insolvent. Thus a competitive bank internalises the ex ante implicit subsidy provided by the LOLR in its zero profit condition. This in turn reduces the lending rate subsequently affecting the insolvency criteria.

It is clear that the presence of a noisy LOLR is conducive to a moral hazard problem whereby competitive banks in anticipation of a potential bail-out reduce the lending rates which in turn can increase the probability of insolvency of the

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\(^{18}\)The role of the IMF in providing these implicit subsidies to its member countries has also been criticised by the Independent Evaluation Office (IEO) of the IMF in its 2002 report, since the loans provided to insolvent members are seldom recovered. The IEO documents the following: “Some twenty-five countries have been indebted to the Fund for more than thirty years out of the last fifty. Sixteen countries have been under Fund-supported programs for twelve years or more out of the last eighteen. Such prolonged use risks turning the Fund into a source of long term financing.”—Independent Evaluation Office (IEO) of the International Monetary Fund (IMF), 2002.
banking system. This moral hazard problem will be more severe the lower the precision of the LOLR’s signal. It follows from Proposition 4 that if the precision of the LOLR’s signal is low enough, then the severity of the moral hazard problem will increase the riskiness of the banking system. Thus we have the following corollary to Proposition 4.

**Corollary 2** The moral hazard problem is directly proportional to the amount of noise in the information set of the LOLR. Thus the probability of insolvency of the banking system increases as the degree of bank transparency worsens.

In the above corollary, transparency refers to the flow of information from banks to external agents including the LOLR. A higher degree of transparency will translate into more precise signals obtained by investors, and in particular the LOLR. On the other hand, a lower degree of transparency would imply that the probabilities of Type I and Type II errors will increase which would worsen the severity of the moral hazard problem.

### 7.2 Impact on entrepreneurial investment

Endogenising the asset side of the bank in the model, enables us to examine the impact a noisy LOLR will have on entrepreneurial investment. Taking the first order condition of expression (6) yields:

\[
\hat{\pi} [X - (1 + \rho)] = b.
\]  

(17)

The above condition states that entrepreneurs will invest as long as the expected marginal benefits from investment just equal the marginal costs. It is noteworthy from (17) that a reduction in the lending rate relaxes the incentive compatibility condition of entrepreneurs and subsequently increases entrepreneurial incentive to undertake risky projects. Hence, entrepreneurial investment, \( N \), increases as the lending rate declines.

Given the moral hazard problem that arises in the presence of a noisy LOLR as stated in Corollary 2, it is clear that as the asymmetry of information between the bank and the LOLR worsens, the lending rate declines and the investment level increases. Indeed, if the noise level is high enough, the investment level in the presence of a noisy LOLR may overshoot the second best level, which will be conducive to an overinvestment problem. Thus \( N > N_{sb} \) for high enough \( \epsilon_L \), since the value of the internalised subsidy will be high and thus lending rates will be low, which in turn will affect entrepreneurial incentives. Hence, in the presence of a noisy LOLR there exists a possibility that there might be overinvestment as entrepreneurs may undertake socially negative NPV projects given distorted incentives. We state this in the following remark.

**Remark 2** If the moral hazard problem is severe enough, then there will be overinvestment in the economy.

Given Corollary 2 and Remark 2, it is clear that the likelihood of overinvestment increases as the precision of the LOLR’s information worsens.
7.3 Ex ante versus ex post efficiency

We have seen that the presence of a noisy LOLR is conducive to a moral hazard problem and that the probability of bank insolvency increases with noise. Thus the presence of a noisy LOLR is ex ante inefficient as it distorts the incentives of both banks and entrepreneurs. Nevertheless, ex post the LOLR can prevent inefficient liquidation and thus the cost of panic runs may be avoided since the LOLR is likely to bail out a solvent bank if it is sufficiently well informed. There therefore exists a trade-off between ex ante inefficiency and ex post efficiency.

Since premature liquidations are costly, banks have to hold reserves to ensure against liquidity problems. However, reserves have an opportunity cost of \(1 + r\) per unit since they can alternatively be invested in the risk free storage technology. However, the presence of a LOLR induces banks to reduce their reserve levels. Thus the reduction in the reserve cost with the presence of a LOLR is a measure of ex post efficiency. The reduction in the reserve cost with the presence of a noisy LOLR is \(\left[\Omega_{^\infty \text{LOLR}} - \Omega\left(\epsilon_L\right)_{\text{LOLR}}\right](1 + r)\). Note that as discussed earlier, the reserve level with the presence of a LOLR is a function of the noise, \(\epsilon_L\), in the LOLR’s information.

