An Estimated DSGE Model with a Deflation Steady State: An Application to the Japanese Economy*

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Abstract

Benhabib, Schmitt-Grohé, and Uribe (2001) argue the existence of a deflation steady state when the zero lower bound on the nominal interest rate is considered in a Taylor-type monetary policy rule. This paper estimates a medium-scale DSGE model with the deflation steady state for the Japanese economy during the period from 1999 to 2013, when the Bank of Japan conducted the zero interest rate policy and the inflation rate was almost always negative. Although the model exhibits equilibrium indeterminacy around the deflation steady state, a set of specific equilibria is selected by Bayesian methods. According to the estimated model, shocks to the demand side of the economy do not necessarily have an inflationary effect, in contrast to a standard model with a targeted-inflation steady state. An economy in the deflation equilibrium could be unexpectedly volatile due to sunspot fluctuations, but it turns out that the effect of sunspot shocks to Japan’s business cycles is marginal and that macroeconomic stability during the period is maintained by good luck.

Keywords: Deflation, Zero interest rate, Japanese economy, Indeterminacy, Bayesian estimation

JEL Classification: E31, E32, E52

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1 Introduction

Dynamic stochastic general equilibrium (DSGE) models have become a popular tool in macroeconomics. In particular, following the development of Bayesian estimation and evaluation techniques, an increased number of researchers have estimated DSGE models for empirical research as well as quantitative policy analysis. These models typically consist of optimizing behavior of households and firms and a monetary policy rule, along the lines of King (2000) and Woodford (2003). In this class of models, a central bank follows an active monetary policy rule, that is, the nominal interest rate is adjusted more than one for one when inflation deviates from a given target, and the economy fluctuates around the steady state where actual inflation coincides with the targeted inflation. In addition to such a target-inflation steady state, Benhabib, Schmitt-Grohé, and Uribe (2001) argue that the combination of an active monetary policy rule and the zero lower bound on the nominal interest rate gives rise to another long-run equilibrium, called a deflation steady state, where the inflation rate is negative and the nominal interest rate is very close to zero.

The primary contribution of this paper is to build and estimate a DSGE model with the deflation steady state for the Japanese economy, whereas almost all the existing studies have estimated DSGE models with the targeted-inflation steady state. The analysis in the present paper is motivated by Bullard (2010), who points out the possibility that the Japanese economy has been stuck in the deflation equilibrium. Figure 1 plots the nominal interest rate and inflation in Japan during the period from 1981Q1 to 2013Q1. In the figure, following Bullard (2010), a nonlinear Taylor-type monetary policy rule (thick solid line) is fitted to the data. Moreover, two long-run Fisher relations are added: One (dotted line) is the Fisher equation where the real interest rate \( r \) is fixed at 3.30, which is the mean of the ex-post real interest rate for the sample from 1981Q1 to 1998Q4. The other (dashed line) is the one where \( r = 1.42 \), which is the same mean for the sample from 1999Q1 to 2013Q1. During the latter sample period, the Bank of Japan conducted the virtually zero interest rate policy, with the exception of August 2000–March 2001 and July 2006–December 2008, and the inflation rate was almost always negative. As argued in Benhabib, Schmitt-Grohé, and Uribe (2001), two steady states emerge as the intersections of the nonlinear policy rule and the Fisher equations, that is, the steady states for the

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1The only exception is Hirose (2007), who estimates a prototypical New Keynesian DSGE model for the Japanese economy around the deflation steady state with the nominal interest rate being exogenous.

2The overnight call rate, which is the monetary policy rate in Japan, is used for the nominal interest rate. Inflation is measured as the percent change in the GDP deflator from one year earlier.
pre- and post-1999 period correspond to the targeted-inflation and deflation steady state, respectively. Therefore, Japan’s macroeconomic fluctuations during the zero interest rate period are possibly well characterized as equilibrium dynamics near the deflation steady state.

