Investigating the Impacts of Introducing Emission Trading Scheme to Maritime Markets

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Background:
Many emission control mechanisms are being discussed for the maritime industry. One prominent method is to use the Emission Trading Schemes (ETS) which has been adopted in the Kyoto Protocol as well the aviation sector in Europe in the Maritime sector (METS). If EU extends the ETS to the maritime sector, shipping firms will be given certain amounts of free quota of emission. A firm needs to purchase additional quota / credit if her emission level exceeds such a quota. On the other hand, a firm can sell extra credit if her emission level is reduced below the pre-allocated level.

There are however many issues yet to be agreed with respect to ETS. For example, one un-answered question is whether the maritime sector should have its own trading scheme (Closed METS) or they should trade with other all sectors via a common ETS scheme (Open METS). In addition, within the maritime industry there are different sectors, predominantly the container liner group vs. the bulk shipping group. Since goods carried and cost structures are different across these carrier groups, any proposed ETS mechanism is likely to have differential impacts on these groups. However, few studies have invested such issues analytically.

Issues: (1) What is the impact of ETS on individual shipping sector?
(2) What is the impact of Open ETS on the emission reduction objective in international shipping.

Tasks: (1) compare the output and optimal shipping fleet number with/without ETS.
(2) different impacts with closed ETS in international shipping, and the open ETS.

Objective:
This study aims to achieve following objectives via economic modeling on the Asia –Europe maritime markets:

(1) To investigate and benchmark the effects of the two different ETS mechanisms, namely (a) a maritime industry only ETS scheme vs. (b) a common ETS scheme open to other sectors.

(2) To quantify the differential impacts of ETS on various maritime sectors including the container shipping and bulking shipping groups.
The Economic Model:

We consider the case when there are $N_1$ carriers providing homogenous container shipping services and $N_2$ carriers providing homogenous bulk shipping services. Before ETS is introduced, the annual demands for container shipping and bulking shipping are independent (not substitutable), which can be modeled with the following demand functions

\begin{equation}
P_1 = a_1 - b_1 \sum_{i=1}^{N_1} q_{1,i} \quad i = 1, \ldots N_1 \text{ and } N_1 \geq 1
\end{equation}

\begin{equation}
P_2 = a_2 - b_2 \sum_{i=1}^{N_2} q_{2,i} \quad i = 1, \ldots N_2 \text{ and } N_2 \geq 1
\end{equation}

where $q_{r,i}$ is carrier $i$’s outputs / traffic volumes ($r = 1$ for container carrier; $r = 2$ for bulk carrier), while $P_r$ is the market shipping price. $t_{r,i}$ is the transport/shipping time for carrier $i$. If the average distance travelled by a ship is $D_r$; the average cruising speed of a ship is $S_{r,i}$ for carrier $i$, then we have $t_{r,i} = D_r / S_{r,i}$.

Before ETS scheme is introduced, a carrier’s cost for one ship per year is the sum of fuel cost $f_{r,i}$ and capital cost of the ship $y_{r,i}$. Following Psaraftis (2008, 2009), fuel cost is a cubic function of ship cruising speed as specified in equation (2), where $\lambda_r$ is a coefficient representing energy efficiency which depends on ship operation. And $\eta$ is fuel price.

\begin{equation}
f_{r,i} = \eta \lambda_r S_{r,i}^3
\end{equation}

Where $r = 1$ for container carrier $i$ and $r = 2$ for bulk carrier. The lower the value of $\lambda_r$, the higher the energy efficiency, because it requires lower cost for given $S_{r,i}$. The annualized fixed cost of a ship includes the capital and financial costs, periodical maintenance cost, and operation cost, which can be assumed fixed $y_r$ for each group.

Therefore, a carrier needs to balance the two countervailing factors in choosing its optimal speeds: with increased speed, fixed capital cost reduces due to less number of ships required. On the other hand, increased speed will lead to higher fuel consumption and thus higher fuel costs.

\footnote{There may be other cost proportional to the number of ships per year, such as labor cost, insurance cost etc. Considering these costs explicitly will not change our model except that in such a case $y$ would be sum of such cost and capital costs of ships.}
For a carrier, before ETS scheme its objective is to maximize profit as (3) by choosing the optimal quantity and cruising speed. This can be represented with the following objective function, where $U_r$ is the average capacity of a ship.

\[
\text{max}_{q_{r,i},s_{r,i}} \pi_{r,i} = p_{r,i} q_{r,i} - (f_{r,i} + \gamma_r) \frac{q_{r,i}}{u_r s_{r,i} \rho / d_r}
\]

Where $r = 1$ for container carrier; $r = 2$ is for bulk carrier. $\rho$ is a ship’s average proportion of working days in a year. $p_{r,i} q_{r,i}$ is the yearly total revenue. $f_{r,i}$ plus $\gamma_r$ is the total operating cost per ship in one year, equaling fuel cost plus the ship capital cost. $\frac{q_{r,i}}{u_r s_{r,i} \rho / d_r}$ is total the number of ships utilized per year\(^3\). The corresponding first order conditions (FOCs) for (3) are derived as (4).

\[
\frac{\partial \pi_{r,i}}{\partial q_{r,i}} = a_r - 2b_r q_{r,i} - b_r \sum_{j \neq i}^N q_{r,j} - b_r q_{r,i} \sum_{j \neq i}^N \frac{\partial q_{r,j}}{\partial q_{r,i}} - \frac{1}{u_r s_{r,i} \rho / d_r} [\eta \lambda r S_{r,i}^3 + \gamma_r] = 0
\]

\[
\frac{\partial \pi_{r,i}}{\partial s_{r,i}} = - \frac{q_{r,i}}{u_r \rho / d_r} \left[ 2 \eta \lambda r S_{r,i} - \frac{\gamma_r}{S_{r,i}^2} \right] = 0
\]

Referring to Brander and Zhang (1990, 1993) and Fu et al (2006), we introduce Conduct parameter $\nu_{r,i,j}$ in FOC (4.1), and we assume $\nu_{r,i,j}$ is a constant value\(^4\).

\[
\nu_{r,i,j} = \sum_{j \neq i}^N \frac{\partial q_{r,j}}{\partial q_{r,i}}, \quad -1 \leq \nu_{r,i,j} \leq N_r - 1
\]

\(^{2}\)This is equivalent to the assumption that carriers are conducting Cournot competition. This assumption is used as capacity of the shipping industry cannot be changed within a short period. If firms compete with capacity then price, such a competition game would be equivalent to Cournot competition.

\(^{3}\)In reality, ship number is incremental, for example, when one ship is fully loaded, marginal additional cargo may require one more ship to be deployed. To make our analysis tractable, here, we assume that load factor of ship is 100% and the number of ship is continuous. This assumption should not alter the discussion and conclusions obtained in following part of our study.