As far as the moral hazard costs are concerned, in the model, these are borne by the LOLR itself. This is because in the case of insolvency, the bank is unable to repay its loan, if any, to the LOLR. On the other hand, ex ante the depositors on average still receive their reservation utility. The expected loss to the LOLR, \(\ell(\epsilon_L)\), is thus a measure of ex ante inefficiency, where \(\ell(\epsilon_L) = \int_{\hat{\phi}}^{\epsilon_L} \int_{\tilde{\phi}}^{\phi} (nD(1 + r_1) - \Omega_{LOLR}) f(\phi) f(\tilde{\phi}) d\phi d\tilde{\phi}\).

Let \(\epsilon^*\) be the level of noise at which the expected gains from a LOLR are just equal to the expected costs. Then \(\epsilon^*\) solves the following:

\[
\left[\Omega_{^\infty \text{LOLR}} - \Omega\left(\epsilon_L^*\right)_{\text{LOLR}}\right](1 + r) - \ell(\epsilon_L) = 0. \tag{18}
\]

We show in the Appendix that the expected loss is increasing in \(\epsilon_L\). The intuition is that as the noise level increases, the probability of a Type II error increases and hence the LOLR is more likely to make a bad loan. On the other hand, the expected gain is decreasing in \(\epsilon_L\). The reason why the expected gains are decreasing in noise is that the optimal reserve level in the presence of a LOLR increases as noise increases. This is because, as discussed earlier, an increase in the noise level increases the probability of a Type I error and hence banks realise that they might not be bailed out even if they are solvent and subsequently they hold higher reserves to insure against a situation where they are not bailed out. Thus reserve costs increase as the noise level increases.

Since the expected gains are decreasing in \(\epsilon_L\), while the expected loss increases in \(\epsilon_L\), there therefore exists a critical \(\epsilon^*\) that solves equation (18). Hence if \(\epsilon_L < \epsilon^*\), then the expected gains from ex post efficiency outweigh the costs of ex ante inefficiency and vice versa. We thus have the following proposition.

---

19 This is why it is often argued that the IMF loans to insolvent member countries are eventually borne by the taxpayers of industrial countries, which are the IMF’s main shareholders.
 Proposition 5 An imperfectly informed LOLR will be productive if and only if $e_L < e^*$, where $e^*$ solves equation (18).

It can be argued that one policy implication that emerges from the analysis is that the LOLR should choose a bail-out strategy, $s^*_l$, such that given the distribution of the noise term, there is a zero probability of a Type II error. This is because we know that the moral hazard problem is directly proportional to the probability of a Type II error. Nevertheless such a statement is misleading and is subject to two qualifications. First, since the probabilities of a Type I and Type II error are inversely related, decreasing the probability of a Type II error to zero will significantly increase the probability of a Type I error. The costs associated with a Type I error should not be understated. By not bailing out a solvent bank, the LOLR will not only suffer reputational and credibility costs but more importantly there are likely to be contagious costs which will spill over to the entrepreneurial sector. Thus the cost of a Type I error cannot be overemphasised.\textsuperscript{20} The LOLR therefore needs to be very careful before deciding not to bail out a bank given that the immediate costs in case of a Type I error can be substantial.

Second, we have assumed that the noise term in the LOLR’s signal has a bounded uniform distribution. This assumption was made for tractability. However, if the noise term followed an unbounded distribution, say normality, then there will always exist a positive probability of both a Type I and Type II error. In reality there is always likely to be a chance that the LOLR might make mistakes in both directions. Thus the sensible policy implication which follows is that the LOLR should improve the quality of bank monitoring and supervision so as to minimise the probabilities of errors.

8 Discussion

As mentioned before, the model that we have developed can be applied both to a domestic and international setting. In a domestic setting, the central bank will carry out LOLR operations. In an international setting, an institution like the IMF would play the role of international lender of last resort.

Our model shows the interlinkages between different sectors of the economy and how shocks are transmitted within sectors via the banking system. We showed that there exists a one to one mapping from the depositors’ equilibrium strategy to an optimal contract prevailing in the economy. In the presence of imperfect information, depositors in our model run only if their signals about the underlying fundamentals are below a certain threshold. Thus banking crises are not unrelated to the real economy as long as depositors do not have perfect information. The sunspot theories are thus a special case which hold validity only if the depositors have perfect information.

\textsuperscript{20}Goodhart and Huang (2003) show that the central bank will have an incentive to carry out LOLR operations, even in the presence of moral hazard, given the sizeable cost of contagion.
In the case of perfect information, we showed that it is indeed true that there may exist multiple equilibria, where either everyone runs or no one runs. However, we showed that if depositors observe only noisy signals of the fundamentals, then in general there will exist a unique equilibrium. Thus perfect information can actually be destabilising relative to a small amount of noise. This result was also obtained by Morris and Shin (2001) and Rochet and Vives (2002) who argued that a small amount of noise in the fundamentals can actually be stabilising. However, we showed in our model that this result only holds in the absence of the LOLR. We thus argue that in the presence of another institution, like the LOLR, which is perfectly informed, the good equilibrium will prevail. Hence, under such a scenario the presence of the LOLR is clearly a Pareto improvement.

