

# Preferences and Equilibrium in Monopoly and Duopoly\*

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**Abstract.** This paper takes the new approach of using a copula to characterize consumer preferences in a discrete choice model of product differentiation, and applies it to the economics of monopoly and duopoly. The comparative statics of demand strength and preference diversity, both properties of the marginal distribution of values for each product variety, are strikingly similar across market structures. Preference dependence, a property of the copula and an indicator of product differentiation, is a key determinant of whether prices are higher in multiproduct industries compared to single-product monopoly. Furthermore, the effects of preference on prices and profits influence equilibrium product selection. Remarkably, a horizontally-differentiated duopoly sometimes can foreclose a higher-quality monopoly to the detriment of consumer and social welfare.

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## 1. INTRODUCTION

This paper develops a new framework for studying how consumer preferences determine equilibrium firm conduct and market performance in the context of monopoly and duopoly markets. At the heart of this new framework is the copula approach to modelling the distribution of consumer preferences in a discrete choice model of product differentiation. This approach separates the effects of the marginal distributions of consumer values for a product variety from the dependence relations between varieties captured by a copula. Our analysis uncovers some unifying principles in the economics of monopoly and duopoly, extending and developing new insights for economic literatures on business strategy, product differentiation, pricing, and market structure.

Our main model of product differentiation considers two symmetric varieties of a good. We focus on three dimensions of consumer preferences: demand strength, preference diversity, and preference dependence. Demand strength and preference diversity are measured respectively by the mean ( $\mu$ ) and variance ( $\sigma$ ) of the marginal distribution. Preference dependence is measured by a parameter ( $\theta$ ) ordering a copula family according to conditional stochastic dominance. Loosely speaking, greater preference dependence means that more consumers regard the two varieties as close substitutes. Thus preference dependence is an intuitive measure of the degree of product differentiation.

With this new approach to product differentiation, we re-examine firm conduct and market performance in a standard discrete choice model of consumer demand. The market can be either a monopoly or a duopoly, and each firm produces either one or two goods. This gives rise to three possible market structures of interest: single-product monopoly, horizontally-differentiated multiproduct monopoly, and horizontally-differentiated duopoly. The strategic variables are prices, which firms choose simultaneously under duopoly competition.

We show how demand strength and preference diversity affect equilibrium price, profit, and consumer welfare for monopoly and duopoly market structures. First, prices, profit and consumer surplus are all higher with stronger demand. Second, firm profits increase in preference diversity for "low-demand" products ( $\mu \leq 0$ ); while for "high-demand" products ( $\mu > 0$ ) profits exhibit a U-shaped relationship with  $\sigma$ , first

decreasing and then increasing. Third, prices and consumer surplus increase in preference diversity if  $\mu \leq 0$  or if  $\sigma$  is sufficiently large. These comparative-static results are similar across all three market structures and under various dependence relations,<sup>1</sup> providing unifying principles in the economics of monopoly and duopoly. Moreover, the effects of preference diversity on profits clarify a key result in Johnson and Myatt (2006) for "variance-ordered distributions" for a single-product monopoly, and, under general preference dependence, extend their insights to horizontally-differentiated multiproduct monopoly and to horizontally-differentiated price-setting duopoly.<sup>2</sup>

We also consider how preference dependence affects prices and profits within markets and their comparisons across markets. The standard approach to discrete-choice models of differentiated oligopoly, pioneered by Perloff and Salop (1985), typically assumes independence between consumer values for different varieties.<sup>3</sup> Our analysis advances the literature on product differentiation by showing that, under certain sufficient conditions, both prices and profits in multiproduct industries decrease as preferences become more positively dependent or less negatively dependent, suggesting that preference dependence is a useful dimension along which to measure product differentiation.<sup>4</sup> We also contribute to this literature by identifying general conditions comparing prices in horizontally differentiated industries with the price under single-product monopoly. In particular, extending Chen and Riordan (2008), we find that single-monopoly price is *higher (lower)* than symmetric duopoly price if the hazard rate of the marginal distribution is *non-decreasing (non-increasing)* and preferences are *positively (negatively)* dependent. In addition, we show that multiproduct monopoly price is higher than single-product monopoly price if preferences are not too positively dependent.

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<sup>1</sup>The results require appropriate regularity conditions for each market structure. The comparative statics of duopoly profits imposes the additional sufficient condition of an increasing hazard rate for the marginal distribution of consumer values.

<sup>2</sup>Johnson and Myatt (2006) show insightfully that firm profit is maximized with either minimum or maximum preference diversity for a single-product monopolist, and, under the assumption of preference independence between varieties, for a multiproduct monopoly with vertically differentiated products and for a quantity-setting oligopoly. This has interesting implications for business strategy in areas such as advertising, marketing, and product design.

<sup>3</sup>Anderson, dePalma, and Thisse (1992) provides an excellent overview of discrete-choice models of product differentiation.

<sup>4</sup>As we noted above, preference diversity, the usual measure of product differentiation used in the literature under preference independence, has non-monotonic relations with profits if  $\mu > 0$ .

We further apply our framework to characterize equilibrium market structures in a two-stage game, where production of each good requires a fixed cost ( $K$ ) and two rival firms make simultaneous product selections, choosing to produce one, both, or neither of the goods, followed by their price decisions. Early work on the economics of multiproduct firms considered the incentive and ability of an incumbent to use brand proliferation to deter entry (e.g., Schmalensee, 1978; Judd, 1985), as well as the role of cost factors such as economies of scope in giving rise to multiproduct firms (Panzar and Willig, 1981). Shaked and Sutton (1990) directed the literature toward considering how demand may affect the nature of equilibrium with multiproduct firms, focusing especially on empirically testable relationships between market characteristics and market structure. Our analysis complements and extends Shaked and Sutton (1990) by showing how the distribution of preferences affects equilibrium product selections in a general discrete choice model of product differentiation. We find that multiple products are offered (either by competing firms or by a monopoly) as  $\mu$  increases beyond a certain level, and that when  $K$  is large enough the duopoly equilibrium emerges before the multiproduct monopoly equilibrium as  $\mu$  increases. We further identify explicitly the roles played by preference diversity and dependence, together with fixed cost (relative to market size), in determining equilibrium outcomes. In particular, in markets with relatively low demand ( $\mu < 0$ ), multiple products are more likely to be offered when preference diversity is *higher*; but in markets with relatively high demand ( $\mu > 0$ ), if  $\sigma$  is below some critical level, multiple products are more likely to be offered when preference diversity is *lower*. On the other hand, more positive (less negative) dependence makes it more likely that single-product monopoly is the unique equilibrium and less likely that duopoly and/or multiproduct monopoly can be an equilibrium. As in Shaked and Sutton (1990), there are parameter regions where a duopoly equilibrium and a multiproduct monopoly equilibrium coexist, suggesting that product proliferation by a monopoly sometimes can foreclose duopoly competition, resulting in higher prices and lower consumer welfare.

Our framework can be extended to incorporate the characteristics approach to consumer demand (Lancaster, 1966; Berry and Pakes, 2007). We illustrate this with an extended model in which a firm can offer a product that bundles the defining characteristics of two product varieties studied in the main model. This third product

can be viewed as a high-quality product, compared to a low-quality product with only a single characteristic. In this extended model, we consider an additional market structure in which monopolist offers the high-quality product. Our earlier results on how preferences affect price, profit, and consumer welfare also hold for the high-quality monopoly. More surprisingly, when firms first make simultaneous product choices before deciding on prices, it is possible that horizontal competition between low-quality duopolists forecloses the high-quality monopoly to the detriment of industry profit, consumer welfare, and social welfare.

We formulate our main model in Section 2, establish the relationships between preferences and equilibrium outcomes under various market structures in Section 3, characterize equilibrium product selections of the two-stage game in Section 4, and examine bundling and product selection in the extended model in Section 5. Section 6 concludes.

## 2. PREFERENCES

Consider two possible varieties of a good, referred to as X and Y. A consumer's utility for X is  $w(x)$  and for Y is  $w(y)$ , where  $x$  and  $y$  are uniformly distributed on  $[0, 1]$ ,  $w(\cdot)$  is a strictly-increasing twice-differentiable function, and there exists some interval on which  $w(x) > 0$ . The consumer purchases at most one of the two varieties. Let  $p$  denote the price of X and  $r$  the price of Y. When only one product variety is offered in the market, it will be called X and the consumer purchases it if  $w(x) - p \geq 0$ , where zero is the normalized utility of an "outside good". When both varieties are offered in the market, the consumer purchases X if  $w(x) - p \geq \max\{w(y) - r, 0\}$  and Y if  $w(y) - r > \max\{w(x) - p, 0\}$ .

The population of consumers, the size of which is normalized to 1, is described by a symmetric copula  $C(x, y)$ . A copula is a bivariate uniform distribution, satisfying  $C(x, 1) = x$ ,  $C(1, y) = y$ , and  $C(x, 0) = 0 = C(0, y)$ . Additionally, any copula, which we assume to have strictly positive density  $c(x, y) = \frac{\partial^2 C(x, y)}{\partial x \partial y}$  on  $[0, 1]^2$ , lies between the Frechet-Hoeffding lower and upper bounds:

$$\max\{0, x + y - 1\} < C(x, y) < \min\{x, y\}. \tag{1}$$

The copula determines the statistical dependence of consumer values for the two varieties. In particular,  $C_1(x, y) \equiv \frac{\partial C(x, y)}{\partial x}$  is the conditional distribution of  $y$  given  $x$ , and  $C_{11}(x, y) \equiv \frac{\partial^2 C(x, y)}{\partial x^2} < 0$  ( $> 0$ ) indicates positive (negative) stochastic dependence. The independence copula is  $C(x, y) = xy$ . Positive (negative) stochastic dependence also implies  $C(x, y) > xy$  ( $C(x, y) < xy$ ).<sup>5</sup>

A family of copulas,  $C(x, y; \theta)$ , indexed by some parameter  $\theta$ , is said to satisfy the monotonic dependence ranking property (**MDR**) if for  $0 < x, y < 1$

$$\frac{\partial C_{11}(x, y; \theta)}{\partial \theta} \equiv C_{11\theta}(x, y; \theta) < 0. \quad (2)$$

**MDR** implies  $C_\theta(x, y; \theta) \equiv \frac{\partial C(x, y; \theta)}{\partial \theta} > 0$  (Nelson, 2006). Thus if  $C(x, y; \theta)$  satisfies **MDR**, a higher  $\theta$  means that  $C$  has a higher (lower) degree of positive (negative) dependence.

It is useful to define a normalized utility. The mean utility of either good is

$$\mu = \int_0^1 w(x) dx, \quad (3)$$

and the variance is

$$\sigma^2 = \int_0^1 [w(x) - \mu]^2 dx. \quad (4)$$

Let

$$u(x) = \frac{w(x) - \mu}{\sigma}. \quad (5)$$

The normalized utility  $u = u(x)$  is assumed to have the cumulative distribution function  $F(u) = u^{-1}(u)$ , with a corresponding density function  $f(u) > 0$ , on support  $[u(0), u(1)]$ .<sup>6</sup> Thus  $w_x \equiv w(x)$  is distributed according to  $F(\frac{w_x - \mu}{\sigma}) = w^{-1}(w_x)$ ;  $w_y = w(y)$  is distributed similarly. The joint distribution of  $w_x$  and  $w_y$ , which is assumed to be symmetric, is  $C(F(\frac{w_x - \mu}{\sigma}), F(\frac{w_y - \mu}{\sigma}))$ .<sup>7</sup>

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<sup>5</sup>Positive stochastic dependence is in turn implied by positive likelihood ratio dependence or  $\frac{\partial \ln c(u, v)}{\partial u \partial v} > 0$  (Nelson, 2006).

<sup>6</sup>We extend the support in the usual way, i.e.,  $F(u) = 0$  for  $u < u(0)$  and  $F(u) = 1$  for  $u > u(1)$ , unless  $u(0) = -\infty$  and/or  $u(1) = \infty$ .

<sup>7</sup>By Sklar's Theorem (Nelson, 2006), it is without loss of generality to represent joint distribution of consumers' values for two products by a copula and marginal distributions.

**Bivariate exponential case.** We shall use the following parametric functions to illustrate results:

$$u(x) = -[1 + \ln(1 - x)], \quad (6)$$

$$C(x, y) = xy + \theta xy(1 - x)(1 - y). \quad (7)$$

In this case, the normalized utility  $u = u(x)$  has an exponential distribution

$$F(u) = 1 - e^{-u-1} \quad (8)$$

with  $E\{u\} = 0$ , and  $Var(u) = 1$ . The copula belongs to the Fairlie-Gumbel-Morgenstern (FGM) family for which  $\theta \in [-1, 1]$ . Members of the FGM family exhibit positive stochastic dependence if  $\theta > 0$ , negative dependence if  $\theta < 0$ , and independence if  $\theta = 0$ . The FGM copula density is

$$c(x, y) = 1 + \theta(2x - 1)(2y - 1). \quad (9)$$

The resulting bivariate exponential distribution  $C(F(u), F(u))$  was introduced by Gumbel (1960).<sup>8</sup> Our numerical analyses will focus on the extreme cases of positive and negative dependence ( $\theta = 1$  and  $\theta = -1$ ) and independence ( $\theta = 0$ ).<sup>9</sup>

Summarizing, the demand for the two goods is completely characterized by the marginal distribution function  $F(u)$ , the copula  $C(x, y; \theta)$ ,<sup>10</sup> and parameters  $\mu$  and  $\sigma$  which respectively measure demand strength and preference diversity. The parameter  $\theta$  measures the degree of preference dependence. Introduced by Chen and Riordan (2008), the copula approach to product differentiation has the advantage of separating the effects of the marginal distribution of consumer values for each variety from dependence relationships.

We conclude this section by placing our formulation in the context of standard demand analysis, adopting the following normalizations

$$\bar{p} = \frac{p - \mu}{\sigma}; \bar{r} = \frac{r - \mu}{\sigma}; \bar{\mu} = \frac{\mu}{\sigma}. \quad (10)$$

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<sup>8</sup>Gumbel (1960) introduced several bivariate exponential distributions. This is the second one.