This conduct parameter $v_{t,i,j}$ measures how aggressively one firm competes with another within the same market. When $-1 \leq v_{t,i,j} \leq 0$, the more negative the $v_{t,i,j}$, the more fierce the competition is between two firms. While, when $0 \leq v_{t,i,j} \leq N_t - 1$, the more positive the $v_{t,i,j}$, the more cooperative the two firms are. Specifically, $v_{t,i,j} = 0$ corresponds to Cournot competition; $v_{t,i,j} = -1$ corresponds to Bertrand competition; $v_{t,i,j} = N_t - 1$ corresponds to Perfect collusion to maximize joint profit. (See Appendix 1 for detailed illustration). As well, the second order condition (SOC) for (3) is also checked and proves to satisfy (see Appendix 2).

As we assume non-negative optimal traffic quantity, the FOC (4.2) can be transformed as (4.3)

\[
2\eta \lambda r S_{r,t}^3 - y_r = 0
\]

Imposing symmetry so that $q_{r,1} = q_{r,2} = \cdots = q_{r,n} = q_r$; $S_{r,1} = S_{r,2} = \cdots = S_{r,n} = S_r$; $v_{t,i,j} = v_{t,i,g}$; $v_{t,i,i} \neq j \neq g$; and further $v_{t,1} = v_{t,2} = \cdots v_{t,n} = v_t$, then the equilibrium quantity and cruising speed for a carrier can be solved as:

\[
\begin{align*}
\xi_r &= \frac{3}{2} \sqrt{\frac{y_r}{2\eta \lambda_r}} > 0 \\
\bar{q}_r &= \frac{2a_r U_r \rho - 3D_r \frac{3}{2} \frac{2\eta \lambda_r y_r^2}{2U_r \rho \beta_r [(N_r + 1) + y_r]}}
\end{align*}
\]

(5.1) states the relationship between optimal speed with fixed cost of a ship and energy efficiency of the ship. From this, one can see that ship speed is lower if a ship has lower fixed cost, lower efficiency (higher $\lambda_r$), and higher fuel price.

The fuel consumption volume at equilibrium can be obtained as

\[
\bar{F}_r = \lambda_r S_r^3 \frac{q_r}{U_r \rho S_r / D_r} = \frac{3}{4} \left( \frac{2a_r U_r \rho - 3D_r}{\sqrt{2\eta \lambda_r y_r^2}} \right) \left( \frac{2a_r U_r \rho - 3D_r}{\sqrt{2\eta \lambda_r y_r^2}} \right)^2
\]

The non-negativity of shipping quantity $\bar{q}_r$ and fuel consumption $\bar{F}_r$ implies that

\[
2a_r U_r \rho > 3D_r \sqrt{2\eta \lambda_r y_r^2}
\]
In addition, it is direct to show that (see Appendix 3)

$$\frac{\partial q_r}{\partial \eta} < 0, \frac{\partial q_r}{\partial \lambda_r} < 0, \frac{\partial q_r}{\partial v_r} < 0, \frac{\partial q_r}{\partial \nu_r} > 0, \frac{\partial \delta_x}{\partial \eta} < 0, \frac{\partial \delta_x}{\partial \lambda_r} < 0, \frac{\partial \delta_x}{\partial v_r} > 0; \frac{\partial \delta_y}{\partial \eta} < 0, \frac{\partial \delta_y}{\partial \lambda_r} < 0, \frac{\partial \delta_y}{\partial v_r} > 0; \frac{\partial \delta_z}{\partial \eta} < 0, \frac{\partial \delta_z}{\partial \lambda_r} < 0, \frac{\partial \delta_z}{\partial v_r} < 0;$$

the signs for $\frac{\partial \delta_x}{\partial \lambda_r}, \frac{\partial \delta_x}{\partial v_r}$ are unclear.

The interpretations are straightforward: when fuel price increases or the fuel efficiency is lower, carriers will slow their cruising speed to save fuel cost, thus resulting lower total fuel consumption volume and traffic quantity. When the capital cost increases, carriers increase cruising speed to reduce the number of ships utilized so as to save the ship capital cost. Despite higher speed, total traffic volume decreases as a result of increasing ship capital cost, indicating shipping carriers reduce the number of ships in larger degree. Besides, when market is more collusive, carriers will reduce throughput deployed in the market so as to raise the price, thus achieving higher profit. It is noted that the optimal cruising speed is affected neither by market competition type, average shipping distance nor the average ship size. This is because that cruising speed is not a competitive tool for carrier to interact with each other in the market. Carriers simply set their optimal cruising speed in order to minimize operating cost given throughput, average route distance and ship size (as shown in FOC (4.2)).

**Case I: A common ETS scheme.**

In Case I, maritime carriers need to buy/sell credit with other industries. This implies that the market price of emission credit is (mostly) exogenous. In such a case, ETS is equivalent to a uniform charge, which can be a positive tax/charge (if carriers buy credit) or negative subsidy (if carriers sell credit). Since there is a definite relationship between fuel consumption and gas emission, ETS is equivalent to a tax/subsidy on fuel consumption. Assuming that each carrier is pre-allocated a quota of fuel consumption which is $\theta$ ($0 < \theta < 100\%$) percentage of her existing fuel consumption, a shipping firm’s profit maximization problem is defined as follows for the case of a container carrier $r = 1$ or a bulk carrier $r = 2$, where $\chi > 0$ is the exogenously determined ETS charge per ton of fuel.

$$\max_{q_r,i,S_r,i} \pi_{r,i} = P_r q_r,i - (f_r,i + \gamma_r) \frac{q_r,i}{u_r S_r,i \rho / \bar{d}_r} - \chi [\lambda_r S_r,i \frac{q_r,i}{u_r S_r,i \rho / \bar{d}_r} - \theta \bar{P}_r]$$

Since firms are symmetric, we will restrict to symmetric solutions. In such a case, the solution for container sector and bulk shipping sector will be independent. The market outcome will be a result of the exogenously determined emission credit price $\chi$ and the target of emission reduction percentage ($1 - \theta$).
The corresponding FOCs for maximization problem (8) can be derived as follows

\[
\frac{\partial \pi_{r,i}}{\partial q_{r,i}} = a_r - 2b_r q_{r,i} - b_r \sum_{j \neq i}^{N_r} q_{r,j} - b_r q_{r,i} \sum_{j \neq i}^{N_r} \frac{\partial q_{r,j}}{\partial q_{r,i}} - \frac{1}{u_r S_{r,i} \rho / \nu_r} \left[ (\eta + \chi) \lambda_r S_{r,i}^3 + y_r \right] = 0
\]

(9.1)

\[
\frac{\partial \pi_{r,i}}{\partial S_{r,i}} = -\frac{q_{r,i}}{u_r \rho / \nu_r} \left[ 2(\eta + \chi) \lambda_r S_{r,i}^2 - \frac{v_r}{S_{r,i}^3} \right] = 0
\]

