

Total Factor Productivity Growth and Convergence in Petroleum Industry: An Empirical Analysis Testing for Non-Convexity

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Abstract:

While economic theory acknowledges that some features of technology (e.g., indivisibilities, economies of scale and specialization) can fundamentally violate the traditional convexity assumption, almost all empirical studies take the convexity property by faith. In this contribution, we apply two alternative flexible production technologies to measure total factor productivity growth and test the significance of the convexity axiom using a nonparametric test of closeness between unknown distributions. Using unique field level data on petroleum industry, the empirical results reveal significant differences indicating that this production technology is most likely non-convex. Furthermore, we also show the impact of convexity on answering traditional convergence questions in the productivity growth literature.

Key words: Productivity, Luenberger Productivity Indicator, Non-Convexity, Convergence.
JEL Classification codes: C430, L710

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1. Introduction

Indivisibility implies that inputs and outputs are not necessary perfectly divisible and also that scaling up or down the entire production process in infinitesimal fractions may not be feasible.¹ Economies of scale and specialization (implied by the presence of indivisibilities and other forms of non-convexities in production) entail that a larger per-capita production increases the extent of the market, facilitates the division of labor, and increases the efficiency of production.² These economically important features of technology fundamentally violate convexity of the production possibility set. But, in traditional empirical analysis (e.g., traditional parametric production analysis, or even nonparametric production analysis) these features are assumed away by imposing the convexity axiom. In reality, it is clear that non-convexities in production are sufficiently important to explain behavior in some industries and are critical in the development of the new growth theory (see, e.g., Romer, 1990, on nonrival inputs). In a similar vein, already McFadden (1978) recognized that the importance of convexity in production analysis lies in its analytic convenience rather than its economic realism.

Therefore, for the sake of relevance in both economic theory and associated empirical analysis one cannot ignore the potential impact of non-convexity, even if this implies a risk of producing less “elegant” results compared to standard approaches. However, almost no previous studies have directly tested for the existence of (non-)convexities in production by rigorous statistical techniques. One possible reason might be that the widespread perception of convexity as a theoretically sound regularity property may have discouraged empirical testing. Another possible reason might be that existing specification tests are unsatisfactory for this purpose, because they may confuse

¹ Scarf (1986, 1994) stresses the importance of indivisibilities in selecting among technological options.

² Wibe (1984) shows scale elasticity is generally larger than one for all output levels when reviewing studies of engineering production functions.

non-convexities with the effect of small sample error. However, non-convexities in production play an important role in the theoretical micro-economic literature and have been studied for decades (see Villar, 1999). For instance, the general equilibrium theory of non-convex technologies has been thoroughly analyzed (e.g., Joshi, 1997).

In this paper, we apply two alternative flexible production models defined using nonparametric specifications of technology and test the validity of the non-convexity assumption in production (NCP). One convexity-free specification of production technology is the non-convex Free Disposable Hull (FDH) model (initiated by Deprins et al., 1984). FDH only imposes the assumption of strong or free disposability of both inputs and outputs. Another more common technology specification adds convexity to these strong disposability axioms to form a convex nonparametric production model (CP) (see, e.g., Afriat, 1972 or Färe et al., 1994).

Based on distance functions as representations of technology (and their interpretation as efficiency indicators (introduced by Farrell, 1957)) computed relative to both these non-convex and convex nonparametric specifications of technology, we test the significance of the differences using Li's (1996) nonparametric test of closeness between two unknown distributions resulting from independent or dependent observations. If the true production possibility set is convex, CP and NCP estimators yield approximately the same results in large data sets, though the NCP model might have a slower rate of convergence. However, if technology is non-convex, then the CP model offers an inconsistent approximation and convexity should not be imposed on the specification (see Simar and Wilson, 2008).

These nonparametric specifications require large data sets on production technologies to avoid a small sample error problem. Furthermore, to avoid any aggregation bias the analysis should ideally focus on firm level data with sufficient details on the production process. Here, we apply this test of convexity to a unique field level data set of

the petroleum industry in the US Gulf of Mexico over the period 1947 to 1998. Though the production possibility set of oil and gas development and exploitation is acknowledged to be non-convex in part of the literature (see, e.g., Devine and Lesso (1972) and further arguments below), we are unaware of previous economic studies putting this assumption to an empirical test. Hence, whether the above NCP methodology yields a relevant reference technology in this industry becomes a most interesting empirical question for testing.

Furthermore, a topic that has received widespread attention with the appearance of endogenous growth theories is the question of convergence in productivity levels (see Islam (2003) for a survey). In view of the importance of non-convexities for growth theory (Romer, 1990), we consider the suggestion by Bernard and Jones (1996) that “future work on convergence should focus much more carefully on technology”. In particular, we investigate the issue of convergence/divergence in total factor productivity change using a recent discrete time Luenberger productivity indicator (Chambers, 2002) while testing for the significance of the eventual differences between CP and NCP models. The very length of the observation period provides ample scope to test the impact of the convexity assumption on the eventual convergence of total factor productivity growth rates.

The choice between non-convexity and convexity in measuring total factor productivity change relates to the nature of technical change. The NCP model has the advantage of eventually allowing for local instead of global technical change. While this distinction between local and global technological change plays a role in some theoretical work (see, e.g., Atkinson and Stiglitz, 1969, among others), we are unaware of any empirical work raising this issue. If NCP is the true representation of technology, then previous empirical work on the convergence issue might not have been reliable. Anticipating one of the key results, this study only finds convergence for the NCP model.