<sup>9</sup>The FGM family of copulas has limit range of positive and negative dependence; so the extreme cases of  $\theta = 1$  or  $\theta = -1$  does not approximate perfectly positive or perfectly negative dependence.

<sup>10</sup>We sometimes omit  $\theta$  in expressing the copula function.

If  $u(1) > \bar{p} > u(0)$ , and similarly for  $\bar{r}$ , then there is a positive demand for all goods, including the outside good, and the demand for X is calculated by integrating over the acceptance set:

$$Q(\bar{p}, \bar{r}) = 1 - C(F(\bar{p}), F(\bar{r})) - \int_{F(\bar{r})}^1 C_2(F(u(y) + \bar{p} - \bar{r}), y) dy. \quad (11)$$

The demand for Y is calculated similarly. Thus the two goods are always substitutes because

$$\frac{\partial Q(\bar{p}, \bar{r}; \theta)}{\partial \bar{r}} = \int_{F(\bar{r})}^1 c(F(u(y) + \bar{p} - \bar{r}), y; \theta) dy > 0.$$

Since the cross price elasticity between the two goods is  $\eta = \frac{\partial Q(\bar{p}, \bar{r}; \theta)}{\partial \bar{r}} \frac{\bar{r}}{Q(\bar{p}, \bar{r}; \theta)}$ , in general it is not clear that a higher  $\theta$  will always increase the substitutability of the two goods (in the sense of higher  $\eta$ ), despite  $Q$  being lower with a higher  $\theta$ . However, if  $\bar{p} = \bar{r} \rightarrow u(0)$ , then a sufficient condition for the two goods to become closer substitutes as  $\theta$  increases is

$$\int_0^1 c_\theta(x, x; \theta) f(u(x)) dx \geq 0. \quad (12)$$

This condition states that greater preference dependence is associated with greater density of consumer preferences along the diagonal of the type space, meaning that with greater dependence more consumers value the two goods similarly.<sup>11</sup> We shall return to this condition later.

If only good X is offered in the market, its demand can be obtained by setting  $F(\bar{r}) = 1$  in (11), or

$$q^m(\bar{p}) = 1 - F(\bar{p}). \quad (13)$$

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<sup>11</sup>While this is an intuitive property, we do not know whether it is an implication of MDR. We are not aware of any relationship established between the two conditions in the statistics literature.

### 3. PRICING

We assume that the production of each good requires a constant average variable cost normalized to zero. An appropriate interpretation of the variable cost normalization is that  $p$  is a markup on variable cost, and  $\mu$  is mean demand net of constant average variable cost.

We further assume that equilibrium prices exist uniquely and are interior under all market structures.<sup>12</sup> This, together with the symmetry of  $C(\cdot, \cdot)$ , implies that equilibrium is symmetric when products are symmetric. The maintained assumptions simplify comparative statics and comparisons of prices, profits, and welfare for different market structures.

In this section, we develop the comparative statics of consumer preferences. These results will demonstrate some unifying principles in the economics of monopoly and duopoly, contributing to the theory of price and product differentiation, and providing the basis for our later study of product selection,

#### Monopoly

First, consider a single-product monopolist, who produces X. From (10), the monopolist's (gross) profit function is

$$\pi^m(\bar{p}) = \sigma(\bar{p} + \bar{\mu}) [1 - F(\bar{p})]. \quad (14)$$

The profit-maximizing price satisfies

$$(\bar{p}^m + \bar{\mu})\lambda(\bar{p}^m) = 1, \quad (15)$$

where

$$\lambda(\bar{p}) \equiv \frac{f(\bar{p})}{1 - F(\bar{p})} \quad (16)$$

is the hazard rate. The following regularity condition guarantees a unique local maximum.<sup>13</sup>

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<sup>12</sup>That is, profit functions are differentiable at equilibrium prices. For convenience, we refer to optimal prices under monopoly as equilibrium prices.

<sup>13</sup>Our maintained assumption of interior equilibrium requires  $\bar{p}^m > u(0)$  at the solution of (15).

**A1:**  $\frac{d[(\bar{p}+\bar{\mu})\lambda(\bar{p})]}{d\bar{p}} > 0$ , where  $\bar{p} + \bar{\mu} = p/\sigma > 0$ .

Thus,  $\lambda'(u) \geq 0$ , a familiar assumption in the discrete choice product differentiation literature, is sufficient but not necessary for **A1**. The maximum profit is  $\pi^m \equiv \pi^m(\bar{p}^m)$  and the consumer surplus is

$$w^m = \sigma \int_{\bar{p}^m}^{u(1)} [1 - F(\bar{p})] d\bar{p}. \quad (17)$$

Defining normalized profits and consumer surplus,  $\bar{\pi}^m = \pi^m/\sigma$  and  $\bar{w}^m = w^m/\sigma$ , we have the following result.

**Proposition 1** *Given **A1**, (i)  $\frac{d\bar{p}^m}{d\bar{\mu}} < 0$ , and there exists some  $\bar{\mu}^m > 0$  such that  $\bar{p}^m \geq 0$  if  $\bar{\mu} \leq \bar{\mu}^m$ ; (ii)  $\frac{d\bar{\pi}^m}{d\bar{\mu}} > 0$ ; and (iii)  $\frac{d\bar{w}^m}{d\bar{\mu}} > 0$ .*

**Proof.** Part (i) follows immediately from (15) and **A1**, part (ii) follows from the envelope theorem, and part (iii) follows from simple differentiation. ■

Proposition 1 predicts how single-product monopoly conduct and performance depend on demand strength and product diversity, as in the following two results:

**Corollary 1** *Given **A1**: (i)  $\frac{dp^m}{d\mu} \geq 0$  if  $\lambda'(p) \geq 0$ ; (ii)  $\frac{d\pi^m}{d\mu} > 0$ ; and (iii)  $\frac{dw^m}{d\mu} > 0$ .*

**Proof.** (i) is true because

$$\frac{dp^m}{d\mu} = \frac{1}{\sigma} \frac{dp^m}{d\bar{\mu}} = \frac{d(\bar{p}^m + \bar{\mu})}{d\bar{\mu}} \geq 0 \text{ if } \lambda'(p) \geq 0$$

from (15) and part (i) of Proposition 1. (ii) and (iii) follow directly from parts (ii) and (iii) of Proposition 1 since  $\frac{d\pi^m}{d\mu} = \frac{d\bar{\pi}^m}{d\bar{\mu}}$  and  $\frac{dw^m}{d\mu} = \frac{d\bar{w}^m}{d\bar{\mu}}$ . ■

Therefore, profit and consumer welfare under single-product monopoly are both increasing in demand strength, and so is monopoly price if  $\lambda'(p) > 0$ .

**Corollary 2** *Given **A1**: (i)  $\frac{dp^m}{d\sigma} > 0$  if  $\sigma$  is sufficiently large for any given  $\mu$ , if  $\mu \leq 0$  and  $\lambda'(\cdot) \geq 0$ , or if  $\mu \geq 0$  and  $\lambda'(\cdot) \leq 0$ ; (ii)  $\frac{d\pi^m}{d\sigma} > 0$  when  $\mu \leq 0$ , and  $\pi^m$  first decreases and then increases in  $\sigma$  when  $\mu > 0$ ; (iii)  $\frac{dw^m}{d\sigma} > 0$  if  $\mu \leq 0$  or if  $\sigma$  is sufficiently large for given  $\mu > 0$ .*

**Proof.** (i) From part (i) of Proposition 1, we have

$$\frac{dp^m}{d\sigma} = \bar{p}^m + \sigma \frac{d\bar{p}^m}{d\sigma} = \bar{p}^m - \bar{\mu} \frac{d\bar{p}^m}{d\bar{\mu}} = \frac{1}{\sigma} \left[ p^m - \mu \left( 1 + \frac{d\bar{p}^m}{d\bar{\mu}} \right) \right].$$

Thus  $\frac{dp^m}{d\sigma} > 0$  if  $\bar{\mu}$  is sufficiently close to zero and  $\bar{\mu} < \bar{\mu}^m$ ; i.e., for any given  $\mu$ ,  $\frac{dp^m}{d\sigma} > 0$  if  $\sigma$  is large enough. Furthermore, from **A1**,  $1 + \frac{d\bar{p}^m}{d\bar{\mu}} \geq 0$  if  $\lambda'(p) \geq 0$ . Thus  $\frac{dp^m}{d\sigma} > 0$  if  $\mu \leq 0$  and  $\lambda'(p) \geq 0$  or if  $\mu \geq 0$  and  $\lambda'(\cdot) \leq 0$ .

(ii) Since  $\bar{p}^m \geq 0$  if  $\bar{\mu} \leq \bar{\mu}^m$  from part (i) of Proposition 1 and

$$\frac{d\pi^m}{d\sigma} = \bar{\pi}^m + \sigma \frac{d\bar{\pi}^m}{d\sigma} = \bar{\pi}^m + \sigma \frac{\partial \bar{\pi}^m}{\partial \bar{\mu}} \frac{d\bar{\mu}}{d\sigma} = \bar{\pi}^m - \bar{\mu} [1 - F(\bar{p}^m)] = \bar{p}^m [1 - F(\bar{p}^m)],$$

$\frac{d\pi^m}{d\sigma} > 0$  if  $\mu \leq 0$ . For given  $\mu > 0$ ,  $\frac{d\pi^m}{d\sigma} < 0$  if  $\sigma < \frac{\mu}{\bar{\mu}^m}$ ,  $\frac{d\pi^m}{d\sigma} = 0$  if  $\sigma = \frac{\mu}{\bar{\mu}^m}$ , and  $\frac{d\pi^m}{d\sigma} > 0$  if  $\sigma > \frac{\mu}{\bar{\mu}^m}$ .

(iii) Since  $\frac{\partial \bar{w}^m}{\partial \bar{\mu}} > 0$  from part (iii) of Proposition 1,

$$\frac{dw^m}{d\sigma} = \bar{w}^m + \sigma \frac{d\bar{w}^m}{d\sigma} = \int_{\bar{p}^m}^{u(1)} [1 - F(\bar{p})] d\bar{p} - \bar{\mu} \frac{\partial \bar{w}^m}{\partial \bar{\mu}} > 0$$

if  $\mu \leq 0$  or if  $\sigma$  is sufficiently large for given  $\mu > 0$ . ■

Johnson and Myatt (1983) studied families of "variance-ordered" distributions for which mean value is a differentiable function of variance, i.e.  $\mu = \mu(\sigma)$ , and showed that  $\pi^m$  is a quasi-convex function of  $\sigma$  if  $\mu'(\sigma) \geq 0$ . Part (ii) of Corollary 2 details this result for the special case of a constant mean, i.e.  $\mu'(\sigma) = 0$ . Johnson and Myatt further showed that  $q^m$  is a convex function of  $\sigma$  if  $\mu'(\sigma) \geq 0$ . Our equation (13), however, implies that  $q^m$  is decreasing in  $\bar{p}^m$ . Therefore, part (i) of Proposition 1 clarifies for the constant mean case that  $q^m$  increases with  $\sigma$  if  $\mu < 0$ , and, conversely,  $q^m$  decreases with  $\sigma$  if  $\mu > 0$ . An important implication of Johnson and Myatt's result is that a single-product monopolist seeks either to maximize or minimize preference diversity.<sup>14</sup> Corollary 2 further clarifies that the monopolist always seeks to increase preference diversity for a "weak demand" product for which  $\mu \leq 0$ .

<sup>14</sup>Johnson and Myatt's Proposition 1 establishes a preference for extremes for an even broader family of distributions ordered by a decreasing sequence of "rotation" points.

Our analysis goes further in explicitly considering consumer welfare. Corollary 2 shows that the firm's incentive to increase  $\sigma$  for weak-demand products coincides with the consumer interests. Strong-demand products ( $\mu > 0$ ) lack such general coincidence of interests, because the critical  $\sigma$  at which  $\pi^m$  switches from decreasing to increasing in  $\sigma$  is different from the corresponding critical  $\sigma$  for  $w^m$ . Nevertheless, if  $\sigma$  is large enough, the firm's incentive to further increase diversity does coincide with consumer interests.

Next consider the multiproduct monopolist who produces both varieties of the good. While Johnson and Myatt (2003) studied a multiproduct monopoly producing vertically-differentiated products under independent valuations between products, the following analysis deals with horizontal differentiation under general preference dependence. Under our maintained symmetry assumption, the monopolist charges the same price for both variants. Defining  $\bar{p}$  as before, the monopolist's profit function is

$$\pi^{mm}(\bar{p}) = \sigma(\bar{p} + \bar{\mu}) [1 - C(F(\bar{p}), F(\bar{p}))]. \quad (18)$$

The profit-maximizing normalized price  $\bar{p}^{mm}$  satisfies

$$(\bar{p}^{mm} + \bar{\mu}) \lambda^C(\bar{p}^{mm}) = 1, \quad (19)$$

where

$$\lambda^C(\bar{p}) \equiv \frac{2C_1(F(\bar{p}), F(\bar{p}))}{1 - C(F(\bar{p}), F(\bar{p}))} f(\bar{p}) \quad (20)$$

is the hazard rate corresponding to cumulative distribution function

$$F^C(\bar{p}) \equiv C(F(\bar{p}), F(\bar{p}))$$

on support  $[u(0), u(1)]$ . This distribution function determines the total demand of a monopolist selling two symmetric goods at the same price. The corresponding maximum profit is  $\pi^{mm} \equiv \sigma \bar{\pi}^{mm} = \pi^{mm}(\bar{p}^{mm})$ , and consumer surplus given  $\bar{p}^{mm}$  is

$$w^{mm} \equiv \sigma \bar{w}^{mm} = \sigma \int_{\bar{p}^{mm}}^{u(1)} [1 - C(F(\bar{p}), F(\bar{p}))] d\bar{p}, \quad (21)$$

where  $\bar{\pi}^{mm}$  and  $\bar{w}^{mm}$  denote normalized values. The following regularity condition guarantees a unique local maximum, thus ensuring unique comparative statics.

**A2:**  $\frac{d[(\bar{p}+\bar{\mu})\lambda^C(\bar{p})]}{d\bar{p}} > 0$ .

**A2** is satisfied if  $\lambda^C(\bar{p})$  increases (or does not decrease too quickly) in  $\bar{p}$ .

Note that the first-order condition (19) is the same as (15) for single-product monopoly, except that the hazard rate here,  $\lambda^C(\bar{p}^{mm})$ , corresponds to the cumulative distribution function  $F^C(\bar{p})$ . It follows that comparative statics with respect to  $\bar{\mu}$  are essentially the same as for a single-product monopoly.