(9.2)

Imposing symmetry so that \( q_{r,1} = q_{r,2} = \cdots = q_{r,n} = q_r \), and \( S_{r,1} = S_{r,2} = \cdots = S_{r,n} = S_r \); \( v_{ri,j} = v_{rij} \neq j \neq g \); and further \( v_{r,1} = v_{r,2} = \cdots v_{r,n} = v_r \) the equilibrium quantity and cruising speed for container shipping group can be solved as

(10.1)

\[
\bar{q}_r = \frac{2a_r \nu_r - 3b_r \nu_r^2 (\eta + \chi) \lambda_r \nu_r^2}{2u_r \rho b_r (N_r + 1) + \nu_r}
\]

(10.2)

and fuel consumption:

(10.3)

\[
\bar{F}_r = \frac{3}{4} \frac{2a_r \nu_r^3 - 3b_r \nu_r^5 (\eta + \chi) \lambda_r \nu_r^3}{(\eta + \chi)^2 u_r^2 \rho^2 b_r (N_r + 1) + \nu_r}
\]

The non-negativity of \( \bar{q}_r \) and \( \bar{F}_r \) implies that

\[
2a_r \nu_r \rho > 3D_r \sqrt{2(\eta + \chi) \lambda_r \nu_r^2}
\]

(11)

**Proposition 1.1** Under the common ETS scheme, for any \( \theta < 1 \), the fuel consumption, traffic quantity and cruising speed for the shipping industry will reduce if there is any positive price for the emission credit. And the degree of reduction simply depends on the exogenous determined emission credit price \( \chi \). Specifically, the larger \( \chi \) is the more the fuel consumption, traffic quantity and cruising speed decrease.

Proof:
Comparing solutions in (10) and (5), under the common ETS, it is observed that the equilibrium solutions in (10) are equivalent to adding the emission credit price $\chi$ to fuel price $\eta$. From (7), we know $\frac{\partial q_r}{\partial \eta} < 0$, $\frac{\partial S_r}{\partial \eta} < 0$ and $\frac{\partial F_r}{\partial \eta} < 0$, thus the proposition 1.1 is proved to be true. ■

Although the target emission reduction percentage $(1 - \theta)$ does not affect the equilibrium fuel consumption volume, traffic quantity and cruising speed (as $\theta$ does not enter the FOCs for optimization problem (8)), $\theta$ determines the trading behavior of the shipping industry with other sectors under the common ETS scheme. We define threshold $\theta'^r_t$ as follows, where $r = 1$ for container sector, $r = 2$ for bulk sector.

\[
\theta'^r_t = \frac{F_r}{F_r} = \frac{3}{2} \sqrt{(\frac{\eta}{\eta+\chi})^2 \frac{(2a_rU_r-p^rD_r)^2}{2(2(a_rU_r-p^rD_r)^2)}} < 1
\]

For an emission reduction requirement $(1 - \theta)$ following proposition exists:

**Proposition 1.2.** When $\theta > \theta'^r_t$, the shipping company sells its emission credit to other industries. When $\theta < \theta'^r_t$, the shipping company buys emission credit from other industries. And when $\chi$ is higher, the shipping companies are more likely to sell their emission credit to other industries.

Proof:

Under the common ETS scheme, carrier's fuel actual fuel consumption is $\lambda_r(S_r^3 - \frac{q_r}{U_rS_r D_r}) = \theta'^r_t F_r$. When $\theta > \theta'^r_t$, we have $\lambda_r S_r^3 - \frac{q_r}{U_rS_r D_r} < \theta'^r_t F_r$, indicating the shipping company uses fewer fuel than its freely assigned quota, thus selling the extra emission credit out. When $\theta < \theta'^r_t$, $\lambda_r S_r^3 - \frac{q_r}{U_rS_r D_r} > \theta'^r_t F_r$, indicating that the assigned free quota is not enough and thus the shipping company has to buy extra emission credits from other industries. ■

It is also evident that the critical threshold $\theta'^r_t$ is a decreasing function of $\chi$. When the emission credit price $\chi$ increases, the threshold $\theta'^r_t$ will decreases correspondingly, implying the shipping company is more likely to sell emission credit to other sectors. It is intuitively understandable as when the profit margin of selling the emission credit is higher, the company may have stronger willingness to sell instead of use its emission credit. But it is interesting to note that this critical threshold $\theta'^r_t$ is irrelevant to market competition condition determined by number of shipping firms or their conduct parameters.
The implementation of the common ETS scheme can affect the profit for the shipping industry when compared with the benchmark case (no ETS scheme). The specific impacts are summarized in following proposition 1.3 for both container shipping and bulk carriers.

Proposition 1.3.

1.3.1 Carrier’s profit under open ETS is jointly determined by parameters $\theta$, $\nu_r$, and $N_r$. When shipping markets are less competitive (either with larger $\nu_r$ or smaller $N_r$), shipping firms’ profits are more likely to be lower under open ETS scheme than the case without such scheme.

1.3.2 Specifically, for Perfect collusion case ($\nu_r = N_r - 1$), shipping firms’ profits will decrease with $\chi$ and be lower than benchmark case For Bertrand competition case ($\nu_r = -1$), shipping firms’ profits will always be higher than the benchmark case. For Cournot competition case ($\nu_r = 0$), when $\theta < \frac{2}{N_r+1}$, the profit change pattern is the same as the Perfect collusion case, while for $\theta \geq \frac{2}{N_r+1}$ the profit change pattern is the same as the Bertrand competition case.

Proof:

We totally differentiate the optimal profit $\tilde{\pi}_{r,i}$ by emission credit $\chi$.

\[
\frac{d\tilde{\pi}_{r,i}}{d\chi} = \left( \frac{\nu_r}{N_r-1} - 1 \right) (N_r - 1) b_r \tilde{q}_r \frac{\partial \tilde{q}_r}{\partial \chi} - [\lambda_r \tilde{S}_r \frac{3}{u_r S_r \rho / b_r} \frac{\tilde{q}_r}{\tilde{q}_r} - \theta \tilde{F}_r]
\]