Specifically, this paper estimates a medium-scale DSGE model, along the lines of Christiano, Eichenbaum, and Evans (2005), Smets and Wouters (2003, 2007), and Justiniano, Primiceri, and Tambalotti (2010), approximated around the deflation steady state, using the data from 1999Q1 to 2013Q1 in Japan. The difficulty in estimating the model around the deflation steady state is that the equilibrium is indeterminate, i.e., there exist an infinite number of equilibrium trajectories that converge to the deflation steady state, due to a passive monetary policy which is constrained by the zero lower bound on the nominal interest rate. In this regard, following Lubik and Schorfheide (2004), a set of specific equilibrium paths is selected among an infinite number of equilibria using Bayesian methods.

Through the lens of the estimated model, the characteristics of the Japanese economy during the zero interest rate period are revealed. First, shocks to the demand side of the economy do not necessarily have an inflationary effect, in contrast to a standard model with the targeted-inflation steady state. This finding about the inflation responses to the demand shocks provides a novel view about flattening of the short-run Phillips curve in Japan, which has been argued in the literature such as Nishizaki and Watanabe (2000) and De Veirman (2009). While the analyses in the previous studies are based on the estimation of reduced-form Phillips curves, our full-information-based estimation of the DSGE model offers a structural interpretation about their arguments. According to the estimated structural parameters, the slope of the Phillips curve itself does not become flat. Rather, the ambiguity of the inflation responses to demand shocks, as shown in the impulse response analysis, leads to a weak comovement between inflation and output. This weak comovement can be identified as flattening of the Phillips curve in the estimation of reduced-form equations. Second, while an economy in the deflation equilibrium could be unexpectedly volatile due to sunspot shocks, which are non-fundamental disturbances, our estimation results show that the effect of sunspot shocks to Japan’s business cycle fluctuations is quite marginal. On the contrary, the sunspot shocks contribute to stabilize the economy over the business cycles. Therefore, macroeconomic stability during the zero interest period is maintained by good luck.

The model estimated in this paper can be the first benchmark model to investigate the Japanese economy during the zero interest rate period. In the literature, Sugo and Ueda (2007), Kaihatsu and Kurozumi (2010), Fueki, Fukunaga, Ichiue, and Shirota (2010),
Hirakata, Sudo, and Ueda (2011), Iwata (2011), Hirose and Kurozumi (2012), and Ichiu, Kurozumi, and Sunakawa (2013) estimate medium-scale DSGE models using Japanese data. However, these authors either exclude the zero interest rate period from their samples or ignore the zero lower bound constraint on the nominal interest rate in their estimation because of computational difficulties in the treatment of nonlinearity arising from the bound. Although the model in the present paper does not take account of the zero lower bound explicitly, the effect of ignoring it is mitigated around the deflation steady state, where the slopes of the monetary policy rule with respect to inflation and output are very flat.

This paper also contributes to the literature on the estimation of DSGE models under equilibrium indeterminacy. With several exceptions such as Hirose (2007, 2008, 2013), Belaygorod and Dueker (2009), Bhattarai, Lee, and Park (2012a, 2012b), and Zheng and Guo (2013), there have been still few papers that estimate indeterminate models using the methods developed by Lubik and Schorfheide (2004). This paper is the first empirical work that applies Lubik and Schorfheide’s approach to the estimation of a medium-scale DSGE model under indeterminacy, whereas the previous studies estimate relatively small models. A technical contribution here is that our estimation procedure numerically computes a particular solution for the medium-scale DSGE model such that the impulse responses of endogenous variables to fundamental shocks are continuous at the boundary between the determinacy and indeterminacy region while Lubik and Schorfheide (2004) analytically characterize such a solution for a prototypical New Keynesian model. Moreover, our model is estimated based on another particular solution where the contributions of fundamental and sunspot shocks to endogenous forecast errors are orthogonal, and we find from Bayesian model comparison that the former solution is well fitted to the data.