However, if the information set of the LOLR is noisy, then our earlier conclusion that the LOLR is a Pareto improvement falls into ambiguity. On the one hand, the LOLR is productive ex post as it can avoid inefficient liquidation. On the other hand, the presence of an imperfectly informed LOLR is conducive to the moral hazard problem and is thus ex ante inefficient. Because of this moral hazard problem, we showed that the probability of insolvency of the banking system actually increases in the presence of the LOLR as noise increases.

Furthermore, the moral hazard problem aggravates the adverse selection problem faced by the banks. This is because competitive banks internalise the implicit subsidy provided by the LOLR and thus reduce the lending rates. This in turn affects the incentives of the entrepreneurs and we showed that there exists a possibility that the level of investment might increase above the second best optimal level. In other words, the presence of a LOLR might lead to overinvestment in the economy.

In our model, we had assumed throughout that the strong law of large number applies and hence there was no uncertainty with regards to idiosyncratic entrepreneurial risk. Hence the only source of uncertainty was the macroeconomic shock which hit the economy. However, had we not assumed that the SLLN holds, then our results would have been even stronger. This is because the banks would then face an additional risk regarding the realisation of the idiosyncratic entrepreneurial risk. In the presence of an imperfectly informed LOLR, a reduction in the lending rate will increase the entrepreneurial risk faced by the banks because the average success probability of projects will decline. Under such a scenario, if there is overinvestment in the economy, then the economy will be more fragile relative to the case where the LOLR does not exist.

We believe that this is what happened in the Asian crisis (1997). The East Asian economies had a competitive banking system and the government had provided implicit or explicit insurance to the financial intermediaries. The government in turn expected an IMF bail out in case of a debt crisis.\(^{21}\) Subsequently, there was substantial overinvestment in East Asia. A good example is

\(^{21}\)Note that as we showed in our model, there will exist a moral hazard problem, even if a bail out is not guaranteed. All that is required is that the agent expect a bail out from the principal with a positive probability. Of course, the higher the probability of bailing out an insolvent institution, the more severe would be the moral hazard problem.
Thailand, where the finance companies were largely unregulated and there was massive overinvestment in the real estate and property sector. Thus the Asian economies were already fragile before they were hit by a speculative shock which adversely affected their currencies.

Krugman (1998) argued that the Asian crisis occurred because of the moral hazard problem. However, in the following year, Krugman (1999) asserted that he was wrong, and that the Asian crisis was in fact due to a balance sheet transfer problem. Our model shows that Krugman was actually correct both times. There did exist a moral hazard problem in Asia. Subsequently, when the Asian economies were hit by a shock, the moral hazard problem translated into a balance sheet transfer problem. The adverse impact on the entrepreneurs’ balance sheet was consequently transferred to investors via the banking system.

We do not recommend that a LOLR should not exist. Indeed, we have shown that the LOLR can be ex post efficient. The policy implication that stems from our model is that the banking system be as transparent as possible to the LOLR. In other words, the noise in the LOLR’s information set be at a minimal. We showed that if the noise in the LOLR’s information set is sufficiently low then the gains from ex post efficiency can outweigh the ex ante inefficiency. On the contrary, if the precision of the LOLR’s signal is not high enough, then the net effect of the presence of a LOLR on the economy will be negative. Hence, if the LOLR is to play a productive role, it is imperative that the information set of the LOLR be as precise as possible. This can be achieved by closer monitoring of the banking system. Thus the supervisory process of the banking system should be strict and efficient. It is also vital that the supervisory process be independent and free of any political interference.

This brings into question the increasing trend to delegate supervisory authority to a separate agency as distinct from the central bank. For example, banking supervision in the UK is now under the domain of the Financial Services Authority (FSA). However, the LOLR operations are carried by the central bank. To the extent that there exist conflicts of interest between the supervisory authority and the central bank, the LOLR operations can be adversely affected. This is because the LOLR might not have access to ‘relevant information’. The objective function of the supervisory authority might be different from the central bank and hence the two bodies might have different perspectives on what constitutes ‘relevant information’. As we demonstrated in our model, as the precision of the LOLR’s signal regarding the solvency of the banks worsens, the probability of insolvency of the banking system increases. Hence, the prime objective of the supervisory authorities should be to pass as precise information as possible to the LOLR so that it can carry out its operations effectively. If the LOLR is better able to collect such information directly, then the delegation of banking supervision to another body is not justified. This would therefore largely depend on how well the two bodies can cooperate with each other.