The first expression on the RHS, $\left( \frac{\nu_r}{N_r-1} - 1 \right) (N_r - 1) b_r \tilde{q}_r \frac{\partial \tilde{q}_r}{\partial \chi} > 0$, can be regarded as “Price Rising” effect, namely because the increase in $\chi$ reduces $\tilde{q}_r, (\chi)$, thus increasing market price. The second term, $- [\lambda_r \tilde{S}_r \frac{3}{u_r S_r \rho / b_r} \frac{\tilde{q}_r}{\tilde{q}_r} - \theta \tilde{F}_r] = \frac{\partial \tilde{\pi}_{r,i}}{\partial \chi}$, can be regarded as “Emission Trading” effect, which is negative when shipping company buys credits and positive when shipping company sells credit. Overall, the sign for $\frac{d\tilde{\pi}_{r,i}}{d\chi}$ depends on the sum of the two terms on the RHS of (13.1).
When $\chi = 0$, $\bar{\pi}_{r,i}$ is just the profit under benchmark case without common ETS scheme, which is 

$$\bar{\pi}_{r,i} = \frac{1}{4U_r^2p^2b_1[(N_r+1)+v_r]^2}.$$ 

When, $\chi = \bar{\chi}_r = \left[\frac{4(a_r U_r p)^3}{27 \lambda_r \gamma_r D_r^2} - \eta\right]$, we have $\bar{q}_r = 0$. Therefore, when $\chi > \bar{\chi}_r$, shipping company will sell all its pre-allocated emission credits through the common ETS scheme, thus when $\chi > \bar{\chi}_r$, $\bar{\pi}_{r,i}$ becomes a straight line with slope $\frac{d\bar{\pi}_{r,i}}{d\chi} = \theta \bar{F}_r$.

Therefore, it is apparent that profits for shipping firms under common ETS are jointly determined by $v_r$, $N_r$ and $\theta$. Table 1 summarizes two possible profit function shapes after implementing common ETS. The formal proof for Table 1 is in Appendix 4.

| Case | $\theta$ | $\frac{d^2\bar{\pi}_{r,i}}{d\chi^2}$ | $\frac{d\bar{\pi}_{r,i}}{d\chi}$ $|_{\chi=0}$ | Graph |
|------|-----------------|---------------------------------|-----------------------------|-------|
| Case 1 | $0 < \theta \leq \bar{\theta}_r$ | $> 0$ | $\leq 0$ | Convex (Figure 1) |
| Case 2 | $\bar{\theta}_r < \theta \leq 1$ | $> 0$ | $> 0$ | Convex (Figure 2) |

The threshold $\bar{\theta}_r = \frac{2(v_r+1)}{(N_r+1)+v_r}$ is jointly determined by parameters $v_r$ and $N_r$, which describe market competition intensity. For Perfect collusion case ($v_r = N_r - 1$), $\bar{\theta}_r = 1$, thus no matter how much free emission quota is allocated, shipping firms' profit will reduce after the implementation of open ETS, (possibly can be higher than benchmark case when $\chi$ is very high). For Bertrand competition case ($v_r = -1$), $\bar{\theta}_r = 0$, thus no matter how much free emission quota is allocated, open ETS scheme raises shipping firms' profits. While for other intermediate cases where $-1 < v_r < N_r - 1$, the shape of profit function under open ETS is determined by the threshold $\bar{\theta}_r = \frac{2(v_r+1)}{(N_r+1)+v_r}$ in Table 1. We have $\frac{d\bar{\theta}_r}{dv_r} = \frac{2N_r}{[(N_r+1)+v_r]^2} > 0$ and $\frac{d\bar{\theta}_r}{dN_r} < 0$. So when the competition is more intense ($v_r$ is small or $N_r$ is large), $\bar{\theta}_r$ is smaller, indicating that Case 2 becomes more likely. When firms are more collusive ($v_r$ is large or $N_r$ is small), $\bar{\theta}_r$ is larger, indicating that Case 1 becomes more likely. Thus, Proposition 1.3 is proved. Table 2 gives numerical example of threshold $\bar{\theta}_r$ by assigning different values for $v_r$ and $N_r$. ■
Table 2. Numerical Example of Threshold $\bar{\theta}_r$ in Different Values of $\nu_r$ and $N_r$\textsuperscript{5}

<table>
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<th>$\nu_r$</th>
<th>$N_r$</th>
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<th>4</th>
<th>6</th>
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The intuition of proposition 1.3 is as follows. Common ETS scheme gives rise to two main effects, namely the “Price Rising” effect and “Emission Trading” effect, as we mentioned earlier in FOC (16.1). When market competition is becoming more intense, the positive “Price Rising” effect due to unit cost increase is more prominent\textsuperscript{6}. Similar conclusions can be found in other economics literature, such as Vives (1985), and Milliou and Petrakis (2011). On the other hand, the “Emission Trading” effect can be negative or positive, decided by whether shipping carriers to buy or to sell credits through the open ETS scheme. Thus, when market competition is not that intense and regulator assigns not enough free emission quota, carriers have to incur too much additional cost to buy emission credits from other industries, and this negative “Emission Trading” effect prevails “Price Rising” effect, making shipping firms’ profits lower than benchmark case (shown in Figure 1). In liner shipping industry, strategic alliances are prominent among carriers, such as the Grand alliance and the New World alliance. Especially during the period with high demand uncertainty and excessive capacity, shipping lines also tend to be more cooperative by signing cooperative agreements to alleviate unnecessary fierce competition (Song and Panayides, 2002; Panayides and Wiedmer, 2011). This rather collusive nature of shipping industry will restrict carriers’ “Pricing Rising” effect, thus ETS scheme may damage carriers’ profitability. This conclusion may explain the wide concern among shipping industry and their conservative attitude toward ETS implementation.

However, by examining FOC (9.1), it is interesting to note when shipping carriers decide their market optimal throughput, they consider entire emission credit charge $\chi$ to be fuel price increase, while the actual unit fuel cost should be lower than $\chi$ considering the pre-allocated free emission quota $\theta$. Thus, the price rising may be exaggerated, making that shipping firms’ profit might be higher than benchmark case, as shown in Figure 2. This increase in profit under open ETS scheme is referred as “Windfall Profit”, which is observed by empirical studies by Sijm et al (2006) and Smale et al (2006) in European electricity plants and other energy-intensive sectors under EU ETS. This

\textsuperscript{5} Here, the market in our study is defined as particular shipping route, for example, Europe-Asia, Far East-USEC, Transatlantic and Transpacific, instead of aggregate global market. Although, there might be many shipping carriers in aggregate level (global market), the number of carriers on route level should be smaller.

\textsuperscript{6} For “Perfect collusion” case, market price is already set optimally, thus leaving quite limited room for further price increase due to higher unit cost. While for “Bertrand competition” case, market price is set to be equal to marginal cost, which is far from optimal level, thus making plenty room for price increase.
“Windfall Profit” arouse by regulator’s freely pre-allocated emission quota is a controversy issue among practitioners and academia. It is argued that the customers, shippers in our shipping industry case, pay for the emission cost, while firms actually benefit from such ETS scheme by receiving free pre-allocated emission quota as implicit subsidy, but passing entire cost increase to end users through unreasonable price increase.