**Proposition 2** *Given A2: (i)  $\frac{d\bar{p}^{mm}}{d\bar{\mu}} < 0$ , and there exists some  $\bar{\mu}^{mm} > 0$  such that  $\bar{p}^{mm} \gtrless 0$  if  $\bar{\mu} \gtrless \bar{\mu}^{mm}$ ; (ii)  $\frac{d\pi^{mm}}{d\bar{\mu}} > 0$ ; (iii)  $\frac{dw^{mm}}{d\bar{\mu}} > 0$ .*

As for the single-product case, comparative statics with respect to  $\mu$  and  $\sigma$  are straightforward corollaries, and the comparative statics on  $\sigma$  extends some of the basic results in Johnson and Myatt (2003) to symmetric multiproduct monopoly.

**Corollary 3** *Given A2: (i)  $\frac{dp^{mm}}{d\mu} \gtrless 0$  if  $\lambda^{C'}(p) \gtrless 0$ ; (ii)  $\frac{d\pi^{mm}}{d\mu} > 0$ ; (iii)  $\frac{dw^{mm}}{d\mu} > 0$ .*

**Corollary 4** *Given A2: (i)  $\frac{dp^{mm}}{d\sigma} > 0$  if  $\sigma$  is sufficiently large for any given  $\mu$ , if  $\mu \leq 0$  and  $\lambda^{C'}(\cdot) \geq 0$ , or if  $\mu \geq 0$  and  $\lambda^{C'}(\cdot) \leq 0$ ; (ii)  $\frac{d\pi^{mm}}{d\sigma} > 0$  when  $\mu \leq 0$ ,  $\pi^{mm}$  first decreases and then increases in  $\sigma$  when  $\mu > 0$ ; (iii)  $\frac{dw^{mm}}{d\sigma} > 0$  if  $\mu \leq 0$  or if  $\sigma$  is sufficiently large for given  $\mu > 0$ .*

We can also investigate more generally how the degree of preference dependence affects outcomes under multiproduct monopoly. Toward this end, we consider a family of copulas satisfying **MDR**, and define  $\lambda^C(\bar{p}; \theta)$  accordingly. A useful property of copula is that the conditional copula,  $C_1(x, x; \theta) \equiv \frac{\partial C(x, x; \theta)}{\partial x}$ , increases (decreases) in  $\theta$  when  $x$  is small (large). This implies that greater positive dependence shifts up the hazard rate for the multiproduct monopolist when the market coverage is high.

**Lemma 1** *Given MDR,  $\frac{\partial \lambda^C(\bar{p}; \theta)}{\partial \theta} > 0$  if  $\bar{p} (> u(0))$  is sufficiently close to  $u(0)$ .*

**Proof.** Given **MDR**, it is sufficient to show that there exists some  $x_1$ , with  $0 < x_1 < 1$ , such that  $C_1(x, x; \theta)$  increases in  $\theta$  if  $x < x_1$ . For any  $p > 0$ ,

$$\int_0^p C_1(x, x; \theta) dx = \frac{1}{2} \int_0^p \frac{dC(x, x; \theta)}{dx} dx = \frac{1}{2} C(p, p; \theta)$$

increases in  $\theta$ , which is possible only if there is some  $x_1 > 0$  such that  $C_1(x, x; \theta)$  increases in  $\theta$  if  $x < x_1$ .<sup>15</sup> ■

Furthermore, the market is fully covered, or nearly so, if demand is sufficiently strong. This consideration contributes to part (ii) of the following:

**Proposition 3** *Given **A2** and **MDR**: (i)  $\frac{d\pi^{mm}}{d\theta} < 0$ ; (ii) if  $\bar{\mu}$  is sufficiently large, then  $\frac{dp^{mm}}{d\theta} < 0$ .*

**Proof.** (i) holds from application of the envelope theorem to (18) and  $C_\theta > 0$ . (ii) For  $F(\cdot)$  and  $C(\cdot, \cdot; \theta)$  satisfying **A2**, if  $\bar{\mu}$  is sufficiently high, then (19) implies  $\bar{p}^{mm} < F^{-1}(x_1)$  for all  $\theta$ , where  $x_1$  is defined in the proof for Lemma 1. Therefore, if  $\bar{\mu}$  is sufficiently large, Lemma 1 implies  $\frac{\partial \lambda^C(\bar{p}^{mm}; \theta)}{\partial \theta} > 0$ . It follows from (19) that  $\frac{d\bar{p}^{mm}}{d\theta} < 0$  and hence  $\frac{dp^{mm}}{d\theta} < 0$ . ■

Therefore, a multiproduct monopolist prefers that consumer values for its two products are less positively (more negatively) dependent. This is intuitive, since the more similar are product varieties the less valuable is a choice. A sufficient condition for  $p^{mm}$  to decrease in  $\theta$  is demand being sufficiently strong ( $\bar{\mu}$  sufficiently high). It is possible that  $p^{mm}$  increases with  $\theta$  if demand is sufficiently weak. For example, for the bivariate exponential case based on the FGM copula family, numerical analysis shows that  $p^{mm}$  increases in  $\theta$  if  $\bar{\mu}$  is below a critical value.

The effects of greater preference dependence on consumer welfare appear ambiguous in general. On the one hand, a higher  $\theta$  shifts down the demand curve, thus reducing consumer surplus at any price. On the other hand, a higher  $\theta$  might result in a lower price, as when demand is sufficiently strong, which increases consumer surplus given the demand curve. However, if more dependence leads to higher prices, as it is sometimes the case when  $\bar{\mu}$  is low, then greater dependence reduces consumer welfare. This general ambiguity persists even in the neighborhood of independence

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<sup>15</sup>Similarly, there exists  $x_2$ , with  $x_1 \leq x_2 < 1$ , such that  $C_1(x, x; \theta)$  decreases in  $\theta$  if  $x > x_2$ . For the FGM family,  $x_1 = x_2 = 1/2$ .

and for strong demand. For the bivariate exponential special case, however, numerical analysis shows that consumer welfare increases with preference dependence when demand is sufficiently strong.

## Duopoly

Assume the two products are sold by symmetric single-product firms. The profit function of Firm X is  $\pi^d(\bar{p}, \bar{r}) = \sigma(\bar{p} + \bar{\mu})Q(\bar{p}, \bar{r})$ . In equilibrium,  $\bar{p} = \bar{r} = \bar{p}^d$ , satisfying

$$(\bar{p}^d + \bar{\mu}) h(\bar{p}^d) = 1, \quad (22)$$

where we define the adjusted hazard rate under duopoly competition as

$$h(\bar{p}) \equiv \lambda^C(\bar{p}) + \frac{2}{1 - C(F(\bar{p}), F(\bar{p}))} \int_{F(\bar{p})}^1 c(x, x) f(u(x)) dx, \quad (23)$$

which is the hazard rate under multiproduct monopoly adjusted by an extra term. The extra term measures the business-stealing effect when both firms charge the same price, i.e. the percentage demand increase from a price cut resulting from customers who change allegiance. Notice that if  $\bar{p} \leq u(0)$ , then  $\lambda^C(\bar{p}) = 0$  and  $h(\bar{p}) = 2 \int_0^1 c(x, x) f(u(x)) dx$ . Thus  $u(0)$  is the critical price separating the equilibrium regimes of fully covered versus non-fully covered markets.

To ensure unique comparative statics, we adopt the regularity condition:

**A3:**  $\frac{d[(\bar{p} + \bar{\mu})h(\bar{p})]}{d\bar{p}} > 0$ .

For example,  $h'(\bar{p}) \geq 0$  and hence **A3** holds if  $F(u)$  is exponential and  $C_{11} \geq 0$ , from (26) in the proof of Proposition 5. However, **A3** is also consistent with  $h'(\bar{p}) < 0$  and/or  $C_{11} < 0$ .

Each firm's equilibrium profit and consumer welfare are, respectively:

$$\pi^d \equiv \sigma \bar{\pi}^d = \frac{1}{2} \sigma (\bar{p}^d + \bar{\mu}) [1 - C(F(\bar{p}^d), F(\bar{p}^d))], \quad (24)$$

$$w^d \equiv \sigma \bar{w}^d = \sigma \int_{\bar{p}^d}^{u(1)} [1 - C(F(\bar{p}), F(\bar{p}))] d\bar{p}. \quad (25)$$

The effect of demand strength and preference diversity on firm conduct and consumer welfare under duopoly is qualitatively similar to monopoly. The effect on profit, however, is qualitatively similar under the sufficient condition of a non-decreasing adjusted hazard rate  $h(\cdot)$ .

**Proposition 4** *Given **A3**: (i)  $\frac{d\bar{p}^d}{d\bar{\mu}} < 0$ , and there exists some  $\bar{\mu}^d > 0$  such that  $\bar{p}^d \begin{smallmatrix} \geq \\ \leq \end{smallmatrix} 0$  if  $\bar{\mu} \begin{smallmatrix} \leq \\ \geq \end{smallmatrix} \bar{\mu}^d$ . (ii)  $\frac{d\bar{\pi}^d}{d\bar{\mu}} > 0$  if  $h'(\cdot) \geq 0$ . (iii)  $\frac{d\bar{w}^d}{d\bar{\mu}} > 0$ .*

**Proof.** (i) follows immediately from (22) and **A3**.

(ii) From (22) and **A3**,  $\frac{d(\bar{p}^d + \bar{\mu})}{d\bar{\mu}} \geq 0$  if  $h'(\cdot) \geq 0$ . Thus, from (24):

$$2 \frac{d\bar{\pi}^d}{d\bar{\mu}} = \frac{d(\bar{p}^d + \bar{\mu})}{d\bar{\mu}} [1 - C(\cdot, \cdot)] - 2p^d C_1(\cdot, \cdot) f(\bar{p}^d) \frac{d\bar{p}^d}{d\bar{\mu}} > 0.$$

(iii) follows from differentiating  $\bar{w}^d$  with respect to  $\bar{\mu}$ , using  $\frac{d\bar{p}^d}{d\bar{\mu}} < 0$ . ■

An increase in  $\bar{\mu}$  has the direct effect of increasing demand, with a positive effect on profit; but it also has a strategic effect that can potentially lower equilibrium price  $p^d$ . If  $h'(\cdot) \geq 0$ ,  $\frac{d(\bar{p}^d + \bar{\mu})}{d\bar{\mu}} = \frac{dp^d}{d\bar{\mu}} \geq 0$ , ensuring a positive strategic effect and hence higher  $\pi^d$ . As for the monopoly cases, comparative statics of  $\mu$  and  $\sigma$  follow straightforwardly:

**Corollary 5** *Given **A3**: (i)  $\frac{dp^d}{d\mu} \begin{smallmatrix} \geq \\ \leq \end{smallmatrix} 0$  if  $h'(p) \begin{smallmatrix} \geq \\ \leq \end{smallmatrix} 0$ ; (ii)  $\frac{d\pi^d}{d\mu} > 0$  if  $h'(\cdot) \geq 0$ ; (iii)  $\frac{dw^d}{d\mu} > 0$ .*

**Corollary 6** *Given **A3**: (i)  $\frac{dp^d}{d\sigma} > 0$  if  $\sigma$  is sufficiently large for any given  $\mu$ , if  $\mu \leq 0$  and  $h'(\cdot) \geq 0$ , or if  $\mu \geq 0$  and  $h'(\cdot) \leq 0$ . (ii) Assume  $h'(\cdot) \geq 0$ . Then  $\frac{d\pi^d}{d\sigma} > 0$  when  $\mu \leq 0$ , and  $\pi^d$  first decreases and then increases in  $\sigma$  when  $\mu > 0$ . (iii)  $\frac{dw^d}{d\sigma} > 0$  if  $\mu \leq 0$  or if  $\sigma$  is sufficiently large for given  $\mu > 0$ .*

Part (ii) of Corollary 6 extends the key insight of Johnson and Myatt (2003) to duopoly under price competition.

We further consider how the equilibrium outcomes under duopoly competition are affected by the degree of preference dependence. It is intuitive to expect that prices and profits decline with preference dependence, since greater preference dependence means that more consumers regard the two varieties as close substitutes, intensifying competition. We identify two sufficient conditions under which this is true:

**Proposition 5** *Given **A2**, **A3** and **MDR**: both  $\bar{p}^d$  and  $\pi^d$  decrease in  $\theta$  if either (i)  $h(u) + \frac{f'(u)}{f(u)} \geq 0$  and  $\frac{d^2 \ln f(u)}{du^2} \geq \frac{2f^2(u(x))}{C_\theta(x,x)} C_{11\theta}(x,x)$  (i.e.,  $f(x)$  does not decrease too fast and is not too logconcave); or (ii)  $\bar{\mu}$  is sufficiently large and condition (12) holds.*

**Proof.** (i) Integrating by parts, we can rewrite

$$\begin{aligned}
h(\bar{p}) &= 2 \frac{C_1(F(\bar{p}), F(\bar{p}))f(\bar{p}) + \int_{F(\bar{p})}^1 \frac{dC_1(x,x)}{dx} f(u(x)) dx - \int_{F(\bar{p})}^1 C_{11}(x,x)f(u(x)) dx}{1 - C(F(\bar{p}), F(\bar{p}))} \\
&= 2 \frac{f(u(1)) - \int_{F(\bar{p})}^1 C_1(x,x) \frac{f'(u(x))}{f(u(x))} dx - \int_{F(\bar{p})}^1 C_{11}(x,x)f(u(x)) dx}{1 - C(F(\bar{p}), F(\bar{p}))}. \tag{26}
\end{aligned}$$

Then, since

$$\begin{aligned}
& - \int_{F(\bar{p})}^1 C_1(x,x) \frac{f'(u(x))}{f(u(x))} dx = \int_{F(\bar{p})}^1 \frac{f'(u(x))}{f(u(x))} d\frac{1}{2} [1 - C(x,x)] \\
& = -\frac{f'(\bar{p})}{f(\bar{p})} \frac{1}{2} [1 - C(F(\bar{p}), F(\bar{p}))] - \int_{F(\bar{p})}^1 \frac{1}{2} [1 - C(x,x)] d\left(\frac{f'(u(x))}{f(u(x))}\right), \\
h(\bar{p}) &= -\frac{f'(\bar{p})}{f(\bar{p})} + 2 \frac{f(u(1)) - \int_{F(\bar{p})}^1 \frac{1}{2} [1 - C(x,x)] d\left(\frac{f'(u(x))}{f(u(x))}\right) - \int_{F(\bar{p})}^1 C_{11}(x,x)f(u(x)) dx}{1 - C(F(\bar{p}), F(\bar{p}))}.
\end{aligned}$$

Thus, if  $h(\bar{p}) + \frac{f'(\bar{p})}{f(\bar{p})} \geq 0$  and  $\frac{d^2 \ln f(u)}{du^2} > \frac{2C_{11\theta}(x,x)f^2(u(x))}{C_\theta(x,x)}$ ,  $\frac{\partial h(\bar{p})}{\partial \theta} =$

$$\frac{\int_{F(\bar{p})}^1 \left[ \frac{C_\theta(x,x)}{f(u(x))} \frac{d^2 \ln f(u)}{du^2} - 2C_{11\theta}(x,x)f(u(x)) \right] dx + \left[ h(\bar{p}) + \frac{f'(\bar{p})}{f(\bar{p})} \right] C_\theta(F(\bar{p}), F(\bar{p}))}{1 - C(F(\bar{p}), F(\bar{p}))} > 0$$

because  $C_\theta > 0$  and  $C_{11\theta} < 0$  from (2). From (22) and A3 we thus have  $\frac{d\bar{p}}{d\theta} < 0$  and hence  $\frac{dp^d}{d\theta} < 0$ .