This contribution is structured as follows. Section 2 reviews the background literature. Section 3 presents the Luenberger productivity indicator as well as its underlying

distance functions and the econometric models employed to test for convergence. Section 4 discusses the empirical results and provides the outcomes of the statistical tests. The final section offers some concluding remarks.

2. Non-Convexity in Production: Literature Review

Interestingly, the literature on non-parametric production analysis (see e.g., Afriat, 1972 or Varian, 1984) typically uses convexity only as an instrumental regularity property on technology which is justified by the assumed economic optimization hypotheses. Thus, convexity is motivated from the perspective of the economic objectives (such as cost minimization or profit maximization), not as an inherent feature of technology. Similarly, the parametric approach (see, e.g., Bauer, 1990) sometimes imposes regularity restrictions on the parameters of cost, revenue and profit functions, but it does not systematically test for the convexity assumption of technology.

Thereby, the impact of convexity or not of technology on the cost function is often ignored. While the general property that the cost function is non-decreasing in outputs is well-known, it seems often forgotten that cost functions estimated on convex (non-convex) technologies are furthermore convex (non-convex) in the outputs. Jacobsen (1970) was one of the first to point out that convexity of the cost function in the outputs is due to convexity of the output set (see proposition 5.2).

Several empirical studies suggest violations of convexity in a wide variety of industries. Indivisibilities are an obvious feature of real world production settings (see Scarf 1986, 1994). Phenomena of economies of scale and specialization have equally been empirically tested in the literature. The empirical evidence of process analysis, which derives production relations directly from theoretical and practical engineering knowledge, has found evidence of violations of convexity (see Wibe, 1984). Especially economies of scale are well documented. For instance, Chenery (1949) studied engineering production

functions of the pipeline transportation of natural gas and derived a (non-linear) cost function that exhibits economies of scale. Some evidence of increasing returns has been reported in, for example, chemical industries, manufacturing of process equipment, air pollution control equipment, and in biopharmaceutical equipment (e.g., see the survey in Wibe, 1984). Some other economic analyses documenting these types of violations of convexity include Yang and Rice (1994) and Borland and Yang (1995).

We provide an analysis of the offshore oil and gas industry which faces substantial sunk cost investments in development, exploration and knowledge, which are the main source of non-convexity in production (see Devine and Lesso, 1972, or Frair and Devine, 1975). To enter into some details, once an oil field is found after extensive seismic study via drilling an exploratory well, other “step-out wells” are drilled to obtain more precise information on the characteristics of the field (e.g., size being one critical factor). Assuming entering the production phase is an option (e.g., because of favorable well flow tests), the above information is exploited to decide on possible locations for the production wells (specified by map coordinates and depth).

An offshore oil field is developed by drilling directionally from a series of fixed platforms, whereby drilling and completion costs for a production well are a function of the length and angle of the hole drilled from the platform to the target. Each platform involves a huge investment, the precise costs depending on water depth and the number of wells to be drilled from the platform. Thus, the "platform location problem" is a complex integer optimization problem aiming to minimize the sum of platform and drilling costs by determining the number, size, and location of platforms and the allocation of wells to platforms. Needless to say, small deviations from optimality can generate substantial inefficiencies. For example, injection wells and other capital equipment for extraction are sunk and can not readily be changed because of their geometric features.

Further difficult optimization issues at the level of the oil field are, for instance,

the determination of the optimal production rate for each time period to maximize the total discounted after-tax cash flow over a specific planning horizon in multilayer oil and gas fields by exercising control on the number of wells acting during the field's exploitation period (e.g., Babayev, 1975). Also Neiro and Pinto (2004) argue that the planning and scheduling of subsystems of the petroleum supply chain (oilfield infrastructure, crude oil supply, refinery operations and product transportation) requires non-convex and nonlinear mixed-integer optimization models (see also the survey of Durrer and Slater, 1977). In brief, optimization problems in this sector are hard because of the integer nature of certain decisions, the nonlinearities and dynamics involved, and the intrinsic uncertainties surrounding critical parameters (see, e.g., Dempster, et al., 2000).

However, in most of the rather limited economic literature on the oil sector the issue of convexity seems to be totally ignored. Just to offer one example representative of similar studies, Cuddington and Moss (2001) estimate average cost functions for additional petroleum applying error correction models over the period 1967-1990. Using aggregate data in US, they find the impact of technological change on finding costs for crude oil is large.

3. A Discrete-Time Luenberger Productivity Indicator: Definitions and Technology Specifications

Chambers and Pope (1996) define a discrete-time Luenberger productivity indicator in terms of differences between directional distance functions (see also Chambers, 2002).³ This is the most general primal productivity indicator currently available, since it is based upon the directional distance function as a general representation of technology (Luenberger, 1992; Chambers, Chung, and Färe, 1998). Due to its dual relation with the

³ Indicators (indexes) denote productivity measures based on differences (ratios) (see Diewert, 2005).

profit function, the latter distance function generalizes the traditional input- or output-oriented Shephard (1970) distance functions, which are dual to the cost and the revenue functions, respectively.⁴

This section discusses a three-step approach to analyze total factor productivity in the petroleum industry. In the first step, a discrete time Luenberger total factor productivity indicator is defined. Next, the directional distance functions constituting this indicator are measured relative to two different technology specifications: one non-convex, the other convex. In the second step, these two models are statistically compared using a nonparametric test statistic comparing densities developed by Li (1996). In the third step, we investigate whether there is β - and/or σ -convergence in total factor productivity change for both models using proper econometric models.