Figure 1. Case 1 in Table 1
Case II: A Maritime ETS scheme.
In the case of maritime only ETS, the main difference is that the price of emission credit is no longer exogenously determined. Instead, it is jointly dependent on the decisions of the container sector and bulk sector, as the amount of credit sells (buys) by bulk carriers must be equal to the amount of credit buys (sells) by container carriers. In addition, each carrier is given $\theta$ ($0 < \theta < 100\%$) percentage of her existing fuel consumption for free. Before analyzing the mutual emission credit trading between container and bulk shipping industries, the scenario of such no trading is necessary to be solved first to provide us with direct reference and insight.

It is evident that without emission trading with bulk shipping group, the container carrier ($r = 1$) or the bulk carrier ($r = 2$) is faced with a profit maximization problem under a fuel consumption constraint. The mathematical formulation is as follows. It can be proved that, at optimal, the choices of $q_{r,i}$ and $S_{r,i}$ make the constraint binding. (See proof in Appendix 5)

\begin{equation}
\max_{q_{r,i}S_{r,i}} \pi_{r,i} = P_r q_{r,i} - (f_{r,i} + \gamma_r) \frac{q_{r,i}}{u_r S_{r,i} \rho / \rho_r} \\
\text{s.t. } \lambda_r S_{r,i}^3 \frac{q_{r,i}}{u_r S_{r,i} \rho / \rho_r} = \theta \tilde{F}_r
\end{equation}

By introducing the Lagrangian multiplier $\phi_{r,i} > 0$, we can specify the corresponding Lagrangian function as follows.

\begin{equation}
L_{\phi_{r,i}} = P_r q_{r,i} - (f_{r,i} + \gamma_r) \frac{q_{r,i}}{u_r S_{r,i} \rho / \rho_r} - \phi_{r,i} \left[ \lambda_r S_{r,i}^3 \frac{q_{r,i}}{u_r S_{r,i} \rho / \rho_r} - \theta \tilde{F}_r \right]
\end{equation}

The corresponding FOCs for the Lagrangian function (15) in $q_{r,i}$, $S_{r,i}$, and $\phi_{r,i}$ can be derived as follows:
Imposing symmetry, and solving (16.1) and (16.2), the optimal traffic quantity and cruising speed can be expressed as the function of the Lagrangian multiplier $\lambda$. Substituting $\lambda$ into (16.3) we have following important equation.

\[
\frac{\partial L_{\phi_{r,i}}}{\partial q_{r,i}} = a_r - 2b_r q_{r,i} - b_r \sum_{j \neq i}^{N_r} q_{r,j} - b_r q_{r,i} \sum_{j \neq i}^{N_r} \frac{\partial q_{r,j}}{\partial q_{r,i}} - \frac{1}{u_r s_{r,i} \rho / D_r} [(\eta + \phi_{r,i}) \lambda + y_r] = 0
\]  

(16.2) \[ \frac{\partial L_{\phi_{r,i}}}{\partial s_{r,i}} = - \frac{q_{r,i}}{u_r \rho / D_r} 2(\eta + \phi_{r,i}) s_{r,i} = 0 \]

(16.3) \[ \frac{\partial L_{\phi_{r,i}}}{\partial \phi_{r,i}} = \lambda s_{r,i} \frac{q_{r,i}}{u_r s_{r,i} \rho / D_r} = \theta \tilde{P}_r = 0 \]

In equation (17), the fuel consumption constraint (LHS) and $\phi_{r,i}$ has one-to-one relationship. This can be seen by treating LHS of (17) as function of $\phi_{r,i}$, thus the RHS of (17) is monotonically decreasing with $\phi_{r,i}$. Therefore we know that the optimal Lagrangian multiplier $\lambda$ corresponding to $\theta \tilde{P}_r$ must exist and be unique due to this one-to-one relation between LHS and $\phi_{r,i}$.

The parameter $\phi_{r,i}$ has the economic meaning as shadow price, indicating that one unit relaxation of the fuel consumption constraint can bring $\phi_{r,i}$ unit increase of companies’ profit (i.e., $\frac{d\phi_{r,i}}{d(\theta \tilde{P}_r)} = \tilde{\phi}_{r,i}$).

Due to symmetry assumption, we have $\tilde{\phi}_{1,1} = \tilde{\phi}_{1,2} = \cdots = \tilde{\phi}_{1,n} = \tilde{\phi}_1$ for container shipping group and $\tilde{\phi}_{2,1} = \tilde{\phi}_{2,2} = \cdots = \tilde{\phi}_{2,m} = \tilde{\phi}_2$ for bulk shipping group.

**Proposition 2.1.** When $\tilde{\phi}_1$ and $\tilde{\phi}_2$ are different, both container and bulk sectors have incentive to trade with each other. The shipping group with higher $\tilde{\phi}_r$ will buy emission credits from the other group. Any trading price $h$ between $\tilde{\phi}_1$ and $\tilde{\phi}_2$ will lead to a Pareto improvement compared with no trading, leading to win-win outcome for both the container and bulk sectors.

Proof:
WLOG, we assume $\hat{\phi}_1 > \hat{\phi}_2$. With a negotiated emission credit trading price $h \in (\hat{\phi}_2, \hat{\phi}_1)$, container shipping company is willing to buy additional emission credit to expand his fuel consumption constraint because $d\hat{F}_1 = (\hat{\phi}_1 - h)d(\theta \hat{F}_1) > 0$. In addition, as shown in (17), the shadow price $\phi_1$ and the fuel consumption constrain (LHS) has one-to-one relationship. Specifically, $\phi_1$ monotonically decreases with the increase of the LHS fuel consumption constraint when container carrier continues to buy emission credit at price $h$. Finally, the container company will stop to buy from the bulk company with the credit purchase quantity of $\Delta_1$, such that

$$\theta \hat{F}_1 + \Delta_1 = \frac{3\sqrt{2}a_1y_1^2D_1(2a_1U_1\rho - 3b_1\sqrt{2(\eta + h)})\lambda_1y_1^2}{4\sqrt{(\eta + h)^2U_1^2p^2b_1[(N_1 + 1) + v_1]}}$$

meaning that the shadow price of fuel consumption for container carrier reduces from $\phi_1$ to $h$.

For the bulk shipping company, it also has incentive to sell emission credit to the container shipping company as $d\hat{F}_2 = (h - \hat{\phi}_2)d(\theta \hat{F}_2) > 0$. As also shown in (17), the bulk company’s shadow price $\phi_2$ monotonically increases with the decrease of the LHS fuel consumption constraint when bulk shipping company continues to sell the fuel consumption credit. Finally, the bulking shipping company will stop to sell to container shipping company with the trading quantity $\Delta_2$, such that

$$\theta \hat{F}_2 - \Delta_2 = \frac{3\sqrt{2}a_2y_2^2D_2(2a_2U_2\rho - 3b_2\sqrt{2(\eta + h)})\lambda_2y_2^2}{4\sqrt{(\eta + h)^2U_2^2p^2b_2[(N_2 + 1) + v_2]}}$$

meaning that the shadow price of fuel consumption for bulk shipping company increases from $\phi_2$ to $h$.