Next,  $\pi^d$  decreases in  $\theta$  when  $\frac{d\bar{p}^d}{d\theta} < 0$ , because

$$\begin{aligned} \frac{d\pi^d}{d\theta} &= \frac{\partial \pi^d}{\partial \theta} + \frac{\partial \pi^d}{\partial \bar{p}^d} \frac{d\bar{p}^d}{d\theta} \\ &= -\frac{1}{2} (\bar{p}^d \sigma + \mu) C_\theta(F(\bar{p}^d), F(\bar{p}^d); \theta) + \frac{\partial \pi^d}{\partial \bar{p}^d} \frac{d\bar{p}^d}{d\theta} < 0, \end{aligned}$$

where  $C_\theta(F(\bar{p}^d), F(\bar{p}^d); \theta) > 0$  from (2), and  $\frac{\partial \pi^d}{\partial \bar{p}^d} > 0$  by the envelope theorem and by the fact that a firm's demand increases in the other firm's price.

(ii) The proof of Proposition 3 shows that  $\lambda^C(\bar{p}; \theta)$  increases with  $\theta$  if  $\bar{\mu}$  is sufficiently large. Therefore, from **MDR** and (23), (12) implies  $h(\bar{p}; \theta)$  increases with  $\theta$  if  $F(\bar{p})$  is sufficiently close to zero. From (22),  $\bar{p}^d$  and also  $p^d$  decrease with  $\theta$ . As in (i), it follows that  $\frac{d\pi^d}{d\theta} < 0$ . ■

To see why additional conditions are needed to ensure that prices monotonically decrease with  $\theta$ , we notice that while a higher  $\theta$  results in a lower output for a duopolist at a symmetric equilibrium, motivating a lower price, it also affects the slope of the residual demand curve of the duopolist that can potentially give the incentive to raise price. Sufficient condition (i) in Proposition 5 is satisfied if  $f$  has limited logcurvature and  $C$  is positively dependent, or if  $f$  does not decrease too fast and is not too logconcave. Also, recall that (12), together with demand being sufficiently strong, ensures that the substitutability of the two products increases as  $\theta$  increases, which explains sufficient condition (ii).

The standard discrete choice oligopoly theory of product differentiation, pioneered by Perloff and Salop (1985), typically assumes independence between values for different varieties. We contribute to this literature by showing that preference depen-

dence is another useful measure of product differentiation. In fact, while profits are U-shaped in  $\sigma$  when  $\mu > 0$ , profits always monotonically decreases in  $\theta$  under multi-product monopoly, and profits also monotonically decrease in  $\theta$  under duopoly for all  $\mu$  when  $f$  satisfies some fairly mild restrictions.

Proposition 5 suggests that two competing single-product firms have a mutual incentive to design or promote their products so that consumer values for them are less positively dependent or more negatively dependent. We return to this issue when later considering the characteristics approach to product selection.

The effect of more preference dependence on consumer welfare appears to be generally ambiguous, with a higher  $\theta$  lowering both consumer demand and equilibrium price. In the bivariate exponential special case, however,  $w^d$  increases in  $\theta$ .

### Comparing Prices across Market Structures

We now compare the pricing behaviors of firms across market structures, and show how these comparisons might relate to preference dependence.

We start with comparing the profit-maximizing price under single-product monopoly with the equilibrium price under duopoly. While Chen and Riordan (2008) finds a sufficient condition for  $p^m \gtrless p^d$  when the marginal distribution is exponential (i.e.  $\lambda'(\cdot) = 0$ ), it has been an open question how the prices compare for arbitrary marginal distributions, which we can now answer with the following result:

**Proposition 6** *Given A1 and A3: if  $C_{11} < 0$  (positive dependence) and  $\lambda'(p) \geq 0$ , then  $p^m > p^d$ ; and if  $C_{11} > 0$  (negative dependence) and  $\lambda'(p) \leq 0$ , then  $p^m < p^d$ .*

**Proof.** If  $\lambda'(p) \geq 0$ , then, from (23) and  $u(x) = F^{-1}(x)$ , we have, for any  $\bar{p}$ :

$$\begin{aligned}
 h(\bar{p}) &= \frac{2C_1(F(\bar{p}), F(\bar{p})) f(\bar{p})}{1 - C(F(\bar{p}), F(\bar{p}))} + \frac{2 \int_0^1 (1-x) c(x, x) \frac{f(u(x))}{1-F(u(x))} dx}{1 - C(F(\bar{p}), F(\bar{p}))} \\
 &\geq \frac{2C_1(F(\bar{p}), F(\bar{p})) f(\bar{p})}{1 - C(F(\bar{p}), F(\bar{p}))} + \frac{f(\bar{p})}{1 - F(\bar{p})} \frac{2 \int_0^1 (1-x) c(x, x) dx}{1 - C(F(\bar{p}), F(\bar{p}))}. \quad (27)
 \end{aligned}$$

Since  $\lambda(\bar{p}) \equiv \frac{f(\bar{p})}{1-F(\bar{p})}$  and

$$\begin{aligned}
& \int_{F(\bar{p})}^1 (1-x) c(x, x) dx = \int_{F(\bar{p})}^1 (1-x) \left[ \frac{dC_1(x, x)}{dx} - C_{11}(x, x) \right] dx \\
&= (1-x) C_1(x, x) \Big|_{F(\bar{p})}^1 + \int_{F(\bar{p})}^1 C_1(x, x) dx - \int_{F(\bar{p})}^1 (1-x) C_{11}(x, x) dx \\
&= -[1-F(\bar{p})] C_1(F(\bar{p}), F(\bar{p})) + \frac{1-C(F(\bar{p}), F(\bar{p}))}{2} - \int_{F(\bar{p})}^1 (1-x) C_{11}(x, x) dx
\end{aligned}$$

(27) becomes

$$\begin{aligned}
h(\bar{p}) &\geq \frac{2C_1(F(\bar{p}), F(\bar{p})) f(\bar{p})}{1-C(F(\bar{p}), F(\bar{p}))} + 2 \frac{f(\bar{p})}{1-F(\bar{p})} \\
&\quad - \frac{[1-F(\bar{p})] C_1(F(\bar{p}), F(\bar{p})) + \frac{1-C(F(\bar{p}), F(\bar{p}))}{2} - \int_{F(\bar{p})}^1 (1-x) C_{11}(x, x) dx}{1-C(F(\bar{p}), F(\bar{p}))} \\
&= \lambda(\bar{p}) \left[ 1 - \frac{2 \int_{F(\bar{p})}^1 (1-x) C_{11}(x, x) dx}{1-C(F(\bar{p}), F(\bar{p}))} \right] > \lambda(\bar{p}) \text{ if } C_{11}(x, x) < 0. \quad (29)
\end{aligned}$$

Thus, from (15) and (22), using A1 and A3, we have  $\bar{p}^m > \bar{p}^d$  and hence  $p^m > p^d$  if  $\lambda'(\cdot) \geq 0$  and  $C_{11} < 0$  (positive dependence).

Similarly, if  $\lambda'(\cdot) \leq 0$  and  $C_{11} > 0$  (negative dependence), inequalities in (27) and (29) will be reversed, which proves  $h(\bar{p}) < \lambda(\bar{p})$ , implying that  $\bar{p}^m < \bar{p}^d$  and hence  $p^m < p^d$ . ■

Thus positive dependence and a non-decreasing hazard rate for the marginal distribu-

tion ensures that duopoly competition lowers prices; conversely, negative dependence and a non-increasing hazard rate ensures that competition raises prices.<sup>16</sup>

Next, we compare the price under the multiproduct monopoly with those under single-product monopoly and under symmetric duopoly.

**Proposition 7** *Given A1-A3: (i)  $\bar{p}^d < \bar{p}^{mm}$ . (ii)  $p^{mm} \begin{smallmatrix} \geq \\ \leq \end{smallmatrix} p^m$  if  $\forall \bar{p} > u(0)$ :*

$$\int_{F(\bar{p})}^1 (1-x) [C_{11}(x, x) + c(x, x)] dx \begin{smallmatrix} \geq \\ \leq \end{smallmatrix} 0. \quad (30)$$

*In particular, (iii)  $p^{mm} > p^m$  if  $-C_{11}(x, x) < c(x, x)$  for  $x \in (0, 1)$  (i.e., if  $C$  is not too positively dependent); and  $p^{mm} < p^m$  if  $-C_{11}(x, x) > c(x, x)$  for  $x \in (0, 1)$ .*

**Proof.** (i) Since  $h(\bar{p}) > \lambda^C(\bar{p})$  from (23), comparing (19) and (22) leads to  $\bar{p}^d < \bar{p}^{mm}$ .

(ii) Since, from (28),

$$\begin{aligned} & \int_{F(\bar{p})}^1 (1-x) c(x, x) dx \\ &= -[1 - F(\bar{p})] C_1(F(\bar{p}), F(\bar{p})) + \frac{1 - C(F(\bar{p}), F(\bar{p}))}{2} - \int_{F(\bar{p})}^1 (1-x) C_{11}(x, x) dx, \\ \frac{\lambda^C(\bar{p})}{\lambda(\bar{p})} &= \frac{\frac{2C_1(F(\bar{p}), F(\bar{p}))f(\bar{p})}{1 - C(F(\bar{p}), F(\bar{p}))}}{\frac{f(\bar{p})}{1 - F(\bar{p})}} = \frac{2[1 - F(\bar{p})] C_1(F(\bar{p}), F(\bar{p}))}{1 - C(F(\bar{p}), F(\bar{p}))} \\ &= \frac{[1 - C(F(\bar{p}), F(\bar{p}))] - 2 \int_{F(\bar{p})}^1 (1-x) C_{11}(x, x) dx - 2 \int_{F(\bar{p})}^1 (1-x) c(x, x) dx}{1 - C(F(\bar{p}), F(\bar{p}))} \end{aligned}$$

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<sup>16</sup>Chen and Riordan (2007) and Perloff, Suslow, and Sequin (1995) present more specific models in which entry can result in higher prices.

$$= 1 - \frac{2 \int_{F(\bar{p})}^1 (1-x) [C_{11}(x,x) + c(x,x)] dx}{1 - C(F(\bar{p}), F(\bar{p}))} \stackrel{\leq}{\geq} 1 \text{ iff (30) holds.}$$

The conclusion follows from comparing (15) and (19) under assumptions A1 and A2.

(iii) These are the sufficient conditions for (30). ■

Therefore,  $p^{mm} > p^m$ , provided that consumer values for two products are not too positively dependent. If two products were perfectly positively dependent, we would have  $p^{mm} = p^m$ , which suggests that some limit on the degree of positive dependence might be needed for  $p^{mm} > p^m$ . While the sufficient condition for  $p^{mm} < p^m$  is a theoretical possibility, we have not found an example for which it holds.

Interestingly, similar considerations are at work in comparing  $p^m$  with  $p^d$  and comparing  $p^m$  with  $p^{mm}$ . Both comparisons are not obvious, because the introduction of a second product can have two opposite effects on price incentives for a duopolist or for a multiproduct monopolist. The market share effect is that a price increase applies to a smaller customer base for a product when multiple products are produced, which motivates a firm to lower price. The price sensitivity effect is that the multiproduct firm loses fewer customers from a price increase for one of the products (a duopolist might also lose fewer customers if the introduction of a second product steepens the firm's residual demand curve), which motivates the firm to raise price.<sup>17</sup> Proposition 6 gives sufficient conditions under which  $p^m > p^d$  and vice versa, revealing the critical roles played by preference dependence (together with the hazard rate of the marginal distribution). On the other hand, the sufficient condition comparing  $p^m$  and  $p^{mm}$  in Proposition 7 is entirely determined by the dependence property of the copula.<sup>18</sup>

Although preference dependence and the number of firms are rather different economic concepts, our analysis reveals a common theme between their effects on equilibrium prices. Both more preference dependence and more firms are in a sense increasing competition. Each has a market share effect—lower output—that favors lower prices, but each may also have a price sensitivity effect—potentially steepening

<sup>17</sup>See Chen and Riordan (2008) for a more general discussion of these two effects in comparing a symmetric duopoly with a single-product monopoly.

<sup>18</sup>The comparisons of  $p^d$  and  $p^{mm}$  is more obvious and reflects the familiar intuition that a multiproduct internalizes the externalities between products (so the businee stealing effect arises under duopoly but not under monopoly).

the residual demand curve—that favors higher prices. Propositions 6 and 5 give the respective sufficient conditions that the net effect is to lower prices.