Definitions of Technology, Distance Function and Luenberger Productivity Indicator

Using the index set $K = \{1, \dots, K\}$ for production units, let $x = (x^1, \dots, x^N) \in \square_+^N$ and $y = (y^1, \dots, y^M) \in \square_+^M$ be the vectors of inputs and outputs, respectively, and define the technology or production possibility set as follows:

$$T_t = \{(x_t, y_t) \in \square_+^{N+M} \mid x_t \text{ can produce } y_t\} \quad (1)$$

This technology set T consists of all feasible input vectors x_t and output vectors y_t at time period t and satisfies certain minimal axioms sufficient to define meaningful distance functions (see Afriat, 1972). Multi-input, multi-output production technologies and their boundaries (frontiers) can be characterized by distance or gauge functions, without any assumption of optimizing behavior on the part of individual observations.⁵

⁴ Hence, the more popular Malmquist input- and output-oriented productivity indexes (Caves, Christensen and Diewert, 1982) based upon these Shephardian distance functions are less general than the Luenberger productivity indicator.

⁵ In economics, distance functions are related to the notion of the coefficient of resource utilization (Debreu, 1951) and to efficiency measures (Farrell, 1957). Avoiding these maintained hypotheses may be an advantage,

Luenberger (1992) generalizes the traditional Shephardian notion of distance functions by introducing the shortage function that provides a flexible tool capable of taking account both input contractions and output improvements when measuring efficiency. Following Chambers et al. (1998), the proportional distance function is defined as follows:

$$D^t(x_t, y_t; -x_t, y_t) = \max \{ \delta : ((1-\delta)x_t, (1+\delta)y_t) \in T_t \} \quad (2)$$

This distance function completely characterizes the technology at period t . Note that this proportional distance function is a special case of the shortage or directional distance function. The latter is defined using a general directional vector $(-g_i, g_o)$, whereas the proportional distance function employs the special case $(-g_i, g_o) = (-x, y)$.

Following Chambers (2002), the Luenberger productivity indicator (total factor productivity, TFP) in discrete time is defined as follows:

$$TFP = \frac{1}{2} \left[\left(D^t(x_t, y_t) - D^t(x_{t+1}, y_{t+1}) \right) + \left(D^{t+1}(x_t, y_t) - D^{t+1}(x_{t+1}, y_{t+1}) \right) \right]. \quad (5)$$

Several proportional distance functions are needed to estimate this productivity indicator. This formulation represents an arithmetic mean between the period t (first difference) and the period $t+1$ (second difference) Luenberger indicators, whereby each indicator consists of a difference between proportional distance functions evaluating observations in period t and $t+1$ with respect to a technology in period t respectively period $t+1$. Taking an arithmetic mean avoids an arbitrary selection among base years.

This Luenberger productivity indicator can be decomposed into two components:

$$TFP = \left[D^t(x_t, y_t) - D^{t+1}(x_{t+1}, y_{t+1}) \right] + \frac{1}{2} \left[\left(D^{t+1}(x_{t+1}, y_{t+1}) - D^t(x_{t+1}, y_{t+1}) \right) + \left(D^{t+1}(x_t, y_t) - D^t(x_t, y_t) \right) \right], \quad (6)$$

particularly for micro-level analyses that extend over a long time series with significant uncertainty, irreversibility and fixed (and/or sunk) costs. In such cases, assumptions of static efficiency of every production unit in all time periods are likely suspect.

where the first difference represents efficiency change (EC) and the second term, which is an arithmetic mean of two differences, represents technological change (TC). While EC measures changes in the relative position of a production unit relative to the changing frontier, the TC component provides a local measure of the change in the production frontier or productivity changes that are due to innovation. To be more precise, TC measures the arithmetic mean of the productivity change measured from the observation in period t respectively period $t+1$ relative to the production frontiers in both periods.

To estimate productivity change over time, four distance functions are needed, including both within-period and mixed-period distance functions for each field and each time period. For the mixed-period distance function, we have two years, t and $t+1$. For example, $D^t(x_{t+1}, y_{t+1})$ is the value of the proportional distance function for the input–output vector for period $t+1$ and technology in period t .

Now we turn to the specification of technology relative to which these distance functions are estimated. Given the focus on testing the convexity assumption, we look for a framework allowing defining both a convex and a non-convex representation of technology.

Non-Convex and Convex Technologies

In principle, distance functions can be estimated using either parametric or nonparametric specifications of the directional distance function representing technology. The vast majority of empirical productivity studies seem to employ deterministic, nonparametric technologies. However, an example of an empirical productivity study using both nonparametric and parametric technologies is Atkinson, Cornwell and Honerkamp (2003). While it is common to employ traditional convex nonparametric frontier technology specifications, in this study we also compute a directional distance function

relative to a non-convex technology. Therefore, we are able to compare these two measurements in their effect on the Luenberger productivity indicator and test for the validity of the convexity axiom. Note that the estimated production frontiers (both CP and NCP) are based on relative benchmarking among observed units in the marketplace rather than absolute production possibilities (e.g., engineering estimates). Thus, allowing for any eventual inefficiency among observations, these frontier estimates provide inner approximations, i.e., the (non-)convex hull of the underlying unknown true technology.