The final trading quantity between container and bulk shipping groups is $\min (N_1 \Delta_1, N_2 \Delta_2)$. But under such a mechanism, the trading of emission credits can still keep on by choosing another price lower or higher than $h$, as one side of the transaction parties does not have its shadow price equal to $h$. Therefore, we consider a special choice of trading price $\bar{h}$, such that when the container and bulk shipping industries finishes the trading at price $\bar{h}$, the market is cleared leaving no incentive for either side to trade (both sides reach the condition that their shadow price equals to $\bar{h}$). $\bar{h}$ and the equilibrium credit trading quantity $\Delta$ can be solved from the following system of equations.

\begin{align}
\theta \bar{F}_1 + \Delta &= \frac{3\sqrt{2}a_1y_1^2D_1(2a_1U_1\rho - 3b_1\sqrt{2(\eta + h)})\lambda_1y_1^2}{4\sqrt{(\eta + h)^2U_1^2p^2b_1[(N_1 + 1) + v_1]}} \tag{18.1} \\
\theta \bar{F}_2 - \frac{N_1 \Delta}{N_2} &= \frac{3\sqrt{2}a_2y_2^2D_2(2a_2U_2\rho - 3b_2\sqrt{2(\eta + h)})\lambda_2y_2^2}{4\sqrt{(\eta + h)^2U_2^2p^2b_2[(N_2 + 1) + v_2]}} \tag{18.2}
\end{align}

Although it is difficult to obtain closed form solution for $\bar{h}$ from (18), we know that $\phi_1$ is decreasing function of $\Delta_1$, while $\phi_2$ is increasing function of $\Delta_1$, thus the above system of equations has positive solution $\bar{h}$, as shown in figure 3.
When container and bulk groups trade at price $\tilde{h}$, the traffic quantity, cruising speed and fuel consumption at the equilibrium for container group ($r = 1$) and bulk group ($r = 2$) are demonstrated in (19).

$$
\hat{q}_r = \frac{2a_rU_r\rho - 3D_r^2(2(\eta + h)\lambda_r\gamma_r)^2}{2U_r\rho b_r[(N_r + 1) + \nu_r]}
$$

$$
\hat{\phi}_r = \frac{3\sqrt{2(\eta + h)\lambda_r\gamma_r}}{2(\eta + h)\lambda_r} > 0
$$

$$
\hat{\phi}_r = \frac{3\sqrt{2(\eta + h)\lambda_r\gamma_r}2a_rU_r\rho - 3D_r^2[2(\eta + h)\lambda_r\gamma_r]}{4\sqrt{2}(\eta + h)\lambda_r\gamma_r/2U_r^2\rho^2 b_r[(N_r + 1) + \nu_r]}
$$

**Proposition 2.2.** The trading price $\tilde{h}$ under Maritime only ETS increases when the freely allocated emission quota $\theta$ to shipping industry decreases; For the shipping group that buys (sells) credit in the Maritime only ETS, its shadow price $\hat{\phi}_r$ and the resultant trading price $\tilde{h}$ increases with decreasing (increasing) market conduct parameter $\nu_r$.

Proof:

We still assume that container sector buys emission credit from bulk sector ($\hat{\phi}_1 > \hat{\phi}_2$). The $\phi_1$ and $\Delta_1$ has the relation as $\theta F_1 + \Delta_1 = \frac{3\sqrt{2(\eta + h)\lambda_1\gamma_1}2a_1U_1\rho - 3D_1^2[2(\eta + \phi_1)\lambda_1\gamma_1]}{4\sqrt{2}(\eta + \phi_1)\lambda_1\gamma_1/2U_1^2\rho^2 b_1[(N_1 + 1) + \nu_1]}$ as shown in (22.1); $\phi_2$ and $\Delta_1$ has the relation as $\theta F_2 = \frac{N_1\Delta_1}{N_2} = \frac{3\sqrt{2(\eta + h)\lambda_2\gamma_2}2a_2U_2\rho - 3D_2^2[2(\eta + \phi_2)\lambda_2\gamma_2]}{4\sqrt{2}(\eta + \phi_2)\lambda_2\gamma_2/2U_2^2\rho^2 b_2[(N_2 + 1) + \nu_2]}$ as shown in (22.2). Thus it is
easy to observe that $\phi_1$ and $\phi_2$ increase when $\theta$ decreases, for all $\Delta_1 > 0$. From figure 4, when $\theta_1 < \theta_2$, we have $\phi_1(\theta_1) > \phi_1(\theta_2)$ and $\phi_2(\theta_1) > \phi_2(\theta_2), \forall \Delta_1 > 0$, thus we have $\tilde{h}_1 > \tilde{h}_2$.

By using (5.3), (18.1) can be rearranged into

$$\frac{3\sqrt{2} \lambda_1 y_1^2 \rho_1}{4 \gamma_1^2 \rho^2 b_1 ([\lambda_1 + 1] + \nu_1)} \left[ \frac{2a_1 u_1 \rho - 3 b_1 \sqrt{2(\eta + \phi_1) \lambda_1 y_1^2}}{\sqrt{(\eta + \phi_1)^2}} \right] - 2 a U_1 \rho - 3 D 132 \gamma_1 \gamma_2 = \Delta 1.$$ 

Thus, $\forall \Delta 1 > 0$, it is apparent that $\phi 1$ increases when $\nu 1$ decreases (see figure (5)), thus the resultant $\tilde{h}$ is higher. Similarly, by rearranging (18.2), it can be proved that $\phi_2$ and resultant $\tilde{h}$ increases when $\nu_2$ increases. ■

![Figure 4. Change of $\tilde{h}$ with $\theta$ ($\theta_1 < \theta_2$)](image)

![Another figure showing the relationship between $\phi$ and $\Delta_1$](image)
The intuition for proposition 2.2 is straightforward. First, when regulator assigns fewer emission quotas to shipping industry, this emission quota then becomes more valuable due to its scarcity, thus pushing up trading price under Maritime only ETS scheme. Second, for shipping group that buys emission quota, when its market conduct parameter is smaller, firms compete more fiercely with each other, thus making them more aggressive to acquire emission quota. Then, buyers’ valuation of emission quota increases and so does the trading price. On the contrary, for the shipping group that sells emission quota, when its conduct parameter is larger, the shipping firms are more collusive and then have larger bargaining power to sell their emission credit to the other group, thus also raising the emission trading price.

If we assume that the container and bulk shipping carriers trade their emission credit via price $\bar{h}$, we can compare the optimal traffic quantity, cruising speed, fuel consumption and most importantly the profit under three cases, namely the benchmark case without ETS scheme, the case of Common ETS scheme and the Maritime only ETS scheme.