22 For example as argued by Goodhart (2000), the supervisory body might be more concerned about the conduct of business and issues of consumer protection, rather than systemic financial crises per se.
9 Summary and conclusions

We obtain the following results with respect to the role of the LOLR in banking crises: (a) the presence of a perfectly informed LOLR can be a Pareto improvement as costly liquidations can be avoided; (b) perfect transparency between banks and depositors is good if and only if there exists a perfectly informed LOLR; (c) in the presence of an imperfectly informed LOLR, the probability of insolvency of the banking system increases as the noise in the LOLR’s information set increases; (d) the moral hazard problem is directly proportional to the noise in the LOLR’s information set; (e) the presence of an imperfectly informed LOLR can be productive as long as the noise in its information set is sufficiently low; (g) the banking system should be as much transparent as possible to the LOLR. We have therefore provided a framework which places considerable emphasis on the role of information in the evolution of crises.

Appendix

Second best optimal contract

The ‘no runs’ optimal contract is characterised by the following:

\[
(1 + \rho)^*_sb = \left(1 + \frac{r}{\hat{\rho}_{sb}}\right) \left(1 + \frac{\theta D r_1}{N_{sb}}\right) \tag{19}
\]

\[
\Omega^*_sb = \theta D (1 + r_1) \tag{20}
\]

\[
I^*_sb = D \left[1 - \theta (1 + r_1)\right] \tag{21}
\]

\[
p^*_sb = \frac{b}{\hat{\phi} \left[X - (1 + \rho)^*_sb\right]} \tag{22}
\]

**Proof.** Since \(X > (1 + \rho) + b\), we have a problem with four constraints (3), (4), (5), (6) and a concave objective function (2). Starting with \(p^*_sb\), we have already shown that constraint (6) is equivalent to equation (7). This gives us the threshold \(p^*_sb\) in (22). Next, note that the objective is monotonically decreasing in \((1 + \rho)\). Thus constraint (3) must be binding; otherwise given a competitive banking sector, a bank could lower the lending rate and attract all the entrepreneurs. Thus in a competitive market, constraint (3) will be binding and will actually be the zero profit condition of the bank. Then substituting equation (1) for \(R(\phi)\) in (3) and inserting \(I = D - \Omega\) from constraint (4) into (3) and solving for \((1 + \rho)\), we get the following:

\[
(1 + \rho)^*_sb = \left(1 + \frac{r}{\hat{\rho}_{sb}}\right) \left(1 + \frac{\Omega^*_s - \theta D}{N_{sb}}\right) - \frac{\Omega^*_s - \theta D (1 + r_1)}{\hat{\rho}_{sb}}. 
\]
Substituting $p_{sb}^*$ and the above equation in the objective function we can solve for $\Omega^*$ by forming the following Lagrangian:

$$L = (1 - p^*) \left[ \frac{\hat{\lambda}}{\pi} (X - (1 + \rho)) \right] + p^* b + \lambda [\Omega - \theta D (1 + r_1)]$$

where $\lambda$ is the Lagrange multiplier. Taking the partial derivative of the above Lagrangian with respect to $\Omega$ and $\lambda$ and solving for the first order condition for $\Omega^*$ we get equation (20) above. Solving for $\lambda^*$ we find that $\lambda^* = (1 + r) / T$. The multiplier has a very simple interpretation. It is the marginal cost of holding reserves. Hence in this case, the marginal cost of holding reserves per entrepreneur is $1 + r$, as expected. Inserting the value of $\Omega^*$ in $(1 + \rho)^* sb$ above we get equation (19). Finally, given $\Omega^*$, we can solve for $I^*$ from the budget constraint (4). This will give us equation (21).

**Proof of Proposition 2**

Let us first reconsider the case where all agents have perfect information. As we saw in section (5) if all agents are perfectly informed, then in the absence of LOLR, we have multiple equilibria, and the good equilibrium may not necessarily be realised. However, once the LOLR is introduced, there is a unique equilibrium which is the good equilibrium. Thus in the case of perfectly informed agents, the presence of a LOLR (with complete information) is a Pareto improvement.

Next consider the case where the agents have imperfect information but the LOLR is perfectly informed. The investor rationality constraint in this case will be as follows:

$$s^* - \phi \int \left[ (1 + r_1) f (\phi) f (\tilde{e}) d\phi d\tilde{e} + \frac{\phi^R}{n D} f (\phi) f (\tilde{e}) d\phi d\tilde{e} \right] + \int \phi \int \frac{R (1 - 1)}{(1 - n) D} f (\phi) f (\tilde{e}) d\phi d\tilde{e}$$

$$+ \int \phi \int \frac{R I + \Omega I - L}{(1 - n) D} f (\phi) f (\tilde{e}) d\phi d\tilde{e} \geq \tilde{r}.$$ (23)

In the above constraint $s^* = \phi_L$ since as discussed earlier, agents follow a switching strategy around $\phi_L$ as long as the LOLR is perfectly informed. The above constraint says that if $\phi < \phi_L$, then the depositors who run will either get the full interim payment or the proceeds from bankruptcy depending on whether or not the bank is bankrupt; the depositors who wait will share the bank’s returns if any post liquidation. If $\phi \geq \phi_L$, then the depositors who run get $1 + r_1$, while those who wait share among themselves the bank’s final period return minus the LOLR loan $L$, where $L = \max \left[ n D (1 + r_1) - \Omega, 0 \right]$. Note in the absence of insolvency, there will be no costly liquidation and the bank will never be bankrupt. Given a competitive banking sector, the above constraint will hold with equality and thus the depositors on average will recieve $\tilde{r}$.