The construction of nonparametric, deterministic technologies is based on the minimum extrapolation principle: envelop all observations and extend these using production axioms about what is considered feasible. Imposing convexity, strong disposability of inputs and outputs as well as variable returns to scale to obtain a flexible representation of technology, the proportional distance function at t for CP is defined as:

$$D_c^t(x_t, y_t; -x_t, y_t) = \max_{\delta} \{ \delta_t | (1 - \delta_t)x_t \geq \lambda_t X_t; (1 + \delta_t)y_t \leq \lambda_t Y_t; \lambda_t e = 1 \in T_t \}, \quad (3)$$

where δ is the maximal proportional amount by which outputs, y_t , can be expanded and inputs, x_t , can be reduced simultaneously given the technology, T_t . Y_t and X_t are the matrices of outputs, y_t , and inputs, x_t , respectively, while λ is a vector of activity variables. This CP model involves mathematical programming techniques to estimate the relative efficiency of all production units relative to best-practice frontiers. Notice that free disposability is an almost generally accepted assumption in production economics. This implies that marginal products of inputs, marginal rates of substitution between inputs, and marginal rates of transformation between outputs are assumed to be non-negative.⁶

⁶ Obviously, congestion of production factors can violate this assumption. However, we can interpret monotonicity as a congestion adjustment to the production set, *i.e.* the distance functions for monotonized technologies include both pure technical efficiency and congestion components (see Färe and Grosskopf, 1983). Alternatively, monotonicity can be motivated by the fact that it does not interfere with the Pareto-Koopmans classification of technical efficiency.

Next, the NCP can be formulated as follows. The non-convex technology is obtained from two minimal assumptions: all inputs and outputs are freely (or strongly) disposable (i.e., monotonicity), and variable returns to scale. Thus, the proportional distance function at t for the NCP technology set is defined as (see Deprins *et al.*, 1984):

$$D_n^t(x_t, y_t; -x_t, y_t) = \max_{\sigma} \left\{ \delta_t \mid (1 - \delta_t)x_t \geq \lambda_t X_t, (1 + \delta_t)y_t \leq \lambda_t Y_t, \lambda_t e = 1; \lambda_t^j \in \{0, 1\} \forall j \in S \right\}$$

(4)

Notice that the convex monotone hull (3) is obtained by eliminating the binary integer constraint on the activity variables $\lambda^j \in \{0, 1\} \forall j \in S$ by $\lambda \geq 0$. Actual calculations of the distance functions under CP and NCP models are provided in the Appendix.

Statistical Analysis of Productivity Growth and Convergence

The estimates of NCP are statistically consistent for a wide range of statistical distributions (e.g., Park *et al.*, 2000). Consistency also applies for the CP approximation, but only if the true production set is convex. Hence, if the true production set is convex, CP and NCP models generally yield approximately the same results in large data sets. However, if the production set is non-convex, the CP set yields an inconsistent approximation. Therefore, in large-scale applications, convexity constitutes a potential source of specification error, but cannot improve the statistical fit.

The convexity axioms in productivity have almost never been exposed to rigorous empirical testing (see, e.g., Grifell-Tatjé and Kerstens, 2008 for an exception). However, test procedures for testing the convexity hypothesis in CP model are available. In particular, we apply the nonparametric Li's (1996) test to examine the differences in the distribution of the efficiency scores. Li's (1996) method tests the closeness of two distributions using sample distributions based on the kernel density method. The convexity hypothesis is accepted if there is no statistically significant difference in the efficiency estimates of both

CP and NCP models. This approach requires large samples; in small samples it can confuse non-convexities with the small sample error associated with the relaxed model. The specification tests can test production assumptions only under conditions in which those assumptions cannot improve the fit of the estimators.

Following the convergence literature, we also report estimates on the β -convergence and σ -convergence (e.g., Barro and Sala-i-Martin 1992). The β -convergence notion refers to a tendency of sectors with relatively low initial productivity levels to grow relatively fast, while the σ -convergence suggests a decreasing variance of cross-field differences in productivity levels, building upon the proposition that growth rates tend to decline as sectors approach their steady state.

Despite the popularity of productivity estimates using frontier technologies, notice that few frontier technology studies have analyzed questions surrounding convergence. Available frontier-based convergence studies focus most of the times on countries (e.g., Henderson and Russell, 2005; Kumar and Russell, 2002; or Kumbhakar and Wang, 2005), regions (e.g., Salinas-Jiménez, 2003), or sectoral analysis (e.g., Gouyette and Perelman, 1997). Among the exceptions focusing on firm level data within a given industry, one can notice the articles of Alam and Sickles (2000) on the U.S. airlines industry and Bonaccorsi, Giuri and Pierotti (2005) analyzing convergence over a relatively long period in the jet aero-engine industry. Our study focuses on a single industry observed over a long time period.