### Numerical analysis

We now use the FGM-exponential case (so  $f(\cdot)$  is loglinear and  $\lambda(p)$  is a constant) to further illustrate the patterns of comparative statics and the comparisons of conduct and performance across markets.

**[Insert Figure 1 about here]**

Figure 1 shows how normalized price ( $\bar{p}$ ) vary with preferences and across markets for the FGM-exponential case. There are three panels, corresponding to negative dependence ( $\theta = -1$ ), independence ( $\theta = 0$ ), and positive dependence ( $\theta = 1$ ). The horizontal axis in each graph is measured with respect to  $q^m$ , the profit-maximizing single-product monopoly market share. From (15) and **A1**, there is a positive monotonic relationship between  $q^m$  and  $\bar{\mu}$ ; consequently, the graphs effectively describe how industry outcomes vary with normalized mean demand  $\bar{\mu}$  over the relevant range.<sup>19</sup>

Confirming our analytical results, we observe: (1)  $\bar{p}$  decrease with  $\bar{\mu}$  under all three market structures. (2)  $\bar{p}^d$  decreases in  $\theta$ ; and while Proposition 3 predicts that  $p^{mm}$  decreases in  $\theta$  for  $\mu$  sufficiently large, a numerical analysis (not shown) goes further, finding that  $p^{mm}$  decreases in  $\theta$  for  $\bar{\mu}$  above a critical value but increases in  $\theta$  for  $\bar{\mu}$  below. (3)  $\bar{p}^{mm}$  is always the highest, whereas  $\bar{p}^m > \bar{p}^d$  for  $\theta = 1$  but  $\bar{p}^m < \bar{p}^d$  for  $\theta = -1$ .

**[Insert Figures 2 and 3 about here]**

Figure 2 and Figure 3 conduct the same exercises for normalized profit ( $\bar{\pi}$ ) and consumer welfare ( $\bar{w}$ ). Again confirming our analytical results, we observe: (1)  $\bar{\pi}$  and  $\bar{w}$  increase with  $\bar{\mu}$  under all three market structures.<sup>20</sup> (2)  $\bar{\pi}^{mm}$  and  $\bar{\pi}^d$  decrease as  $\theta$  increases. A numerical analysis (not shown) also confirms that the effects of  $\theta$

<sup>19</sup>In this case,  $\bar{p}^m = 1 - \bar{\mu}$  and  $q^m = e^{-(2-\bar{\mu})}$ .

<sup>20</sup>Since  $\bar{p}^d < \bar{p}^m$  when  $\theta = -1$ , the market is fully covered in duopoly when  $q^m < 1$  but sufficiently high. In this range,  $\bar{\pi}^d$  is flat.

on consumer welfare is ambiguous:  $\bar{w}^{mm}$  is decreasing in  $\theta$  except for large values of  $\bar{\mu}$ , in which case greater positive dependence can deliver more consumer welfare than independence because price is lower. (3) Not surprisingly,  $\bar{\pi}^{mm} > \bar{\pi}^m > \bar{\pi}^d$ . On the other hand, duopoly competition creates the most consumer welfare, but a multiproduct monopoly creates more consumer welfare than single-product monopoly except when demand is high.<sup>21</sup>

#### 4. MARKET STRUCTURE

Now we study how preferences affect equilibrium market structure (product selection), assuming that there is a fixed cost  $K$  of introducing a product.<sup>22</sup> Consider a two stage game in which, in stage one, two firms simultaneously decide whether or not to incur  $K$  for each of the two products, and, in stage two, firms simultaneously set prices for active products. Obviously, there is no pure strategy subgame perfect equilibrium in which both firms produce the same variety. Thus, depending on stage one choices, the possible market structures are single-product monopoly, multiproduct monopoly, and duopoly. Equilibrium prices and profits at the second stage are as discussed in Section 3.

Product choices at the first stage depend on profit comparisons at the second stage. Thus the following preliminary result is useful:

**Lemma 2**  $\pi^d < \pi^m < \pi^{mm} < 2\pi^m$  and  $2\pi^d \leq \pi^{mm}$ .

**Proof.** The upper bound in (1) implies

$$\pi^m = \max_{\bar{p}} \sigma(\bar{p} + \bar{\mu}) [1 - F(\bar{p})] < \max_{\bar{p}} \sigma(\bar{p} + \bar{\mu}) [1 - C(F(\bar{p}), F(\bar{p}))] = \pi^{mm}.$$

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<sup>21</sup>This last qualification is because the higher price of a multiproduct monopoly inefficiently reduces the quantity demand. Even then, however, total welfare clearly is higher under multiproduct monopoly because the widening profit gap offsets the consumer surplus loss.

<sup>22</sup>The analysis here follows closely Shaked and Sutton (1990), which we shall discuss in detail shortly.

The lower bound in (1) implies

$$\begin{aligned}\pi^{mm} &= \max_{\bar{p}} \sigma(\bar{p} + \bar{\mu}) [1 - C(F(\bar{p}), F(\bar{p}))] < \max_{\bar{p}} \sigma(\bar{p} + \bar{\mu}) [1 - (2F(\bar{p}) - 1)] \\ &= 2 \max_{\bar{p}} \sigma(\bar{p} + \bar{\mu}) [1 - F(\bar{p})] = 2\pi^m.\end{aligned}$$

Finally,

$$\pi^d = \frac{1}{2} \sigma(\bar{p}^d + \bar{\mu}) [1 - C(F(\bar{p}^d), F(\bar{p}^d))] \leq \frac{1}{2} \pi^{mm} < \pi^m.$$

■

Shaked and Sutton (1990) insightfully characterized how equilibrium product selection depends on an "expansion effect", measured by  $\frac{\bar{\pi}^{mm} - \bar{\pi}^m}{\bar{\pi}^m}$ , and a "competition effect", measured by  $\frac{\bar{\pi}^{mm} - 2\bar{\pi}^d}{2\bar{\pi}^d}$ . Figure 4 adapts a diagram in Shaked and Sutton (1990) to summarize how equilibrium market structure depends on these two effects, both of which are positive from the above lemma. The diagram assumes an intermediate level of fixed cost satisfying  $\frac{1}{2}\pi^m < K < \pi^m$ . Single-product monopoly is the unique equilibrium if the expansion effect is sufficiently weak given the competition effect. Further, both duopoly and multimarket monopoly are equilibria if the expansion effect is sufficiently strong. Otherwise, duopoly (multimarket monopoly) is the unique equilibrium, if the competition effect is weak (strong). Notice that with multiple equilibria, if **A2** and **A3** are satisfied, then  $p^d < p^{mm}$  from Proposition 4 and consumer welfare is higher under duopoly.

#### Insert Figure 4 about here

Shaked and Sutton (1990) observed that a decrease in  $K$  shifts down the two loci ( $\pi^d = K$  and  $\pi^{mm} - \pi^m = K$ ) which divide the four equilibrium regimes in Figure 4, and, from this, deduced how  $K$  determined equilibria outcomes. The next proposition restates their results, which follow from best responses at the product selection stage.

**Proposition 8** (Shaked and Sutton, 1990) *There exist non-empty sets of  $K \geq 0$  such that: (i) single-product monopoly is the unique equilibrium if and only if  $\max\{\pi^d, \pi^{mm} - \pi^m\} < K \leq \pi^m$ ; (ii) duopoly is an equilibrium if and only if  $K \leq \pi^d$ ; (iii) multiproduct monopoly is an equilibrium if and only if  $K \leq \pi^{mm} - \pi^m$ ; and (iv) both duopoly and multiproduct monopoly are equilibria if and only if  $K \leq$*

$$\min \{ \pi^d, \pi^{mm} - \pi^m \}.^{23}$$

Our framework enables an analysis of how the distribution of preferences determines equilibrium market structure, providing results that complement and extend Shaked and Sutton (1990). The next set of results consider the preference parameters of the marginal distribution; they are straightforward implication of Corollaries 1-6 and Proposition 8. The corollary below states how equilibrium market structures vary with demand strength, similarly as in Shaked and Sutton (1990).

**Corollary 7** *Assume **A1,A2**, and  $h'(\cdot) \geq 0$ . There exist an intermediate range of  $K$  and critical values  $\mu_1^d$ ,  $\mu_1^m$ , and  $\mu_1^{mm}$ , with  $\mu_1^m < \min\{\mu_1^d, \mu_1^{mm}\}$ , such that: (i) single-product monopoly is the unique equilibrium if and only if  $\min\{\mu_1^d, \mu_1^{mm}\} > \mu > \mu_1^m$ ; (ii) duopoly is an equilibrium if and only if  $\mu \geq \mu_1^d$ ; (iii) multiproduct monopoly is an equilibrium if and only if  $\mu \geq \mu_1^{mm}$ ; and (iv) both duopoly and multiproduct monopoly are equilibrium market structures if and only if  $\mu \geq \max\{\mu_1^d, \mu_1^{mm}\}$ , in which case consumer welfare is higher under duopoly.*

The next corollary further clarifies how preference diversity affects market structure for given  $\mu$ .

**Corollary 8** *Assume **A1,A2**, and  $h'(\cdot) \geq 0$ . (i) if  $\mu < 0$  or if  $\mu > 0$  and  $\sigma$  is large enough, then the range of  $K$  values supporting duopoly equilibrium and/or multiproduct monopoly equilibrium expands as  $\sigma$  increases; (ii) if  $\mu > 0$  but  $\sigma$  is small enough, then the range of  $K$  values supporting duopoly equilibrium and/or multiproduct monopoly equilibrium expands as  $\sigma$  decreases.*

Therefore, in markets with relatively low demand ( $\mu < 0$ ), multiple products are more likely to be offered (by multiple single-product firms or by multiproduct monopoly) when preference diversity is *higher*; but in markets with relatively higher demand ( $\mu > 0$ ), if  $\sigma$  is below some critical value, multiple products are more likely to be offered when preference diversity is *lower*.

The possibility of multiple equilibria suggests that multiproduct monopoly might foreclose a duopoly that delivers lower prices and greater consumer welfare. Put

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<sup>23</sup>All the relations in this proposition can be equivalently expressed as relations between  $\bar{K}$  and  $\bar{\pi}^m, \bar{\pi}^{mm}, \bar{\pi}^d$ , if we normalize  $K$  to  $\bar{K} \equiv K/\sigma$ .

another way, if  $K \leq \min \{ \pi^d, \pi^{mm} - \pi^m \}$ , then a policy that prohibited a monopolist from offering multiple varieties would destroy an unwanted multiproduct monopoly equilibrium and allow a superior duopoly equilibrium.<sup>24</sup> Some care must be taken with this policy prescription. For if  $\pi^d < K \leq \pi^{mm} - \pi^m$ , then a prohibition of multimarket equilibrium would result only in an inferior single-product monopoly. Therefore, it is for important policy reasons to know when  $\pi^{mm} - \pi^m < \pi^d$ , a sufficient condition of which is given in the result below, which strengthens the equilibrium characterization:

**Proposition 9** *Assume **A1, A2**, and  $h'(\cdot) \geq 0$ . If  $\bar{\mu}$  is sufficiently large, then  $\pi^{mm} - \pi^m < \pi^d$ , and: (i) single-product monopoly is the unique equilibrium if and only if  $\pi^d < K < \pi^m$ ; (ii) duopoly is the unique equilibrium if and only if  $\pi^{mm} - \pi^m < K < \pi^d$ ; (iii) both duopoly and multiproduct monopoly are equilibria if and only if  $K \leq \pi^{mm} - \pi^m$ .*

**Proof.** If  $\bar{\mu}$  is sufficiently large,  $\bar{p}^{mm}$  can be made arbitrarily close to  $u(0)$  and hence

$$\begin{aligned} \pi^{mm} - \pi^m &= \sigma \{ (\bar{p}^{mm} + \bar{\mu}) [1 - C(F(\bar{p}^{mm}), F(\bar{p}^{mm}))] - (\bar{p}^m + \bar{\mu}) [1 - F(\bar{p}^m)] \} \\ &\leq \sigma (\bar{p}^{mm} + \bar{\mu}) [F(\bar{p}^{mm}) - C(F(\bar{p}^{mm}), F(\bar{p}^{mm}))] \rightarrow 0; \end{aligned}$$

but  $\pi^d = \frac{1}{2}\sigma (\bar{p}^d + \bar{\mu}) [1 - C(F(\bar{p}^d), F(\bar{p}^d))]$  is bounded above zero as  $\bar{p}^{mm} \rightarrow u(0)$ . Thus  $\pi^{mm} - \pi^m < \pi^d$  if  $\bar{\mu}$  is sufficiently large. (i)-(iii) then follow from Proposition 8. ■

Thus, when  $K$  is large enough so that  $\bar{\mu}$  is also sufficiently large (which is needed to ensure  $\pi^m \geq K$ ), as  $\bar{\mu}$  increases, duopoly emerges as equilibrium before multiproduct monopoly does.

We further consider how preference dependence affects the equilibrium product selection, holding the marginal distribution constant. The result below follows straightforwardly from Propositions 3, 5, and 8:

**Corollary 9** *Given **A2, A3**, and **MDR**: (i) the range of  $K$  values supporting multiproduct monopoly equilibrium contracts as  $\theta$  increases; (ii) if  $f(x)$  does not decrease*

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<sup>24</sup>This conclusion is reminiscent of Schmalensee (1978)'s argument that product proliferation by a monopolist can deter entry.

*too fast and is not too logconcave, or if  $\bar{\mu}$  is sufficiently large and condition (12) holds, then, as  $\theta$  increases the range of  $K$  values supporting duopoly equilibrium also contracts, but the range of  $K$  values supporting single-product monopoly as the unique equilibrium expands.*

Therefore, with certain qualifications, multiple products are more likely to be available in markets when their values are more negatively dependent (or less positively dependent).