Comparing constraints (15) and (23) it can be deduced that the lending rate will be lower in the presence of a LOLR. This is the case because of two
reasons. Firstly, a bank realises that in the presence of the LOLR, it will not have to face inefficient liquidation as long as it is solvent. Thus the probability of liquidations is lower since the bank is always bailed out when it is solvent. Hence even though, depositors on average still receive \( \tilde{r} \), but nevertheless the entrepreneurs are better off as competition among banks lowers interest rates.

Secondly, in the presence of the LOLR, fewer depositors are expected to run since they follow a switching strategy around \( \phi_L \), where \( s^*_{L,LOLR} = \phi_L < s^*_{LOLR} \). This is because agents realise that the bank will be bailed out as long as it is solvent.

Furthermore, in the presence of a LOLR, entrepreneur \( i \)'s probability of success is given by \( \pi_i = F(\phi_L) p_i E[\phi (1 - \xi) | \phi < \phi_L] + (1 - F(\phi_L)) p_i \), while in the absence of LOLR it is given by \( \pi_i = p_i E[\phi (1 - \xi)] \). The success probability is higher in the presence of LOLR as liquidations only result in the case of insolvency.

Thus, in the presence of a perfectly informed LOLR, entrepreneurs are always better off; depositors still on average recieve their reservation utility; banks still make zero profits; while the LOLR get its loan back in the event of a bail out. Thus, entrepreneurs are better off, while no one is worse off in the presence of a LOLR. Hence a perfectly informed LOLR is a Pareto improvement. Q.E.D.

**Proof of Proposition 3**

In the presence of a noisy LOLR the probability of a bail out is given by

\[
\int_{s^*_L - \phi}^{s^*_L - \phi} \int_{-\epsilon}^{1} f(\phi) f(\tilde{\epsilon_i}) d\phi d\tilde{\epsilon_i},
\]

while the probability of no bail out is given by

\[
\int_{s^*_L - \phi}^{s^*_L - \phi} \int_{-\epsilon}^{1} f(\phi) f(\tilde{\epsilon_i}) d\phi d\tilde{\epsilon_i}.
\]

If the LOLR does not bail out, then the depositors who recieve a signal below \( s^* \) will either recieve the full interim payment or the proceeds from bankruptcy depending on whether or not the bank is bankrupt. Conversely, depositors who recieve a signal above \( s^* \) will wait and will receive whatever returns the bank generates from its investments at \( t = 2 \). On the contrary, if the LOLR bails out, then agents who run always recieve \((1 + r_1)\), while agents who wait will recieve the bank’s returns from investment (with no liquidation) plus any left-over reserves net of any payments to the LOLR for its possible loan. Overall, the expected return of the depositors should at least be equal to their reservation utility and hence the bank will set \( \Omega \) and \((1 + \rho)\) such that the following investor rationality constraint is satisfied:

\[
\int_{s^*_L - \phi}^{s^*_L - \phi} \int_{-\epsilon}^{1} (1 + r_1) dF(\phi) dH_1(\tilde{\epsilon}) dH_2(\tilde{\epsilon}_i) + \int_{\phi_B}^{\phi_B} \frac{\Omega + R I}{n D} dF(\phi) dH_1(\tilde{\epsilon}) dH_2(\tilde{\epsilon}_i) + \int_{s^*_L - \phi}^{s^*_L - \phi} \int_{-\epsilon}^{1} \frac{R(\phi)(1 - \xi) I + \Omega_2}{(1 - n) D} dF(\phi) dH_1(\tilde{\epsilon}) dH_2(\tilde{\epsilon}_i)
\]
neurial utility is maximised. Thus the lending rate will be as low as possible.
The above constraint will hold with equality as this will ensure that entrepre-

and $(1 + \rho)$, the threshold signal of the investors will be such that the expected payo

The above equation says that in the presence of a LOLR, the threshold strategy of the investors will be such that the expected payo from waiting just equals the marginal cost of holding an additional unit of reserves. Stating this in terms of the FOC, the optimal reserve level will be such that the marginal benefits of holding reserves just equal the marginal costs.