4. Empirical Estimation Results

Data used in this analysis are obtained from the U.S. Department of the Interior, Minerals Management Service (MMS), Gulf of Mexico OCS Regional Office. We have developed a unique micro (i.e., field) level database using three MMS data files: (1) production data including well-level monthly oil and gas outputs from 1947 to 1998 (a total of 5,064,843

observations for 28,946 production wells); (2) borehole data describing drilling activity of each well from 1947 to 1998 (a total of 37,075 observations); (3) field reserve data including oil and gas reserve sizes and discovery year of each field from 1947 to 1998 (a total of 957 observations). Relevant variables were extracted from these data files and merged by year and field across wells, because the well level is not a good unit to measure productivity. Due to spillover effects across wells within a given field, the field level is a more appropriate unit for measuring performance than the well. This explains, for instance, why companies often engage in joint ventures to exploit a field to internalize the externalities between wells within the field.

The final data set comprises annual data from 933 fields over a 50-year time horizon. On average there are 370 fields operating in any particular year, and a total of 18,117 observations. Thus, the database includes field-level annual data over the period 1947 to 1998 for the following variables. Output variables are oil output and gas output. Input variables are number of exploration and development wells drilled, total drilling distance of exploration and development wells, number of platforms, water depth, oil reserve, gas reserve, untreated produced water.

Furthermore, we measure productivity change by looking at relative productivity across fields of different vintages. In so doing, we are able to separate productivity effects associated with aging of the field from effects due to differences in the state of technology. This vintage model differs from the conventional nonparametric model specification in that the mixed period distance functions compare fields of different vintages for a given field year, so that the model compares outputs and inputs holding fixed the number of years that the fields have been operating. Thus, we use cumulative values for inputs and outputs, because for this nonrenewable industry it is more appropriate to express the production technology in cumulative terms. For example, for a field, the production at t is determined by cumulative inputs (e.g., drilling) and outputs (i.e., stock depletion) up to $t-1$. See the

Appendix for a more detailed description for this vintage model.

We first examine the convexity hypothesis by comparing the convex and the convexity-free production models. The TFP in Figure 1 shows how the gross productivity in offshore oil and gas grows 24.9 and 29.7 % over the study period (a yearly growth rate of 0.50 and 0.60 %) under the assumption of NCP and CP, respectively. Though the general trend of these productivities is very similar, their magnitude is clearly different. On average, these results reflect the productivity-enhancing effects of new technologies becoming available.

Turning to specific periods, growth is moderate during the 1980s. The latter result is consistent with common reports of Gulf of Mexico production: it was referred to as the 'Dead Sea' in the 1980s. More recent TFP growth has probably resulted because production has moved to very great water depths in about the last decade or so. While by 1997 production occurred at over a mile deep, by 2001 exploratory wells were being drilled in nearly 10,000 feet of water. This deep-water production has allowed the discovery of larger fields.

The lower TFP for NCP might be because the production frontier reacts to the change in localized productivity change. Atkinson and Stiglitz (1969) analyze the generation of new technologies and introduced the hypothesis that technological change can take place only in a limited technical space, defined in terms of both factor intensity and scale. Technological change is localized because it has limited externalities and affects only a limited span of the techniques, contained by a given isoquant (see also Stiglitz, 1987; Foray, 1997; Antonelli, 2008). Since technological improvements are in fact associated with a specific input space, the convexity assumption is likely to overestimate the true changes of technology. This could explain the higher growth rate under CP.

We also decompose TFP effects into those associated with new innovation (TC) and catch-up effects (EC). We find significant differences in their sources of TC and EC.

The result of TC assuming both convexity and non-convexity is provided in Figure 2. TC increases 15.7 and 27.8 % over the study periods under the assumption of NCP and CP, respectively. Thus, there are significant increases in TC when imposing the convexity assumption, while TC is relatively smaller under non-convexity. These disparities can be explained by the localized nature of technological change (as in the case of TFPs above).

By contrast, we find a significant increase of EC in the non-convex model, while EC is smaller in convexity model. EC increases 9.2 and 1.9 % over the study periods under the assumption of NCP and CP, respectively. This might be because the convexity assumption overestimates the true production frontier shift and therefore EC is obscured in the case of CP. However, if existing resources are not fully utilized in production initially (due to technical inefficiencies, variations in capacity utilization, among others), then one can expect significant scope for variations in EC as revealed by NCP.

Thus, assuming NCP represents the true production frontier, we find TC contributes less to TFP than EC, while the reverse results hold true for CP.

Non-convex specification can shed light on the local nature of technical change. Table 1 reports the descriptive statistics over the years and also indicates the number of observations experiencing progress, regress, and no change. We find more efficient units and more of these push the frontier up (or down) under NCP than CP for all of TFP, TC, and EC. Also, focusing on the TC component, we find the number of observations that are efficient in each period t and that push the frontier forward (or backward) in the period $t/t+1$ is larger under NCP than CP. Thus, it is important to focus on individual results in the non-convex world.

Table 2 shows the results of the Li (1996) test to see whether or not each of the production models NCP and CP has a different distribution of values for the proportional distance functions (i.e., “efficiency scores”), as well as the resulting productivity indicator, and its TC and EC components. In all of these cases, the empirical results indicate that the

two different distributions of CP and NCP provide statistically significant different patterns. Therefore, we reject the null hypothesis of distribution closeness between NCP and CP.