The most closely related paper is Aruoba and Schorfheide (2013). They consider a Markov switching between the targeted-inflation and deflation steady state in a simple sticky price DSGE model and estimate whether the U.S. economy has been in either the targeted-inflation or deflation equilibrium during and after the 2008–09 recession, given the model parameters estimated for the sample from 1984 to 2007. Their switching assumption is appropriate for the U.S. economy because the U.S. never experienced a prolonged deflation period. In contrast, Japan has experienced long-lasting deflation since 1999, which validates the assumption in the present paper that the Japanese economy has

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3Empirical studies that estimate DSGE models with the zero lower bound are still scarce. A remarkable exception is Gust, López-Salido, and Smith (2012) who estimate a nonlinear DSGE model in which the interest-rate lower bound is occasionally binding for the U.S. economy.

4This particular solution is directly obtained with the algorithm described in Sims (2002).
been stuck in the deflation equilibrium during the sample period.

The remainder of this paper proceeds as follows. Section 2 presents a DSGE model with the deflation steady state. Section 3 describes the solution and econometric strategy for estimating the model. Section 4 reports estimation results. Section 5 concludes.

2 The Model

The model is a medium-scale DSGE model along the lines of Christiano, Eichenbaum, and Evans (2005), Smets and Wouters (2003, 2007), and Justiniano, Primiceri, and Tambalotti (2010), but differs from these models in the following respects. First, households’ preferences are specified as in Erceg Guerrieri, and Gust (2006), which ensures the existence of the balanced growth under the constant relative risk aversion (CRRA) utility function. Second, following Greenwood, Hercowitz, and Huffman (1988), the model assumes that a higher utilization rate of capital leads to a higher depreciation rate of capital. This assumption is supported by Sugo and Ueda (2007) who estimate a Christiano, Eichenbaum, and Evans-type model with the same assumption for the Japanese economy and successfully replicate a negative correlation between capital utilization and rental cost observed in the data. Last but not least, the equilibrium conditions are approximated around the deflation steady state, which most distinguishes our model from those in the existing studies.

In the model economy, there are a continuum of households, a representative final-good firm, a continuum of intermediate-good firms, and a central bank. Their optimization problems and equilibrium conditions are presented below.

2.1 Households

There is a continuum of households $h \in [0, 1]$, each of which purchases consumption goods $C_t(h)$ and one-period riskless bonds $B_t(h)$ and supplies one kind of differentiated labor services $l_t(h)$ to intermediate-good firms. Each household’s preferences are represented by the utility function

$$E_0 \sum_{t=0}^{\infty} \beta^t e^{b_t} \left\{ \frac{(C_t(h) - \gamma C_{t-1}(h))^{1-\sigma}}{1 - \sigma} - \frac{Z_t^{1-\sigma} e^{b_t} l_t(h)^{1+\chi}}{1 + \chi} \right\},$$

where $\beta \in (0, 1)$ is the subjective discount factor, $\sigma > 0$ measures the risk aversion, $\gamma \in (0, 1)$ represents habit persistence in consumption preferences, $\chi > 0$ is the inverse of the labor supply elasticity, $Z_t$ represents the level of neutral technology, and $z_t^b, z_t^l$ denote shocks relevant to the subjective discount factor and to labor supply. As in Erceg
Guerrieri, and Gust (2006), we assume the presence of $Z_t^{1–\sigma}$ in the labor disutility, which ensures the existence of the balanced growth path in the model economy.