Given the above it is easy to prove part (i) of Proposition 3. The marginal cost of holding an additional unit of reserves is $1 + \rho$, while the expected marginal benefits of holding reserves are lower compared to the case where there is no noisy LOLR. This is because there now exists a positive probability that the bank will be bailed out and hence for any given level of reserves, the probability of bankruptcy will be lower. Thus $\Omega_{LOR}^* < \Omega_{no LOLR}^*$.

The reserve level will decrease further because the presence of the LOLR will also affect the equilibrium strategy of the depositors. In the presence of the LOLR, the threshold signal $s^*$ will be such that it solves the following:

$$
\int_{-\epsilon_L}^{\epsilon_L} \left[ \int_{-\epsilon_L}^{\epsilon_L} (1 + r_1) dF (\phi) dH (\tilde{\xi}) + \int_{-\epsilon_L}^{\epsilon_L} \int_{-\epsilon_L}^{\epsilon_L} (1 + r_1) dF (\phi) dH (\tilde{\xi}) \right] + \epsilon_L \int_{-\epsilon_L}^{\epsilon_L} \int_{-\epsilon_L}^{\epsilon_L} (1 + r_1) dF (\phi) dH (\tilde{\xi})
= \int_{-\epsilon_L}^{\epsilon_L} \int_{-\epsilon_L}^{\epsilon_L} \int_{-\epsilon_L}^{\epsilon_L} \int_{-\epsilon_L}^{\epsilon_L} (1 + r_1) dF (\phi) dH (\tilde{\xi}) + \epsilon_L \int_{-\epsilon_L}^{\epsilon_L} \int_{-\epsilon_L}^{\epsilon_L} \int_{-\epsilon_L}^{\epsilon_L} \int_{-\epsilon_L}^{\epsilon_L} (1 + r_1) dF (\phi) dH (\tilde{\xi})
$$

(25)

The above equation says that in the presence of a LOLR, the threshold strategy of the investors will be such that the expected payoff from running just equals the expected payoff from waiting. A straightforward comparison of equation (25) with equation (13) implies that $s^*_{LOR} < s^*_{no LOLR}$. This is because from the above equation it is clear that the agents’ expected payoff from waiting is higher in the presence of the LOLR and thus the investors have a lower incentive to run. This in turn will lower the threshold strategy of the investors and thus the expected proportion of depositors who will run will decrease. This in turn will further reduce the optimal reserve level of banks.
The lending rate will be lower because of two effects. First, comparing (24) with (15) it is clear that the bank faces lower risk (of liquidation) in the presence of a noisy LOLR for the same distribution of fundamentals as there is a probability that the bank will be bailed out. Second, there exists a probability that the LOLR will make a Type II error and as is clear from (24), the bank’s IR constraint takes into account the implied subsidy, \( S \). A competitive bank will internalise these two effects in its zero profit conditions and subsequently \( (1 + \rho)_{LOLR}^* < (1 + \rho)_{no \, LOLR}^* \). This proves part (ii) of the proposition. Q.E.D.

**Proof of Proposition 4**

We know from the insolvency criteria described in Case 2, section 4, that a bank is insolvent if its patient depositors are worse off from waiting rather than running, even if only the impatient depositors withdraw in the interim period. Thus the bank is insolvent if and only if

\[
\tilde{RI} + \Omega_2 < (1 - \theta) D (1 + r_1)
\]

where \( \tilde{R} \) is as defined by equation (1). Then at the insolvency point the following holds:

\[
\left[ \omega_p (1 + \rho) \tilde{p} \phi_L + (1 - \omega_p) (1 + r) \right] I + [\Omega - \theta D (1 + r_1)] = (1 - \theta) D (1 + r_1).
\]

Inserting the expression for \( I \) and noting that \( \omega_p = N/I \), after some simplification we get

\[
\phi_L = \frac{D (1 + r_1) + \Omega r - (1 + r) (D - N)}{N (1 + \rho) \tilde{p}}.
\]

We have already established that \( (1 + \rho)_{LOLR}^* < (1 + \rho)_{no \, LOLR}^* \) and \( \Omega_{LOLR}^* < \Omega_{no \, LOLR}^* \). The presence of the LOLR has two effects on the insolvency point. First, the reduction in the lending rate increases the insolvency point and thus increases the probability of insolvency. Second, the advantage of having a LOLR is that bank reserves are lower. From the above equation it is clear that the reduction in reserves decreases the insolvency point and hence decreases the probability of insolvency. Thus in the imperfect information setting, the probability of insolvency will be higher in the presence of a LOLR relative to the case where there is no LOLR, if and only if the first effect outweighs the second effect.