To the best of our knowledge, there appear to be no valid theoretical arguments for assuming a priori that production possibilities are truly convex (see also McFadden, 1978). In this empirical study, the economically important industry of petroleum exploitation reveals violations of the convexity hypothesis. Therefore, NCP seems to have a comparative advantage for analyzing TFP.

Lastly, we investigate the convergence phenomenon in the petroleum industry. In particular, we estimate the following model consisting of a simple speed-of-convergence equation (β -convergence, see Steger, 2000).

$$\Delta \ln y_{it(i)} = \alpha + \beta \ln y_{i0} + e_i,$$

where $\Delta \ln y_{it(i)}$ shows the indicators of average productivity changes, technological changes (TC), and efficiency changes (EC) of field i from year 0 to year t for each field i , $\Delta \ln y_{i0}$ indicators represents the initial level (field discovered) of the indicators, and e_i represents an error term. The indicators might be resulting from either CP or NCP. A negative value of β is interpreted as a support for the convergence hypothesis, since it means that those with lower initial productivity have grown faster over time. Whether or not we observe convergence might depend crucially on the choice of (non) convexity assumption.

Table 3 presents the estimation results of convergence in average productivity changes, technological changes, and efficiency changes for a change in a crucial technology assumption. The results show that productivity change as well as its both components converge in NCP over the observation horizon. By contrast, productivity and efficiency changes do not seem to be converging in CP, though the EC is converging. Therefore, we have confirmed strong evidence for productivity convergence among

petroleum fields assuming NCP provides a true measurement. These results might imply that technology diffusion behind productivity convergence expands opportunities for secondary firms to catch up to the leading firms. Applying the standard assumption of CP yields altogether different conclusions.

In Table 4, we test for the eventual existence of σ -convergence for TFP, TC, and EC in NCP and CP models (see de la Fuente (2002) for a review). Table 4 reports cross-sectional standard deviations of productivity changes for the two years 1947 and 1998 at the beginning and end of the sample period. The 1998 standard deviations are greater than those of 1947 for the CP model, while the reverse is true for NCP. Therefore, assuming NCP, we are able to find the tendency of poor fields to grow faster in a cross-section bivariate regression of growth rates on initial productivity level and also tendency of sample dispersion of productivities to diminish. By contrast, no sign of σ -convergence emerges under the traditional CP.

5. Conclusions

This study raises doubts regarding the ability of traditional convex production functions to explain the real world phenomenon of observed data. We examined data on petroleum industry using unique field level data. In this paper, we found that the traditional convex production model is in trouble and the shapes of the functions are non-convex. In light of the empirical evidence presented in this study, there is no good reason for considering convexity of production sets or convexity of input/output sets as generally realistic axioms. The evidence we have find suggests that non-convexities exist.

It is important to note that the shape of the production function, whether convex or non-convex, is an important determinant, but not the only determinant, of whether a production function is an optimal behavior. If the results implying non-convex production functions are not valid, serious implications for the standard micro-economic theory of the

firm follow. The equilibrium of the firm and the existence of competitive markets normally depend on the convexity of the production functions. It is, therefore, necessary for researchers to avoid a prior assumption of convexity when the true empirically estimated functions are non-convex.

Appendix: Efficiency Estimation based on (Non-) Convex Production Assumption

When analyzing productive efficiency for extraction of non-renewable resources such as in the petroleum industry, one faces challenges not met in typical applications to the single period production of goods and services. For example, production from an oil field at some point in time depends upon cumulative past production from the field due to depletion effects, in addition to the technology employed and the attributes of the field (e.g., field size, porosity, water depth). Holding inputs constant, output from a given field follows a well-known pattern of an initially increasing output rate, obtaining a peak after some years of production, then following a long path of declining output (e.g., Pindyck, 1978). This implies that, for purposes of measuring changes in the productivity, it is inappropriate to compare contemporaneous levels of output from a newly producing field to a field that has been producing for ten years or fifty years. Rather, comparisons across fields should be done holding constant the number of years the fields have been in operation.

Thus, we measure productivity change by looking at relative productivity across fields of different vintages. By doing so, we separate productivity effects associated with aging of the field from effects due to differences in the state of technology. The CP formulation with the vintage model differs from the conventional CP formulation (as described in, e.g., Färe et al., 1994). Similar to the approach of Managi *et al.* (2004), our CP formulation calculates the distance function by solving the following optimization problem:⁷

$$D_c^i(\mathbf{x}_{k^j}^i, \mathbf{y}_{k^j}^i) = \max_{\delta^{k^j}, \lambda} \delta^{k^j}$$

subject to

⁷ Note that Managi et al. (2004) apply a more traditional Malmquist productivity index method.

$$\begin{aligned}
\sum_{k \in K(i)} \sum_{j=0}^{J(k)} \lambda_{kj} y_{kjin}^i &\geq (1 + \delta^{k',j'}) y_{k',j'n}^i, \quad n = 1, \dots, N, \\
\sum_{k \in K(i)} \sum_{j=0}^{J(k)} \lambda_{kj} x_{kjm}^i &\leq (1 - \delta^{k',j'}) x_{k',j'm}^i, \quad m = 1, \dots, M, \\
\sum_{k \in K(i)} \sum_{j=0}^{J(k)} \lambda_{kj} &= 1, \\
\lambda_{kj} &\geq 0, \quad k \in K(i), \quad j = 1, \dots, J(k).
\end{aligned} \tag{A1}$$

where j is the field year, k is the field number, $K(i)$ includes all fields of vintage i (i.e., discovered in year i), $J(k)$ is the final year of production for field k , and λ_{kj} is the weight for field k at field year j .