Johnson and Myatt (2006) has suggested that there are empirically observable ways with which firms can influence consumers' preference diversity. Their logic may be extended to preference dependence, and the business strategies that they suggest may also affect consumer preference dependence. By providing a complete characterization of how market outcomes depend on consumer preference, in terms of mean value for each variety, preference diversity, and preference dependence, our theory of equilibrium product selection connects Shaked and Sutton (1990) with Johnson and Myatt (2006), extending their insights and offering new theoretical predictions. Our analysis in Section 3 suggests that a single-product monopolist has an incentive (i) to increase  $\mu$ ; and, (ii) to increase  $\sigma$  if  $\mu \leq 0$  or if  $\mu > 0$  and  $\sigma$  is large enough, but to reduce  $\sigma$  if  $\mu > 0$  and  $\sigma$  is small enough. With endogenous product selection, however, a higher  $\bar{\mu}$  may switch the equilibrium from single-product monopoly to duopoly equilibrium, which reduces the firm's profit.<sup>25</sup> Thus the firm may seek neither to maximize or minimize  $\sigma$  when an extreme-seeking strategy would change equilibrium from monopoly to duopoly.

Figure 5 continues our analysis of the bivariate exponential special case with graphs that indicate certain critical normalized fixed costs ( $\bar{K} = K/\sigma$ ) for product selection. As before, graphs for negative and positive dependence correspond to the extremes of the FGM family,  $\theta = -1$  and  $\theta = 1$ . If  $\bar{K} \leq \bar{\pi}^d$ , then duopoly is an equilibrium. If  $\bar{K} \leq \bar{\pi}^{mm} - \bar{\pi}^m$ , then multiproduct monopoly is also an equilibrium. If  $\bar{\pi}^m \geq \bar{K} > \bar{\pi}^d$ , then single-product monopoly is a unique equilibrium; and if  $\bar{\pi}^d \geq \bar{K} > \bar{\pi}^{mm} - \bar{\pi}^m$ , then duopoly is a unique equilibrium. Finally, for  $\bar{\pi}^{mm} - \bar{\pi}^m \geq \bar{K}$ , duopoly and multiproduct monopoly coexist as equilibria, and prices are lower and consumers are better off with duopoly, as shown in Figure 1.

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<sup>25</sup>This assumes that another firm will be able to produce a symmetric product with the same  $\bar{\mu}$ .

## Insert Figure 5 about here

The single-product monopoly share is an increasing function of normalized demand strength ( $\bar{\mu}$ ). Therefore reading the diagrams in Figure 5 from left to right indicates how increasing demand strength changes the structure of equilibrium for the bivariate exponential case. When  $\bar{K}$  is low, equilibrium market structure shifts from no production, to single-product monopoly, to duopoly, to multiple equilibria as demand strength increases. Thus duopoly equilibrium always emerges before the multiproduct monopoly equilibrium as  $\bar{\mu}$  increases. For intermediate  $\bar{K}$ , there is a non-monotonicity; when demand strength becomes sufficiently great, multiple equilibria give way to a unique duopoly equilibrium. For higher  $\bar{K}$ , the multiproduct monopoly becomes impossible for any relevant  $\bar{\mu}$ , and for even higher  $\bar{K}$  so does the duopoly equilibrium. These conclusions hold over the entire range of preference dependence for the bivariate exponential case.<sup>26</sup> Furthermore, comparing the three panels, positive (negative) dependence decreases (increases) the range of  $\bar{K}$  supporting multiproduct market structures.

## 5. BUNDLING

We now consider an extended model in which a firm can offer a high-quality product that combines the features of the two product varieties discussed in the previous sections. We suppose that the two varieties are distinguished by product characteristics X and Y, and interpret  $w(x)$  and  $w(y)$  as a consumer's willingness to pay for these characteristics.<sup>27</sup> We interpret the outside good as a competitively-supplied basic product providing consumers a net value of  $\alpha$ , and interpret varieties X and Y as enhanced goods providing values  $\alpha + w(x)$  and  $\alpha + w(y)$  respectively. A high-quality product that bundles both characteristics provides value  $\alpha + w(x) + w(y)$ .<sup>28</sup>

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<sup>26</sup>Shaked and Sutton (1990) showed in a linear-demand representative consumer model similarly that a multimarket equilibrium is never a unique equilibrium.

<sup>27</sup>The characteristics approach to demand goes back to Lancaster (1971). Our formulation here follows more directly Berry and Pakes (2007).

<sup>28</sup>For example, the basic good is a computer with X and Y being two application software; the basic good is cable or Satelight TV subscription with X and Y being two different sets of programs; and the basic good is a vacation trip with X and Y being different sets of activities that can be included in the package.

We maintain the assumption that each consumer buys only one of these products,<sup>29</sup> and normalize constant average variable costs to zero.<sup>30</sup> For convenience, we further normalize  $\alpha = 0$ .

There are two additional relevant market structures in this extended model: a monopolist offering a high-quality product that bundles both characteristics, and a vertically differentiated market where one firm offers the high-quality product while another firm offers a low-quality product with only one characteristic.<sup>31</sup> We shall focus on the analysis of monopoly bundling, but will also consider firm profits in the vertically differentiated market when discussing equilibrium product selection.

### Comparative Statics under Monopoly Bundling

For a monopolist who only offers the bundle at price  $p$ , a consumer will purchase if and only if  $w(x) + w(y) \geq p$ , or  $F^{-1}(x) + F^{-1}(y) \geq \frac{p-2\mu}{\sigma}$ . Thus, letting  $\tilde{p} \equiv \frac{p-2\mu}{\sigma}$ , we obtain the firm's interior demand as:

$$q^B(\tilde{p}) = \begin{cases} 1 - \int_0^{F(\tilde{p}-u(0))} \int_0^{F(\tilde{p}-u(x))} c(x, y) dy dx & \text{if } \tilde{p} \leq u(1) + u(0) \\ \int_{F(\tilde{p}-u(1))}^1 \int_{F(\tilde{p}-u(x))}^1 c(x, y) dy dx & \text{if } \tilde{p} > u(1) + u(0) \end{cases} \quad (31)$$

Notice that  $q^B(\tilde{p})$  is continuous at  $\tilde{p} = u(1) + u(0)$ . The monopolist's profit from the bundle is  $\pi^B(\tilde{p}) = \sigma(\tilde{p} + 2\bar{\mu})q^B(\tilde{p})$ . The optimal monopoly price  $\tilde{p}^B$ , which is assumed to be interior, satisfies

$$(\tilde{p} + 2\bar{\mu})h^B(\tilde{p}) = 1, \quad (32)$$

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<sup>29</sup>Suppose that consumers value the basic good at  $\alpha + b$ , which has a competitive price  $b$ . Then surplus from purchasing both X and Y at prices  $p$  and  $r$  is  $\alpha + b + w(x) + w(y) - p - r$ . With  $p \geq b$  and  $r \geq b$ , the consumer surplus from purchasing both varieties is bounded above by  $\alpha + w(1) + w(1) - b$ , which is negative if  $b$  is sufficiently large.

<sup>30</sup>Put another way,  $\alpha$  is the common consumer value of the outside good net of its constant average variable cost, and  $p$  and  $r$  are the prices of the characteristics net of their constant average variable cost.

<sup>31</sup>We consider only pure bundling. Mixed bundling can be ruled out if the fixed cost to offer a single-characteristic good is high enough.

where

$$h^B(\tilde{p}) \equiv \frac{-q^{B'}(\tilde{p})}{q^B(\tilde{p})} = \frac{\varepsilon(\tilde{p})}{\tilde{p}} > 0 \quad (33)$$

plays a similar role as hazard rate  $\lambda(\bar{p})$  under the single-product monopoly, with  $\varepsilon(\tilde{p})$  being the price elasticity of demand.

The following regularity condition ensures unique comparative statics:

**A4.**  $\frac{d[(\tilde{p}+2\bar{\mu})h^B(\tilde{p})]}{d\tilde{p}} > 0.$

A sufficient condition for A4 is that  $h^B(\tilde{p})$  increases in  $\tilde{p}$ . Notice that since  $q^B(\tilde{p})$  decreases in  $\tilde{p}$ ,  $h^B(\tilde{p})$  increases in  $\tilde{p}$  if  $q^B(\tilde{p})$  is not too convex, similar to the requirement under single-product monopoly that  $f(\cdot)$  does not decrease too fast so that  $\lambda(\cdot)$  increases. At the optimal bundle price, the firm's profit is  $\pi^B \equiv \sigma\bar{\pi}^B \equiv \pi^B(\tilde{p}^B)$ , and consumer welfare is

$$w^B \equiv \sigma\bar{w}^B = \sigma \int_{\tilde{p}^B}^{2u(1)} q^B(\tilde{p}) d\tilde{p}. \quad (34)$$

Extending how  $\bar{\mu}$  affects market outcomes under single-product monopoly and noticing from (31) that  $\frac{\partial q^B(\tilde{p})}{\partial \bar{\mu}} > 0$ , we obtain:

**Proposition 10** *Given A4: (i)  $\frac{d\tilde{p}^B}{d\bar{\mu}} < 0$ , and there exists some  $\bar{\mu}^B > 0$  such that  $\tilde{p}^B \gtrless 0$  if  $\bar{\mu} \lesseqgtr \bar{\mu}^B$ ; (ii)  $\frac{\bar{\pi}^B}{d\bar{\mu}} > 0$ ; (iii)  $\frac{d\bar{w}^B}{d\bar{\mu}} > 0$ .*

Similar to the single-product case, comparative statics with respect to  $\mu$  and  $\sigma$  are straightforward corollaries, and the comparative statics for  $\sigma$  extends some of the basic results in Johnson and Myatt (2003) to a monopoly offering a bundle.

**Corollary 10** *Given A4: for any given  $\sigma$ : (i)  $\frac{dp^B}{d\mu} \gtrless 0$  if  $h^{B'}(p) \gtrless 0$ ; (ii)  $\frac{d\pi^B}{d\mu} > 0$ ; (iii)  $\frac{dw^B}{d\mu} > 0$ .*

**Corollary 11** *Given A4: (i)  $\frac{dp^B}{d\sigma} > 0$  if  $\sigma$  is sufficiently large for any given  $\mu$ , if  $\mu \leq 0$  and  $h^{B'}(\cdot) \geq 0$ , or if  $\mu \geq 0$  and  $h^{B'}(\cdot) \leq 0$ . (ii)  $\frac{d\pi^B}{d\sigma} > 0$  when  $\mu \leq 0$ ; and  $\pi^B$  first decreases and then increases in  $\sigma$  when  $\mu > 0$ . (iii)  $\frac{dw^B}{d\sigma} > 0$  either when  $\mu \leq 0$  or when  $\sigma$  is sufficiently large for given  $\mu > 0$ .*

We can also gain some insights on how preference dependence affects demand and profits under monopoly bundling. In the copula unit square on which  $C(x, y)$  is

defined, the area below the  $u(x) + u(y) = \tilde{p}$  curve contains the square formed by  $x = F(\tilde{p}/2) = y$ , which increases with  $\theta$  by **MDR**. The probability mass in the area below the  $u(x) + u(y) = \tilde{p}$  curve is clearly higher when  $C$  is close to being perfectly dependent than when  $C$  is independent. Thus  $q^B(\tilde{p})$  and  $\pi^B$  tend to be lower when preferences are highly dependent than when they are independent. More generally, if  $\bar{\mu}$  is relatively high (so that  $\tilde{p}^B$  is much lower than  $u(1) + u(0)$ ), more positive dependence tends to increase the probability mass *below* the curve defined by  $u(x) + u(y) = \tilde{p}$  in the copula unit square, lowering  $q^B(\tilde{p})$  and  $\pi^B$ . For our bivariate exponential example, numerical analysis indicates that  $\pi^B$  decreases in  $\theta$ .

### Product Selection

We further consider the issue of endogenous market structure, in a two stage game similarly as in Section 4. Notice that a high-quality firm will incur fixed cost  $2K$  for producing the bundled product with two characteristics. It is useful to compare  $\pi^B$  with profits under other market structures. Since  $\pi^{mm} > \pi^m > \pi^d$ , we focus on finding the sufficient conditions for  $\pi^B > \pi^{mm}$  or  $\pi^B < \pi^d$ :

**Proposition 11**  $\pi^B > \pi^{mm}$  if  $\mu \geq 0$  or if  $C$  is sufficiently positively dependent;  $\pi^B < \pi^d$  if  $\mu (< 0)$  is sufficiently small and  $C$  is sufficiently negatively dependent.

**Proof.** First, for any given  $p > 0$  such that  $q^{mm}(\bar{p}) \geq 0$ , we have  $q^B(\bar{p}) > q^{mm}(\bar{p})$  because the curve  $u(x) + u(y) = F(\bar{p})$  in the copula unit square is inside the square formed by  $x = F(\bar{p}) = y$ . Thus, if  $\bar{\mu} \geq 0$ , since  $\tilde{p} = \bar{p} - \bar{\mu}$ , we have

$$\pi^B = \pi^B(\tilde{p}^B) \geq \pi^B(\bar{p}^{mm}) > \pi^{mm}(\bar{p}^{mm}) = \pi^{mm}.$$

Next, if  $C(\cdot, \cdot; \theta) = \min\{x, y\}$ , we would have  $\Pr(X = Y) = 1$ . Hence, for any given  $p > 0$  such that  $q^{mm}(\bar{p}) \geq 0$ ,

$$q^B(\tilde{p}) = q^B\left(\frac{p}{\sigma} - 2\bar{\mu}\right) > q^B(2\bar{p}) \rightarrow q^{mm}(\bar{p}) \text{ as } C(x, y) \rightarrow \min\{x, y\}.$$

It follows that  $\pi^B > \pi^{mm}$  if  $C$  is sufficiently positively dependent.

Finally, if  $C(\cdot, \cdot; \theta) = \max\{0, x + y - 1\}$ , we would have  $\Pr(X + Y = 1) = 1$ . Hence, if  $C$  is sufficiently negatively dependent, we would have  $x + y \rightarrow 1$  in

probability. Hence, if in addition  $\mu < 0$  is sufficiently small, for almost all  $x$  and  $y$  that satisfy the dependence relationship, we would have  $w(x) + w(y) \leq 0$ , or  $u(x) + u(y) \leq \tilde{p}$  even if  $p = 0$ , implying that  $\pi^B \rightarrow 0 < \pi^d$ , where  $\pi^d$  is bounded above 0 when  $C$  is sufficiently negatively dependent. ■

Thus, pure bundling does poorly when  $\mu$  is low and preferences are highly negatively dependent. In our bivariate exponential example, pure bundling does poorly when  $\mu$  is low even if preferences are independent.