At the beginning of period, each household owns capital stock $K_{t–1}(h)$ and rents utilization-adjusted capital $u_t(h)K_{t–1}(h)$ to intermediate-good firms at the real price $R_t^e(h)$. Then, the capital utilization rate $u_t(h)$ and investment spending $I_t(h)$ are determined subject to the capital accumulation equation

$$K_t(h) = \{1 – \delta(u_t(h))\} K_{t–1}(h) + \left\{1 – S \left( \frac{I_t(h)}{I_{t–1}(h)} \frac{e^{z_i}}{z} \right) \right\} I_t(h). \quad (1)$$

Here, following Greenwood, Hercowitz, and Huffman (1988), the model assumes that a higher utilization rate of capital leads to a higher depreciation rate of capital. Hence, the depreciation rate function $\delta(\cdot)$ has properties of $\delta’ > 0$, $\delta'' > 0$, $\delta(u) = \delta \in (0, 1)$, and $\mu = \delta'(u)/\delta''(u) > 0$, where $u = 1$ is the steady-state capital utilization rate. The function $S(\cdot)$ represents costs involved in changing investment spending, such as financial intermediation costs analyzed by Carlstrom and Fuerst (1997), and takes the quadratic form of $S(x) = (x – 1)^2/(2\zeta)$, where $\zeta$ is a positive constant. The variable $z_i$ is a shock to the investment adjustment costs. The parameters $z > 1$ represents the gross balanced growth rate.

Each household’s budget constraint is given by

$$C_t(h) + I_t(h) + \frac{B_t(h)}{P_t} = W_t(h)l_t(h) + R_t^e(h)u_t(h)K_{t–1}(h) + R_{t–1}^n \frac{B_{t–1}(h)}{P_t} + T_t(h),$$

where $P_t$ is the price of final goods, $W_t(h)$ is real wage, $R_t^n$ is the gross nominal interest rate and $T_t(h)$ consists of a lump-sum public transfer and profits received from firms.

In the presence of complete insurance markets, the decisions are the same for all households, and hence the first-order conditions with respect to consumption, bond-holdings, investment, capital utilization, and capital stock are given by

$$\Lambda_t = e^{z_i} (C_t – \gamma C_{t–1})^{–\sigma} – \beta \gamma E_t e^{\gamma + 1} (C_{t+1} – \gamma C_t)^{–\sigma}, \quad (2)$$

$$\Lambda_t = \beta E_t \Lambda_{t+1} \frac{R_t^n}{\pi_{t+1}} \quad (3)$$

$$R_t^e = Q_t \delta'(u_t), \quad (4)$$

$$1 = Q_t \left\{1 – S \left( \frac{I_t}{I_{t–1}} \frac{e^{z_i}}{z} \right) – S' \left( \frac{I_t}{I_{t–1}} \frac{e^{z_i}}{z} \right) \left( \frac{I_{t+1}}{I_t} \frac{e^{z_{i+1}}}{z} \right) \right\}$$

$$+ \beta E_t \frac{\Lambda_{t+1}}{\Lambda_t} Q_{t+1} S' \left( \frac{I_{t+1}}{I_t} \frac{e^{z_{i+1}}}{z} \right) \left( \frac{I_{t+1}}{I_t} \right)^2 \frac{e^{z_{i+1}}}{z}, \quad (5)$$
where $\Lambda_t$ is the Lagrange multiplier interpreted as marginal utility of income, $\pi_t = P_t/P_{t-1}$, $Q_t$ is the real price of capital.

In monopolistically competitive labor markets, nominal wages are set on a staggered basis à la Calvo (1983) in the face of the labor demand given by

$$l_t(h) = l_t(W_t(h)/W_t)^{-(1+\lambda^w_t)/\lambda^w_t},$$

where $l_t = \left\{ \int_0^1 l_t(h)^{1/(1+\lambda^w_t)}dh \right\}^{1+\lambda^w_t}$ and $\lambda^w_t > 0$ is related to the substitution elasticity between differentiated labor services and represents the exogenous time-varying wage markup. In each period, a fraction $1 - \xi_w \in (0, 1)$ of wages is reoptimized, while the remaining fraction $\xi_w$ is set by indexation to the balanced growth rate $z$ as well as a weighted average of past inflation $\pi_{t-1}$ and steady-state inflation $\pi$. Then, the reoptimized wages solve the following problem