In a similar manner, our NCP formulation calculates the distance function by solving the following optimization problem:

$$D_n^i(\mathbf{x}_{k',j'}, \mathbf{y}_{k',j'}) = \max_{\varpi^{k',j'}, \lambda} \varpi^{k',j'}$$

subject to

$$\begin{aligned}
\sum_{k \in K(i)} \sum_{j=0}^{J(k)} \lambda_{kj} y_{kjin}^i &\geq (1 + \varpi^{k',j'}) y_{k',j'n}^i, \quad n = 1, \dots, N, \\
\sum_{k \in K(i)} \sum_{j=0}^{J(k)} \lambda_{kj} x_{kjm}^i &\leq (1 - \varpi^{k',j'}) x_{k',j'm}^i, \quad m = 1, \dots, M, \\
\sum_{k \in K(i)} \sum_{j=0}^{J(k)} \lambda_{kj} &= 1, \quad \lambda_{kj} \in \{0, 1\} \quad \forall j \in S \\
\lambda_{kj} &\geq 0, \quad k \in K(i), \quad j = 1, \dots, J(k).
\end{aligned} \tag{A2}$$

We refer to this simple algorithm because it points to an important difference in the logic that lays behind convex versus non-convex methodologies. The role of the integrality constraint is indeed essential to identify a dominance relation between observed production plans. On the one hand, an observation is declared efficient and considered as lying on the boundary of the reference production set if it is un-dominated. On the other hand, an observation is declared inefficient, *i.e.* it lies in the interior of the set, if it is

dominated by at least one; and in this case the mixed integer program identifies a most dominating observation that serves as a reference to compute the efficiency degree.

By contrast, the linear programs used in the convex case seek to compute a distance with respect to the frontier of a convex envelope of the data. While dominance also plays some role in identifying this envelope, the additional requirement of convexity induces the possibility that un-dominated observations be declared inefficient because they do not lie on the convex envelope of the data. Note that the NCP model treats only a specific and real peer unit, or a collection of such units in the case of a non-unique solution, as a benchmark at the optimum for the measure of efficiency. Thus, the NCP model provides a more conservative inner approximation and estimation of the production possibility set compared to CP.

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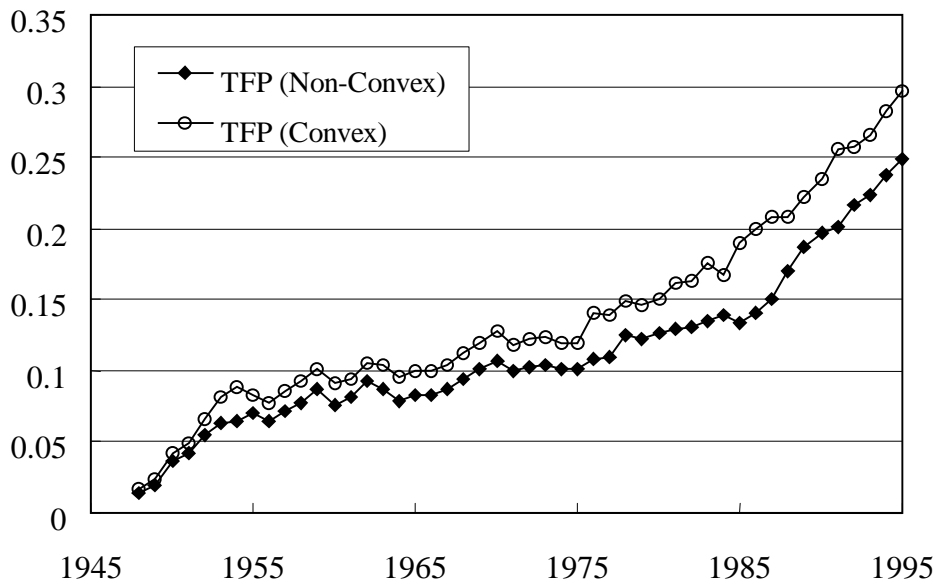


Figure 1. Productivity Change in Petroleum Industry under Non-Convex and Convex Assumptions