**Proposition 2.3.** For optimal traffic quantity, cruising speed, fuel consumption for container group ($r = 1$) and bulk group ($r = 2$), we have,

(a) If $\bar{h} \geq \chi$, $\bar{q}_r(\chi) < \bar{q}_r(\bar{h}) < \bar{q}_r$; $\bar{s}_r(\chi) < \bar{s}_r(\bar{h}) < \bar{s}_r$; $\bar{f}_r(\chi) < \bar{f}_r(\bar{h}) < \bar{f}_r$;

(b) If $\bar{h} < \chi$, $\bar{q}_r(\chi) < \bar{q}_r(\bar{h}) < \bar{q}_r$; $\bar{s}_r(\chi) < \bar{s}_r(\bar{h}) < \bar{s}_r$; $\bar{f}_r(\chi) < \bar{f}_r(\bar{h}) < \bar{f}_r$.

**Proof:**

Proposition 2.3 is the result from the direct comparisons among (5), (10) and (19). ■

**Proposition 2.4.**

(a) When $\theta \geq \hat{\theta} = \frac{2(v_r + 1)}{(N_r + 1) + v_r}$, we have the following relationship among $\bar{\pi}_r$, $\bar{\pi}_r(\chi)$ and $\bar{\pi}_r(\bar{h})$.
(i) When $\bar{h} \geq \chi$, $\bar{\pi}_r < \bar{\pi}_r(\chi) \leq \bar{\pi}_r(\bar{h})$;

(ii) When $\bar{h} < \chi$, $\bar{\pi}_r < \bar{\pi}_r(\bar{h}) < \bar{\pi}_r(\chi)$.

(b) When $\theta < \hat{\theta} = \frac{2(\nu_r+1)}{(N_r+1)+\nu_r}$, and carriers sells credits under both schemes, the relationships between $\bar{\pi}_r(\chi)$ and $\bar{\pi}_r(\bar{h})$.

(i) When $\bar{h} \geq \chi$, $\bar{\pi}_r(\chi) \leq \bar{\pi}_r(\bar{h})$;

(ii) When $\bar{h} < \chi$, $\bar{\pi}_r(\bar{h}) < \bar{\pi}_r(\chi)$.

But the relationship between $\bar{\pi}_r(\chi)$ and $\bar{\pi}_r$, $\bar{\pi}_r(\bar{h})$ and $\bar{\pi}_r$ are unclear.

Proof:

The Maritime only ETS scheme by choosing market clearing trading price $\bar{h}$ is equivalent to the common ETS scheme when the trading price $\bar{h}$ is regarded as exogenously determined by a third party. This is true by examining the solutions in (10) and (19). Thus the Proposition 1.3 can be applied to two different trading price, $\chi$ under Common ETS scheme, and $\bar{h}$ under Maritime only scheme. ■

Proposition 2.3 and 2.4 state the results based on the endogenously determined price $\bar{h}$. As shown in Proposition 2.2, $\bar{h}$ is determined by parameters $\theta$ and $\nu_r$, therefore, we have following immediate corollary for Propositions 2.3 and 2.4 by using the conclusion in Proposition 2.2.

**Corollary for Proposition 2.3 and 2.4**

When regulator aims to reduce emission more aggressively ($\theta$ is smaller) or when the shipping group that sells emission credits are more collusive ($\nu_r$ is larger), the equilibrium traffic volume $\hat{\sigma}_r(\bar{h})$, cruising speed $\hat{S}_r(\bar{h})$ and fuel consumption $\hat{F}_r(\bar{h})$ will decrease under the Maritime only ETS, and Proposition 2.3(a) and Proposition 2.4 b (i) are more likely to hold;
Appendix 1. Shipping carrier’s market conduct with different conduct parameter values.

The FOC for the profit maximization as equation (3.1) reveals that

\[
\frac{\partial \pi_{r,i}}{\partial q_{r,i}} = P_r + p_r'(Q_r)q_{r,i}(1 + \nu_{r,j}) - c_{r,i} = 0
\]

\[c_{r,i} = \frac{1}{\mu_r s_{r,i} p / P_r} \left[ \eta \lambda_r s_{r,i}^3 + y_r \right],\]

which is the marginal cost for shipping company, and \(Q_r = \sum_{j=1}^{N_r} q_{r,j}\), \(j = 1, \ldots, N_r\).

It is clear that when \(\nu_{r,j} = 0\), it is just the FOC for Cournot Competition. When \(\nu_{r,j} = -1\), we have \(P_r = c_{r,i}\), meaning that the Shipping carriers compete in Bertrand type. When \(\nu_{r,j} = N_r - 1\), the FOC becomes \(P_r + p_r'Q_r - c_{r,i} = 0\), which is just the FOC for joint profit maximization (Perfect collusion) case. Since the competition type among the carriers must be between Bertrand and Perfect collusion, the value of \(\nu_{r,j}\) should be between \(-1\) and \(n - 1\).

Appendix 2. Second-order derivative condition for maximization problem (3) and (8)

SOC for the optimization problem in equation (3)

\[
\begin{vmatrix}
\frac{\partial^2 \pi_{r,i}}{\partial q_{r,i}^2} & \frac{\partial^2 \pi_{r,i}}{\partial q_{r,i} \partial s_{r,i}} \\
\frac{\partial^2 \pi_{r,i}}{\partial s_{r,i} \partial q_{r,i}} & \frac{\partial^2 \pi_{r,i}}{\partial s_{r,i}^2}
\end{vmatrix} = \begin{vmatrix}
-b_r(2 + v_r) & -\frac{D_r}{U_r \rho} \left( 2\eta \lambda_r s_{r,i}^3 - y_r s_{r,i}^2 \right) \\
-\frac{D_r}{U_r \rho} \left( 2\eta \lambda_r s_{r,i}^3 - y_r s_{r,i}^2 \right) & -\frac{2q_{r,i} D_r}{U_r \rho} \left( \eta \lambda_r + \frac{y_r}{s_{r,i}^3} \right)
\end{vmatrix}
\]

Off diagonal is equal to zero by the first order condition (6.1). Therefore, for the SOC to hold, the product of the diagonal elements should be positive. \(-\frac{2q_{r,i} D_r}{U_r \rho} \left( \eta \lambda_r + \frac{y_r}{s_{r,i}^3} \right) < 0\), and \(-b_r(2 + v_r)\) is also negative because \(v_r \geq -1\). So the SOC is satisfied.