Since an increase in the probability of a Type II error decreases the lending rate, the proposition will hold if an increase in the probability of a Type I error increases the reserves and an increase in noise increases the probabilities of Type I and Type II errors. Then for a high enough \( \epsilon_L \), the first effect will outweigh the second effect. Thus to prove the proposition we need to show that

\[
\frac{\partial \Omega}{\partial \Pr(s_1 > s^* \mid \phi < \phi_L)} > 0 \quad \text{and} \quad \frac{\partial \Pr(s_1 > s^* \mid \phi < \phi_L)}{\partial L} > 0, \quad \frac{\partial \Pr(s_1 > s^* \mid \phi > \phi_L)}{\partial L} > 0.
\]

If this holds then for a high enough \( \epsilon_L \) it will be the case that \( \phi_{LOLR} > \phi_{no \, LOLR} \).
To show that \( \partial \Omega / \partial \Pr(s_l < s^*_l) \geq 0 \), we first note that the optimal reserve level is increasing in \( E[W | s_l < s^*_l] \), where \( E[W | s_l < s^*_l] \) is the expected withdrawals from the bank if it is not bailed out. This is because an increase in \( E[W | s_l < s^*_l] \) increases the likelihood of inefficient liquidation and thus the marginal benefit of holding higher reserves also increases. \( E[W | s_l < s^*_l] \) is given by:

\[
E[W | s_l < s^*_l] = \Pr(s_l < s^*_l) E(n | s_l < s^*_l) D (1 + r_1) + \Pr(s_l < s^*_l | \phi \geq \phi_L) E(n | \phi \geq \phi_L, s_l < s^*_l) D (1 + r_1)
\]

where the second equality follows from the law of iterated expectations. It follows from (28) that \( \partial E[W | s_l < s^*_l] / \partial \Pr(s_l < s^*_l | \phi \geq \phi_L) > 0 \), which in turn implies \( \partial \Omega / \partial \Pr(s_l < s^*_l | \phi \geq \phi_L) > 0 \).

To show that \( \partial \Pr(s_l \geq s^*_l | \phi < \phi_L) / \partial \phi_L > 0 \), we first need to calculate an analytical expression for \( P_{II} = \Pr(s_l \geq s^*_l | \phi < \phi_L) \). We do this as follows:

\[
P_{II} = \Pr(s_l \geq s^*_l | \phi < \phi_L)
= \Pr(\tilde{\phi} + \tilde{\epsilon}_l \geq s^*_l | \phi < \phi_L)
= \frac{\Pr(\tilde{\epsilon}_l \geq s^*_l - \tilde{\phi}, \phi < \phi_L)}{\Pr(\phi < \phi_L)}.
\]

Given that \( \phi \sim U[\phi, 1] \) and \( \epsilon_l \sim U[-\epsilon_L, \epsilon_L] \), we have

\[
P_{II} = \frac{\int_{\tilde{\phi}}^{\phi_L} \int_{\tilde{\phi}}^{s^*_l - \tilde{\phi}} \frac{1}{2\epsilon_L} \frac{1}{1-\epsilon} d\tilde{\epsilon}_l d\tilde{\phi}}{\int_{\phi}^{\phi_L} \frac{1}{2\epsilon_L} \frac{1}{1-\epsilon} d\phi}
= \frac{\int_{\phi}^{\phi_L} \frac{1}{2\epsilon_L} \frac{1}{1-\epsilon} (\epsilon_L - s^*_l + \tilde{\phi}) d\tilde{\phi}}{\int_{\phi}^{\phi_L} \frac{1}{2\epsilon_L} \frac{1}{1-\epsilon} d\phi}
= \frac{\frac{1}{2\epsilon_L} \frac{1}{1-\epsilon} (\epsilon_L - s^*_l) (\phi_L - \tilde{\phi}) + \frac{1}{2\epsilon_L} \frac{1}{1-\epsilon} (\phi_L - \phi^2)}{\frac{\phi_L - \phi}{1-\epsilon}}.
\]
Simplifying the above, we get the following expression for the probability of a Type II error:

$$P_{II} = \frac{\epsilon_L - s^*_L + \frac{1}{2} (\phi_L + \phi)}{2\epsilon_L}.$$  

(29)

Analogously, it can be shown that the probability of a Type I error is as follows:

$$P_I = \frac{\epsilon_L + s^*_L - \frac{1}{2} (1 + \phi_L)}{2\epsilon_L}. \quad (30)$$

Since $s^*_L$ is such that both $P_I$ and $P_{II}$ are positive, therefore from (29) it follows that

$$P_{II} > 0$$

if and only if

$$s^*_L < \frac{\phi_L + \phi}{2 + \epsilon_L}.$$  

Similarly, from (30) it follows that

$$P_I > 0$$

if and only if

$$s^*_L > \frac{1 + \phi_L}{2 - \epsilon_L}.$$  

Note that for this restriction to make sense it needs to be the case that

$$\left(\phi_L + \phi\right)/2 + \epsilon_L > (1 + \phi_L)/2 - \epsilon_L,$$

which implies $\epsilon_L > \left(1 - \phi\right)/4$. The intuition is that if signals are very precise then it will not be possible to make both Type I and Type II errors given a bounded distribution.