Let  $\pi^H$  and  $\pi^L$  denote respectively the equilibrium profits of the high-quality firm and the low-quality firm under vertical differentiation; their normalized values are respectively  $\bar{\pi}^H = \pi^H/\sigma$  and  $\bar{\pi}^L = \pi^L/\sigma$ . Since the presence of the low-quality product reduces demand for the high-quality product (the bundle), we have  $\pi^H < \pi^B$ . Also, since the presence of the bundle reduces the demand for a single product, we have  $\pi^L < \pi^m$ . Moreover, it can be shown that  $\pi^d < \pi^H$ ; that is, if adding a second characteristic has no additional cost, then a firm in a horizontally differentiated market with a single-characteristic product always has the incentive to add the second characteristic.

### Insert Figure 6 about here

Figure 6 contains three panels drawn for the independent bivariate exponential case for normalized values. Panel (a) graphs critical fixed costs. When  $\bar{\mu}$  is high enough, the curves can be ranked as follows: The highest curve, labelled  $\bar{\pi}^B - \bar{\pi}^m$ , is the difference in profit between a high-quality monopolist, who bundles both characteristics, and a low-quality monopolist who offers only one characteristic. The next three solid lines, labeled  $\bar{\pi}^m$ ,  $\bar{\pi}^d$ , and  $\bar{\pi}^{mm} - \bar{\pi}^m$ , are respectively the profit of a monopolist with a single low-quality product, the profit of a horizontally differentiated low-quality duopolist, and the difference in profits between a multiproduct monopolist with two horizontally differentiated low-quality products and a monopolist with a single low-quality product. The higher unlabeled dashed line is  $\bar{\pi}^H - \bar{\pi}^d$ , the difference in profits between the high-quality firm under vertical differentiation and a horizontally differentiated low-quality duopolist; and the lower unlabeled dashed line is  $\bar{\pi}^L$ , the profit of a low-quality vertically differentiated duopolist. Thus, bundled monopoly is an equilibrium if  $\bar{\pi}^B - \bar{\pi}^m \geq \max\{\bar{\pi}^{mm} - \bar{\pi}^m, K/\sigma\}$ , while horizontal duopoly is an equilibrium if  $\bar{\pi}^d \geq K/\sigma$  and  $\bar{\pi}^H - \bar{\pi}^d < K/\sigma$ . Notice that for an intermediate range

of monopoly market share (i.e. intermediate values of  $\bar{\mu}$ ), both monopoly bundling and symmetric duopoly are equilibrium market structures. However, in some of this intermediate region, panel (b) shows that consumer welfare is higher under monopoly bundling than under horizontal duopoly, and panel (c) further shows that industry profit is higher under monopoly bundling than duopoly unless demand is low. Numerical analyses for cases of positive and negative dependence are similar. We thus have:

**Proposition 12** *In the bivariate exponential example, for  $\bar{\mu}$  sufficiently large there exist an intermediate range of  $K$  for which a high-quality monopoly and horizontally-differentiated low-quality duopoly are both equilibria, with both consumer welfare and industry profit being higher under the monopoly.*

Surprisingly, a horizontal duopoly can foreclose a more efficient monopoly producing a high-quality product that bundles characteristics. This possibility result contrasts with our earlier finding that a horizontal-multiproduct monopoly can foreclose a more efficient horizontal duopoly. Together, these two results illustrate the subtleties of sound antitrust enforcement of horizontal mergers when product selection is endogenous.

## 6. CONCLUSION

Using copulas to describe the distribution of consumer preferences is a convenient and intuitive approach to product differentiation in discrete choice models of consumer demand. The approach leads to several sets of conclusions about how preferences matter for industrial organization. First, with certain qualifications, prices, profits, and consumer welfare all increase in demand strength, and they also all increase in preference diversity if  $\mu \leq 0$  or if  $\sigma$  is sufficiently high; but profit first decreases and then increases in preference diversity if  $\mu > 0$ . These comparative statics are robust to varying degrees of preference dependence across monopoly and duopoly market structures, providing unifying principles in the economics of monopoly and duopoly. Second, under certain conditions greater dependence leads to more product differentiation in multiproduct industries, in the sense of lower prices and profits. Third, for an initially monopolized market, the entry of a competitor with a horizontally

differentiated product lowers market price if preferences are positively dependent and the hazard rate of the marginal distribution is non-decreasing, but raises price under negative dependence and a non-increasing hazard rate. Fourth, when market structures are endogenous, multiproduct industries are more likely to emerge as demand strength increases, as preference diversity increases if  $\mu \leq 0$  or if  $\sigma$  is high enough, and as preferences become less positively dependent. Fifth, when product characteristics are endogenous, it is possible that the exclusion or acquisition of a horizontally differentiated rival leads to a high-quality monopoly that improves consumer and social welfare.

For future research, it would be desirable to extend our analysis to oligopoly markets with arbitrary numbers of firms and product varieties. The copula approach could also provide a convenient framework to study product bundling in a general setting, allowing mixed bundling and general preference dependence relations.<sup>32</sup> Our framework can also be applied to other areas of microeconomics, such as the economics of search (e.g., Anderson and Renault 1999; Schultz and Stahl 1996) and the economic analysis of horizontal mergers. Furthermore, the copula approach and the rich set of predictions our analysis has generated, concerning how market characteristics affect prices, profits, consumer surplus, and market structures, might open up interesting new directions for empirical industrial organization research.

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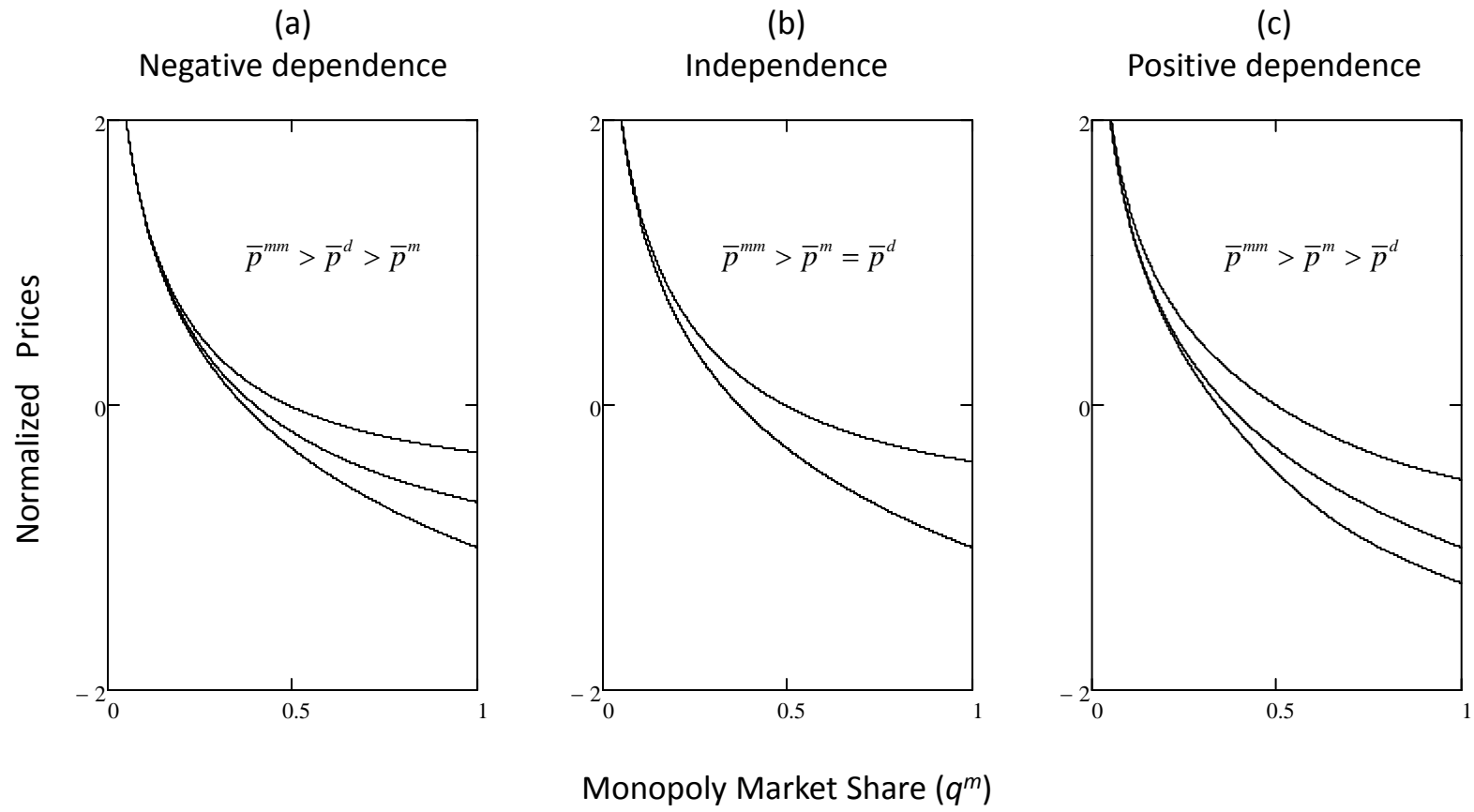
<sup>32</sup>It would be interesting to consider the situation where some consumers may want to purchase both products, as in McAfee, McMillian, and Whinston (1989), which also allows preference dependence. Product bundling when consumer values for two products are correlated have also been studied, for example, in Schmalensee (1984), Nalebuff (2004), and Armstrong and Vickers (2008).

## REFERENCES

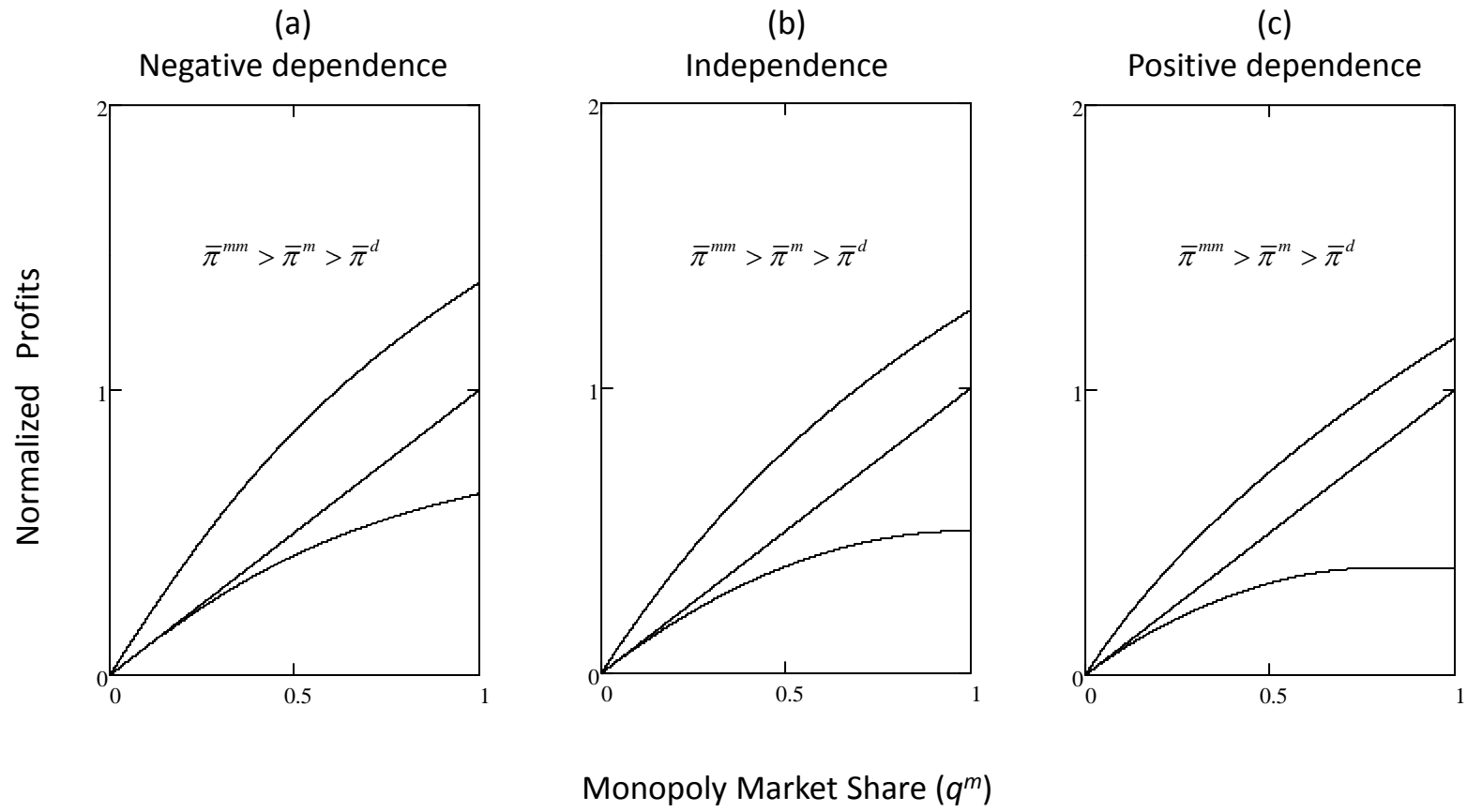
- Anderson, S.P., A. de Palma, and J-F Thisse. *Discrete Choice Theory of Product Differentiation*. Cambridge: The MIT Press, 1992.
- Armstrong, Mark and John Vickers, "Competitive Nonlinear Pricing and Bundling", 2008, working paper.
- Anderson, S.P. and R. Renault. "Pricing, Product Diversity, and Search Costs: A Bertrand-Chamberlin-Diamond Model." *RAND Journal of Economics* 1999, 30(4), 719-35.
- Armstrong, M. and J. Vickers, "Competitive Nonlinear Pricing and Bundling", 2008, working paper.
- Berry, S. and A. Pakes. "The Pure Characteristics Demand Model." *International Economics Review*, 2007, 48(4), Special Issue *Economics to Econometrics: Contributions in Honor of Daniel L. McFadden*, Charles F. Manski and Whitney Newey, eds., pp. 1193-1225.
- Chen, Y. and M.H. Riordan. "Price-increasing Competition." *RAND Journal of Economics*, 2008, 39(4), pp.1042-1058.
- . "Price and Variety in the Spokes Model." *Economic Journal*, 2007, 117 (522), 897-921.
- Joe, H. *Multivariate Models and Dependence Concepts*. London: Chapman and Hall, 1997.
- Johnson, J.P. and D.P. Myatt. "On the Simple Economics of Advertising, Marketing, and Product Design." *American Economic Review*, 2006, 96(3), pp. 756-784.
- Judd, K.L. "Credible Spatial Preemption." *RAND Journal of Economics*, 1985, 16(2), pp. 153-166.
- Lancaster, K.J. "A New Approach to Consumer Theory." *Journal of Political Economy*, 1966, 74(2), pp. 132-157.
- McAfee, R. P., J. McMillan, and M.D. Whinston. "Multiproduct Monopoly, Commodity Bundling, and Correlation of Values." *Quarterly Journal of Economics*, 1989, 104(2), pp. 371-383.