$$\max_{W_t(h)} \sum_{j=0}^{\infty} (\beta \xi_w)^j \left\{ \Lambda_{t+j} l_{t+j}(h) \frac{P_t W_t(h)}{P_{t+j}} \prod_{k=1}^j \left( \pi_{t+k-1} \pi^{1-\gamma_w} \right) - \frac{e^{\xi_{t+j}} Z^{1-\sigma}_{t+j} e^{\xi_{t+j}} l_{t+j}(h)^{1+\chi}}{1 + \chi} \right\},$$

subject to

$$l_{t+j}(h) = l_{t+j} \frac{P_t W_t(h)}{P_{t+j} W_{t+j}} \prod_{k=1}^j \left( \pi_{t+k-1} \pi^{1-\gamma_w} \right)^{-\lambda^w_{t+j} \xi_w},$$

where $\gamma_w \in [0, 1]$ is the weight of wage indexation to past inflation relative to steady-state inflation. The first-order condition for the reoptimized wage $W^o_t$ is given by

$$E_t \sum_{j=0}^{\infty} \left[ (\beta \xi_w)^j \frac{\Lambda_{t+j} l_{t+j}}{\lambda^w_{t+j}} \left\{ \frac{z^{j} W^o_t}{W_{t+j}} \prod_{k=1}^j \left( \pi_{t+k-1} \pi^{1-\gamma_w} \right)^{-\lambda^w_{t+j} \xi_w} \right\} - \frac{e^{\xi_{t+j}} Z^{1-\sigma}_{t+j} e^{\xi_{t+j}} l_{t+j}(h)^{1+\chi}}{1 + \chi} \right] = 0.$$  

The aggregate wage equation $W_t = \left( \int_0^1 W_t(h)^{(1+\lambda^w_t)/\lambda^w_t} dh \right)^{-\lambda^w_t}$ can be expressed as

$$1 = (1 - \xi_w) \left( \frac{W^o_t}{W_t} \right)^{-\frac{1}{\lambda^w_t}} + \sum_{j=1}^{\infty} \xi_w \left( \frac{z^{j} W^o_t}{W_t} \prod_{k=1}^j \left( \pi_{t+k-1} \pi^{1-\gamma_w} \right)^{-\lambda^w_{t+j} \xi_w} \right)^{-\frac{1}{\lambda^w_t}}.$$  

2.2 Firms

2.2.1 Final-good firm

The final-good firm produces output $Y_t$ by choosing a combination of intermediate inputs $\{Y_t(f)\}, f \in [0, 1]$, so as to maximize its profit $P_t Y_t - \int_0^1 P_t(f) Y_t(f) df$ subject to the
production technology \( Y_t = \left\{ \int_0^1 Y_t(f)^{1/(1+\lambda_t^p)} df \right\}^{1+\lambda_t^p} \), where \( P_t(f) \) is the price of intermediate good \( f \) and \( \lambda_t^p > 0 \) is related to the substitution elasticity between differentiated goods and corresponds to the exogenous time-varying price markup.

The first-order condition for profit maximization yields the final-good firm’s demand for each intermediate good given by \( Y_t(f) = Y_t(P_t(f)/P_t)^{-(1+\lambda_t^p)/\lambda_t^p} \), while perfect competition in the final-good market leads to its price \( P_t \) given by

\[
P_t = \left( \int_0^1 P_t(f)^{-\lambda_t^p} df \right)^{-\lambda_t^p}.
\]

(9)

The market clearing condition for final good is

\[
Y_t = C_t + I_t + gZ_t e^{z_t^q},
\]

(10)

where the term \( gZ_t e^{z_t^q} \) represents an external demand component. \( z_t^q \) is an external demand shock, and \( g \) is a scale parameter.