Finally, taking the partial derivative of (29) with respect to $\epsilon_L$ we have

$$\frac{\partial P_{II}}{\partial \epsilon_L} = \frac{2\epsilon_L - 2\left[\epsilon_L - s^*_L + \frac{1}{2} (\phi_L + \phi)\right]}{4\epsilon_L^2}.$$  

Thus $\frac{\partial P_{II}}{\partial \epsilon_L} > 0$ if and only if $2s^*_L - \phi_L - \phi > 0$, or if and only if $s^*_L > \left(\phi_L + \phi\right)/2$.

Since $s^*_L > (1 + \phi_L)/2 - \epsilon_L$ and $\epsilon_L > \left(1 - \phi\right)/4$, it has to be the case that

$s^*_L > (1 + \phi_L)/2 - \left(1 - \phi\right)/4$ which implies that

$s^*_L > \left(1 + 2\phi_L + \phi\right)/4$. 

Thus a sufficient condition for $\frac{\partial P_{II}}{\partial \epsilon_L} > 0$ is that $\left(1 + 2\phi_L + \phi\right)/4 > \left(\phi_L + \phi\right)/2$.

Simplifying the inequality, we find that the sufficient condition holds as long as

$$\left(1 - \phi\right)/4 > 0,$$

which is the case. Using the same line of reasoning it follows that

$$\frac{\partial P_I}{\partial \epsilon_L} > 0.$$  

Q.E.D.

**Proof of Proposition 5**

Since $\Omega_{LOLR}$ is increasing in noise, Proposition 5 will hold as long as $\ell$ is increasing in noise. Thus we need to show that the LOLR’s expected loss is monotonically increasing in noise. Note that
\[ \ell(\varepsilon_L) = \int_{\phi_1^*}^{\phi_L} \int_{\phi_1}^{\phi_L} (nD(1 + r_1) - \Omega) f(\phi) f(\varepsilon) d\phi d\varepsilon_L \] (31)

\[ = P_{II} [D(1 + r_1) E(n|\phi < \phi_L) - \Omega]. \]

where \( [D(1 + r_1) E(n|\phi < \phi_L) - \Omega] > 0 \) since \( \ell(\varepsilon_L) > 0 \). Taking the partial derivative with respect to \( \varepsilon_L \), we have

\[ \frac{\partial \ell}{\partial \varepsilon_L} = [D(1 + r_1) E(n|\phi < \phi_L) - \Omega] \frac{\partial P_{II}}{\partial \varepsilon_L} + P_{II} \left[ D(1 + r_1) \frac{\partial E(n|\phi < \phi_L)}{\partial \varepsilon_L} - \frac{\partial \Omega}{\partial \varepsilon_L} \right]. \]

Note that \( \frac{\partial E(n|\phi < \phi_L)}{\partial \varepsilon_L} > 0 \), since as shown earlier an increase in noise increases the probability of insolvency, \( F(\phi_L) \), which in turn increases \( E(n|\phi < \phi_L) \).

Further, given that in case of insolvency \( E(n|\phi < \phi_L) D(1 + r_1) > \Omega \), and since \( \frac{\partial P_{II}}{\partial \varepsilon_L} > 0 \) and \( \frac{\partial \Omega}{\partial \varepsilon_L} > 0 \), a sufficient condition for \( \frac{\partial \ell}{\partial \varepsilon_L} > 0 \) is \( [D(1 + r_1) \frac{\partial E(n|\phi < \phi_L)}{\partial \varepsilon_L} - \frac{\partial \Omega}{\partial \varepsilon_L}] > 0. \)

We show that the above sufficient condition will be satisfied. We prove this by contradiction. Suppose that \( \frac{\partial \Omega}{\partial \varepsilon_L} > D(1 + r_1) \frac{\partial E(n|\phi < \phi_L)}{\partial \varepsilon_L} \), i.e. reserves increase at a rate faster than \( D(1 + r_1) E(n|\phi < \phi_L) \). Then for a sufficiently high \( \varepsilon_L \), it will be the case that \( \Omega > D(1 + r_1) E(n|\phi < \phi_L) \). However, this is a contradiction since whenever \( \phi < \phi_L \), there is inefficient liquidation and thus \( \Omega < D(1 + r_1) E(n|\phi < \phi_L) \). Thus, it has to be the case that \( \frac{\partial \Omega}{\partial \varepsilon_L} < D(1 + r_1) \frac{\partial E(n|\phi < \phi_L)}{\partial \varepsilon_L} \). Q.E.D.

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