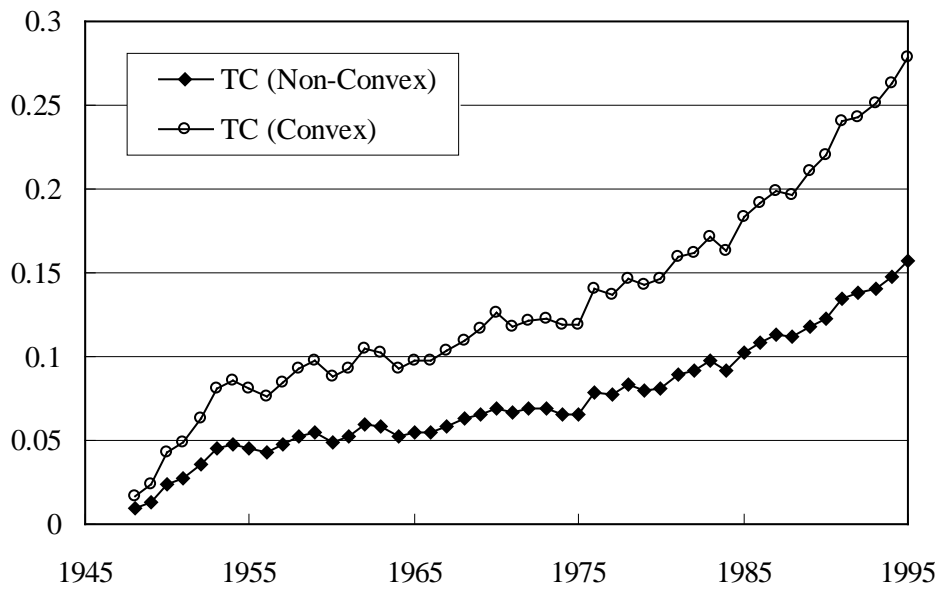


Figure 2. Technological Change in Petroleum Industry under Non-Convex and Convex Assumptions

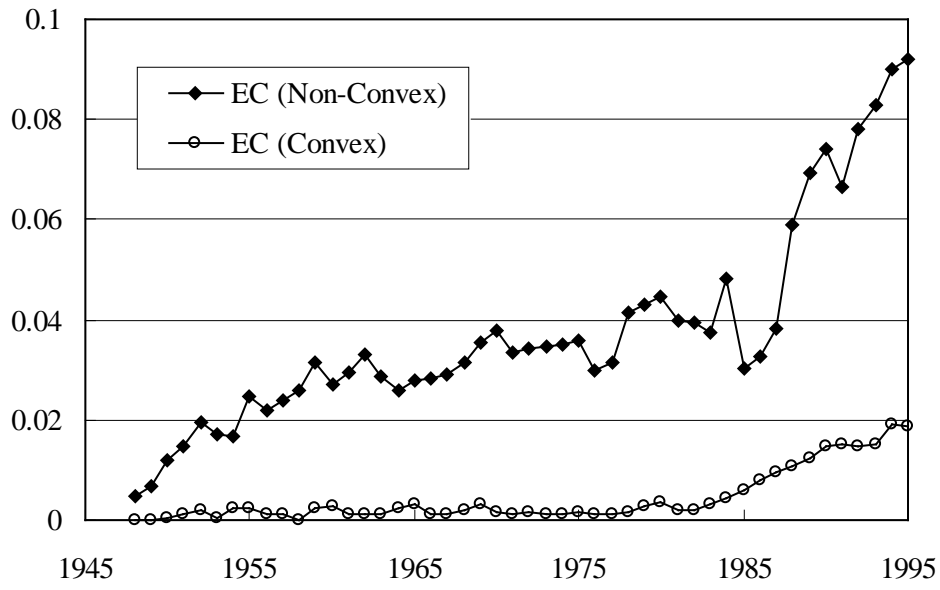


Figure 3. Efficiency Change in Petroleum Industry under Non-Convex and Convex Assumptions

Table 1. Luenberger Productivity Indicator: Descriptive Statistics

Descriptive Statistics	Non-Convex Production			Convex Production		
	TFP	TC	EC	TFP	TC	EC
Mean	0.50	0.31	0.18	0.60	0.56	0.04
Standard deviation	0.23	0.27	0.25	0.25	0.31	0.23
Minimum	-0.74	-0.69	-0.12	-0.81	-0.77	-0.09
Maximum	1.23	1.22	0.67	1.26	1.31	0.43
Mean # obs. with component > 0	204	211	192	185	195	178
Mean # obs. with component < 0	139	134	156	116	124	140
Mean # obs. with component = 0	27	25	22	69	51	52
Mean # obs. (i) efficient in t and (ii) with component > 0 between t and t+1	-	56	-	-	32	-
Mean # obs. (i) efficient in t and (ii) with component < 0 between t and t+1	-	23	-	-	17	-

Table 2. Results of the Closeness of Productivity/Efficiency Distribution

Indicators	z-test statistics	Conclusion
Efficiency Score	3.582***	Reject Null hypotheses
Productivity Change	4.723***	Reject Null hypotheses
Technological Change	5.942***	Reject Null hypotheses
Efficiency Change	3.499***	Reject Null hypotheses

Note: *** Significant at 1 % level.

Table 3. Testing Convergence of Productivity Changes

Parameter	Non-Convex Production		
	TFP	TC	EC
β	-0.009** (-2.69)	-0.012** (-2.39)	-0.007*** (-4.71)
R ²	0.036	0.027	0.052
Parameter	Convex Production		
	TFP	TC	EC
β	-0.004 (-1.63)	-0.003 (-1.52)	-0.006** (-2.86)
R ²	0.012	0.010	0.039

Note.—Values in parentheses are t-values. * Significant at 10% level. ** Significant at 5% level.

*** Significant at 1% level.

Table 4. Testing Convergence of Productivity Changes: σ Convergence

Standard deviations	Non-Convex Production		
	TFP	TC	EC
1947	0.283	0.293	0.262
1998	0.256	0.243	0.250
Standard deviations	Convex Production		
	TFP	TC	EC
1947	0.243	0.302	0.224
1998	0.256	0.312	0.239