- Nelsen, R.B. *An Introduction to Copulas*. New York: Springer, 2006.
- Nalebuff, B. "Bundling as an Entry Barrier." *Quarterly Journal of Economics*, 2004, 119(1), pp. 159-87.
- Patton, A.J. "Copula-Based Models for Financial Time Series." Oxford Financial Center Economics Series, #2008fe21, Oxford University, 2008.
- Panzar, J. and R.D. Willig. "Economies of Scope." *American Economic Review*, 1981, 71, pp. 268-272.
- Perloff, J.M. and S.C. Salop. "Equilibrium with Product Differentiation." *Review of Economic Studies*, 1985, pp. 107-120.
- Perloff, J.M., V.Y. Suslow, and P.M. Seguin. "Higher Prices from Entry: Pricing of Brand-name Drugs." W.P. No. CPC-99-03, Competition Policy Center, University of California at Berkeley, 1995.
- Schmalensee, R. "Entry Deterrence in the Ready-to-Eat Breakfast Cereal Industry." *Bell Journal of Economics*, 1978, 9, pp. 305-327.
- Schmalensee, R. "Gaussian Demand and Commodity Bundling." *Journal of Business*, 1984, 57, pp. 58-73.
- Schultz, N. and K. Stahl. "Do Consumers Search for Higher Price? Oligopoly Equilibrium and Monopoly Optimum in Differentiated-Products Markets." *RAND Journal of Economics*, 1996, 27(3), pp. 542-62.
- Shaked, A. and J. Sutton, "Multiproduct Firms and Market Structure." *RAND Journal of Economics*, 1990, 21(1), pp. 45-62.

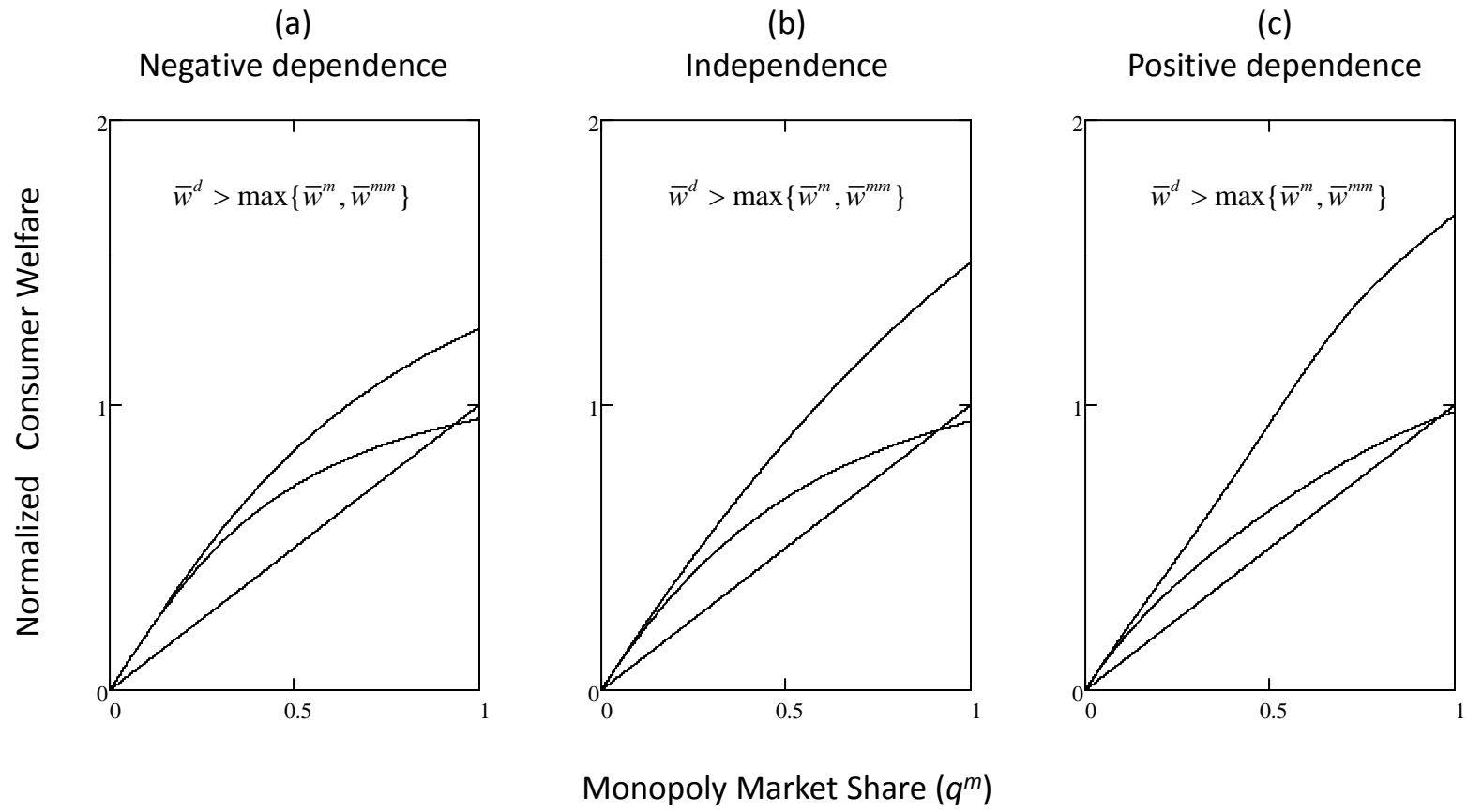
**Figure 1**  
Prices in the Bivariate Exponential Case



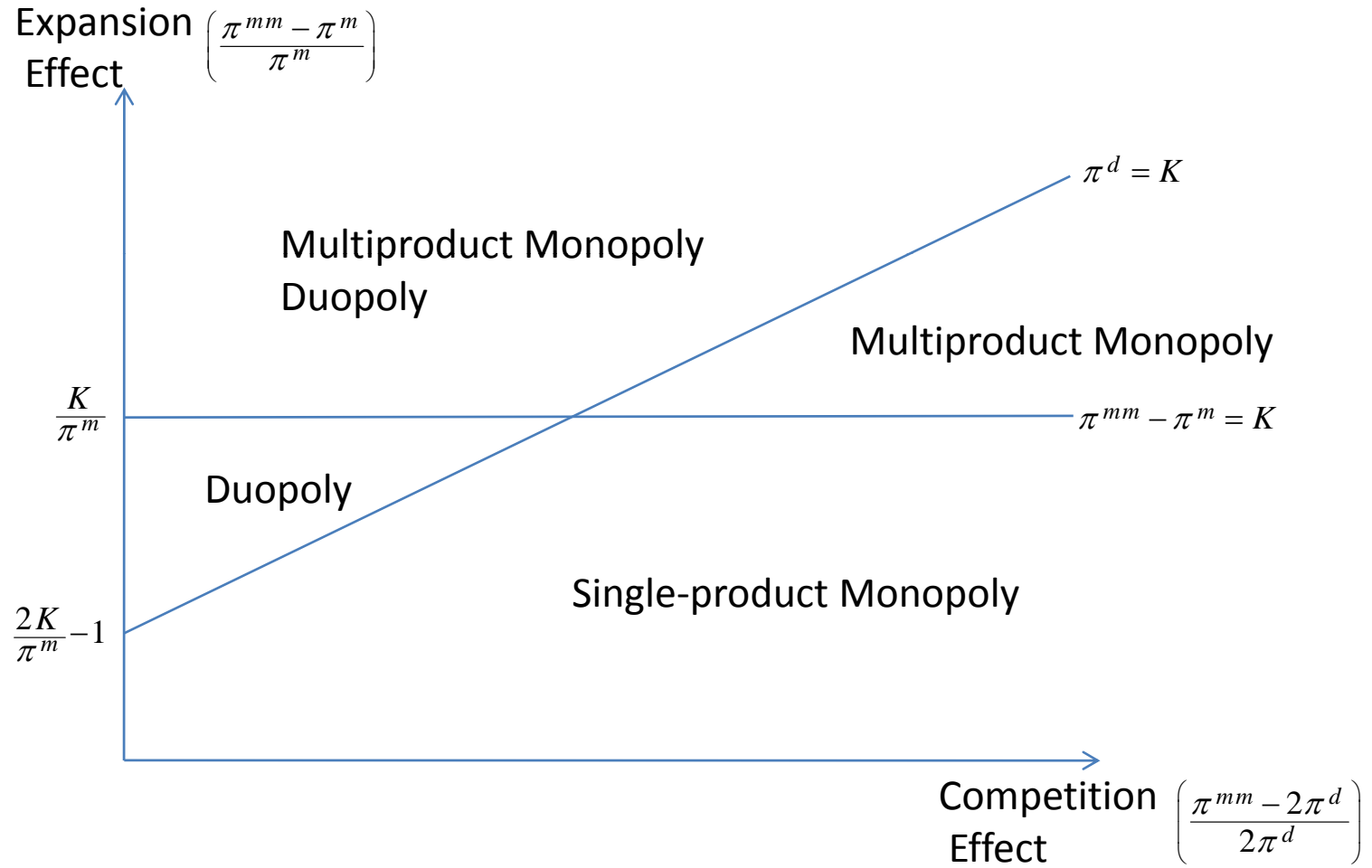
**Figure 2**  
Profits in the Bivariate Exponential Case



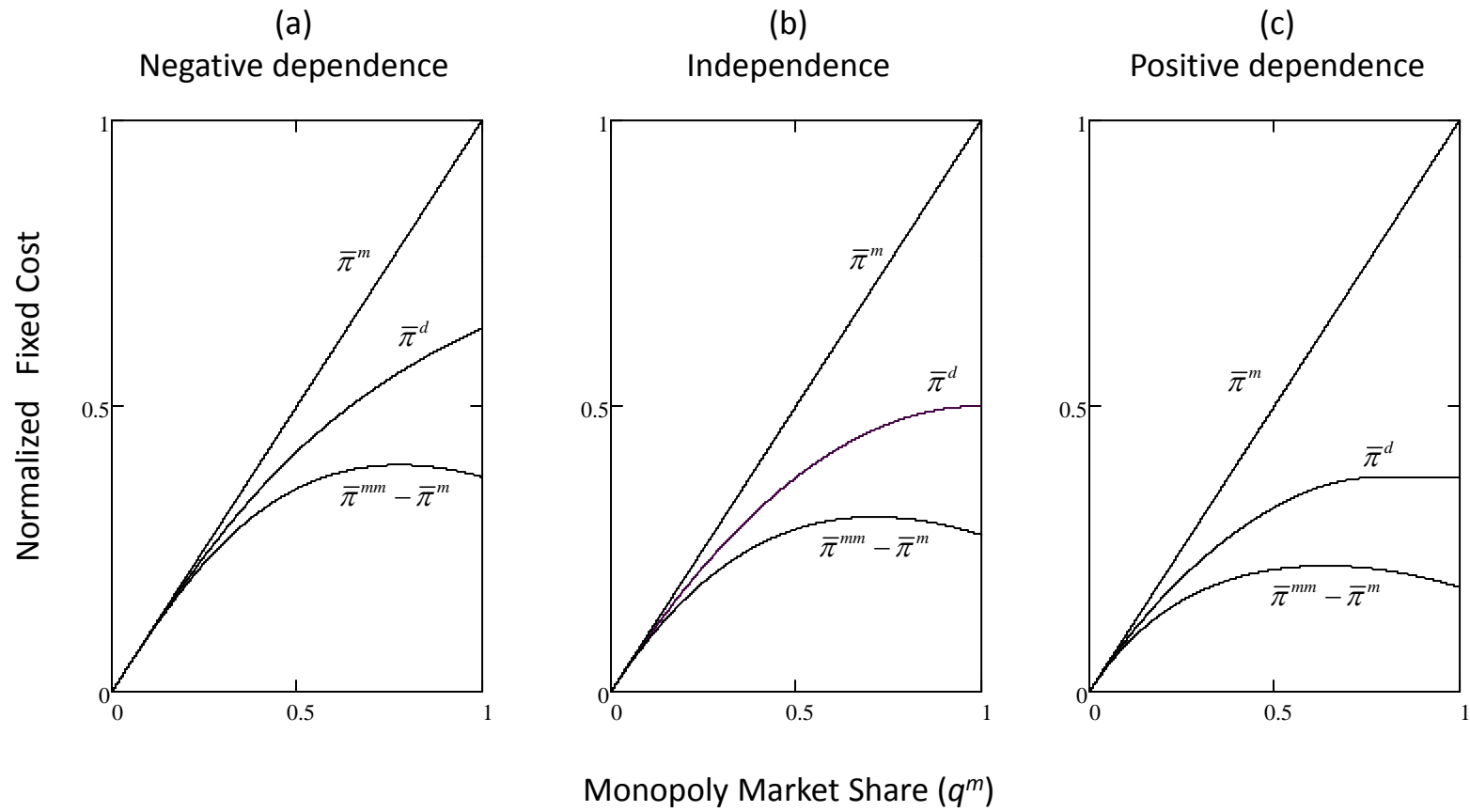
**Figure 3**  
Consumer Welfare in the Bivariate Exponential Case



**Figure 4**  
Equilibrium Market Structure (Shaked and Sutton, 1990)



**Figure 5**  
Critical Fixed Costs in the Bivariate Exponential Case



**Figure 6**  
Welfare-Improving Monopoly in the Independent Bivariate Exponential Case