SOC for maximization problem (8)
Similar to the proof of SOC for \((4)\), it can be proved that the SOC for \((10)\) is also satisfied. ■

Appendix 3. Proof of results in \((7)\)

As \(\bar{q}_r = \frac{2a_rU_r\rho - 3D_r\sqrt{2\eta\lambda_r\gamma_r^2}}{2U_r\rho b_r[(N_r+1)+v_r]}\), it is evident that \(\frac{\partial \bar{q}_r}{\partial \eta} < 0, \frac{\partial \bar{q}_r}{\partial \lambda_r} < 0, \frac{\partial \bar{q}_r}{\partial d_r} < 0, \frac{\partial \bar{q}_r}{\partial v_r} < 0\).

\[
\frac{\partial^2 \lambda_r}{\partial q_r^2} = \begin{vmatrix} -b_r(2 + v_r) & -\frac{D_r}{U_r\rho} \left[2(\eta + \chi)\lambda_r S_{r,i} - \frac{\gamma_r}{S_{r,i}^2}\right] \\ -\frac{D_r}{U_r\rho} \left[2(\eta + \chi)\lambda_r S_{r,i} - \frac{\gamma_r}{S_{r,i}^2}\right] & -\frac{2q_rD_r}{U_r\rho} \left[(\eta + \chi)\lambda_r + \frac{\gamma_r}{S_{r,i}^2}\right] \end{vmatrix}
\]

As \(\bar{S}_r = \frac{3\sqrt{\gamma_r}}{2\eta\lambda_r} > 0\), it is evident that \(\frac{\partial \bar{S}_r}{\partial \eta} < 0, \frac{\partial \bar{S}_r}{\partial \lambda_r} < 0, \frac{\partial \bar{S}_r}{\partial d_r} > 0\).

\(\bar{F}_r = \frac{3\sqrt{2\eta\gamma_r^2D_r(2a_rU_r\rho - 3D_r\sqrt{2\eta\lambda_r\gamma_r^2})}}{4\sqrt{\eta^2U_r^2\rho^2b_r[(N_r+1)+v_r]}}, \) thus, it is clear that \(\frac{\partial \bar{F}_r}{\partial v_r} < 0\)

As we have \(2a_rU_r\rho > 3D_r\sqrt{2\eta\lambda_r\gamma_r^2}\) from \((6)\), thus

\[
\frac{\partial \bar{F}_r}{\partial \eta} = -\frac{3\sqrt{2\eta\gamma_r^2D_r(4a_rU_r\rho - 3D_r\sqrt{2\eta\lambda_r\gamma_r^2})}}{12\eta^2U_r^2\rho^2b_r[(N_r+1)+v_r]} < 0
\]

\[
\frac{\partial \bar{F}_r}{\partial \lambda_r} = \frac{3\sqrt{2\eta\gamma_r^2D_r(2a_rU_r\rho - 3D_r\sqrt{2\eta\lambda_r\gamma_r^2})}}{6\sqrt{\eta\lambda_r\gamma_r^2U_r^2\rho^2b_r[(N_r+1)+v_r]}}, \text{ or } < 0
\]

\[
\frac{\partial \bar{F}_r}{\partial U_r} = \frac{3\sqrt{2\eta\gamma_r^2D_r(2a_rU_r\rho - 3D_r\sqrt{2\eta\lambda_r\gamma_r^2})}}{2\sqrt{\eta^2U_r^2\rho^2b_r[(N_r+1)+v_r]}}, \text{ or } < 0
\]

Appendix 4. Proof for Table 1.

First, we have \(\frac{dF_{r,i}}{dx} \bigg|_{x=0} = \left[\frac{\nu_r}{N_r-1} - 1\right] \left(N_r - 1\right) b_r \frac{\partial \hat{q}_r}{\partial x} \bigg|_{x=0} \hat{q}_r \right) - (1 - \theta) F_r\).

Thus, when \(\theta = 0\),
\[ \frac{d\bar{\pi}_{r,i}}{d\theta} \bigg|_{\theta=0} = \left( \frac{v_r}{N_r-1} - 1 \right) \left( N_r - 1 \right) b_r q_r \frac{\partial q_r}{\partial \theta} \bigg|_{\theta=0} = -\frac{(\eta+1)\sqrt{2\lambda_t^2 r^2 D_r} + 2a_t U_t \rho - 3D_r \sqrt{2\eta\lambda_t^2 r^2}}{2\sqrt{\eta^2 u_t^2 \rho^2 b_r ([N_r+1] + v_r)^2}} \leq 0; \]

When \( \theta = 1 \)

\[ \frac{d\bar{\pi}_{r,i}}{d\theta} \bigg|_{\theta=0} = \left( \frac{v_r}{N_r-1} - 1 \right) \left( N_r - 1 \right) b_r q_r \frac{\partial q_r}{\partial \theta} \bigg|_{\theta=0} = \frac{(1 - \frac{v_r}{N_r-1})\sqrt{2\lambda_t^2 r^2 D_r} + 2a_t U_t \rho - 3D_r \sqrt{2\eta\lambda_t^2 r^2}}{4\sqrt{\eta^2 u_t^2 \rho^2 b_r ([N_r+1] + v_r)^2}} \geq 0. \]

Therefore, \[ \frac{d\bar{\pi}_{r,i}}{d\theta} \bigg|_{\theta=0} \] is a monotone increasing function in \( \theta \) and crosses the x-axis once. Solving \[ \frac{d\bar{\pi}_{r,i}}{d\theta} \bigg|_{\theta=0} (\theta) = 0, \] we get the threshold \( \bar{\theta}_r = \frac{2(v_r+1)}{(N_r+1)+v_r} \) in Table 1.

\[ \frac{d^2\pi_{r,i}}{d\theta^2} = \frac{(\eta+1)\sqrt{2\lambda_t^2 r^2 D_r} + 2a_t U_t \rho - 3D_r \sqrt{2\eta\lambda_t^2 r^2}}{6(\eta+\chi)(\eta+\chi)^2 u_t^2 \rho^2 b_r ([N_r+1] + v_r)^2} \geq 0, \] indicating the profit function is convex. ■

**Appendix 5. Proof of binding inequality constraint for (14)**

The binding of constraint can be proved by contradiction. If we suppose that the constraint is non-binding, then the maximization problem can be solved by ignoring the non-binding constraint momentarily and solving the corresponding unconstrained maximization problem. But the resultant optimal solution must satisfy the non-binding constraint.

\[ \max_{q_r,i,S_{r,i}} \pi_{r,i} = P_r q_r,i - (f_{r,i} + y_r) \frac{q_r,i}{u_r S_{r,i} \rho / D_r} \]

s.t. \[ \bar{\lambda}_r \bar{S}_{r,i} \frac{q_r,i}{u_r S_{r,i} \rho / D_r} < \theta \bar{F}_r \]

However, if we solve the unconstrained problem, the resultant solutions are that in (5), making \( \lambda_r \bar{S}_{r,i} \frac{q_r,i}{u_r S_{r,i} \rho / D_r} = \bar{F}_r > \theta \bar{F}_r \), which violates the non-binding constraint. Therefore, at optimal, the constraint must be binding. ■
References