2.2.2 Intermediate-good firms

Each intermediate-good firm \( f \) produces output \( Y_t(f) \) by choosing a cost-minimizing pair of capital and labor services \( \{u_t K_{t-1}(f), l_t(f)\} \), given their real rental prices \( (R_t^K, W_t) \) and the production function

\[
Y_t(f) = (Z_t l_t(f))^{1-\alpha} (u_t K_{t-1}(f))^{\alpha} - \phi Z_t.
\]

(11)

Here, \( Z_t \) represents the level of neutral technology and is assumed to follow the stochastic process

\[
\log Z_t = \log z + \log Z_{t-1} + z_t^z,
\]

(12)

where \( z > 1 \) is the gross rate of neutral technological changes and \( z_t^z \) represents a shock to the rate of the changes. The parameter \( \alpha \in (0, 1) \) measures the capital elasticity of output. The last term in the production function (11), \( -\phi Z_t \), is a fixed cost of producing intermediate goods, and \( \phi \) is a positive constant.\(^5\)

Combining cost-minimizing conditions with respect to capital and labor services shows that real marginal cost is identical among intermediate-good firms and is given by

\[
m_{c_t} = \left( \frac{W_t}{(1-\alpha)Z_t} \right)^{1-\alpha} \left( \frac{R_t^k}{\alpha} \right)^{\alpha}.
\]

(13)

\(^5\)The zero profit condition for intermediate good firms at the steady state leads to \( \phi = \lambda^p \), where \( \lambda^p \) is the steady-state price markup.
Also, combining the cost-minimizing conditions and aggregating the resulting equation over intermediate-good firms show that the capital-labor ratio is identical among intermediate-good firms and is given by

\[ \frac{u_t K_{t-1}}{l_t} = \frac{\alpha W_t}{(1 - \alpha) R_t^{x_t}} \]  

(14)

where \( K_t = \int_0^1 K_t(f) df \) and \( l_t = \int_0^1 l_t(f) df \). Moreover, using this equation to aggregate the production function (11) over intermediate-good firms yields

\[ Y_t d_t = (Z_t l_t)^{1-\alpha} (u_t K_{t-1})^\alpha - \phi Z_t, \]

(15)

where \( d_t = \int_0^1 (P_t(f)/P_t)^{-(1+\lambda_t^P)/\lambda_t^P} df \) measures the intermediate-good price dispersion and is of second order under the staggered price-setting presented below.

Facing the final-good firm’s demand, each intermediate-good firm sets the price of its product on a staggered basis à la Calvo (1983). Each period a fraction \( 1 - \xi_p \in (0, 1) \) of intermediate-good firms reoptimizes prices, while the remaining fraction \( \xi_p \) indexes prices to a weighted average of past inflation \( \pi_{t-1} \) and steady-state inflation \( \pi \). Then, the firms which reoptimize prices in the current period solve the problem

\[ \max_{P_t(f)} E_t \sum_{j=0}^{\infty} \xi_p^j \left( \beta_j \frac{\Lambda_{t-j}}{\Lambda_t} \right) \left\{ \frac{P_t(f)}{P_{t+j}} \frac{1}{\prod_{k=1}^{j} (\pi_{t+k-1}^{\gamma_p} \pi_{t+k})^{-\frac{\lambda_{t+j}^P}{\lambda_{t+j}}} - mc_{t+j}} \right\} Y_{t+j}(f) \]

subject to

\[ Y_{t+j}(f) = Y_{t+j} \left\{ \frac{P_t(f)}{P_{t+j}} \frac{1}{\prod_{k=1}^{j} (\pi_{t+k-1}^{\gamma_p} \pi_{t+k})^{-\frac{\lambda_{t+j}^P}{\lambda_{t+j}}} - mc_{t+j}} \right\}, \]

where \( \gamma_p \in [0, 1] \) is the weight of price indexation to past inflation relative to steady-state inflation. The first-order condition for the reoptimized price \( P_t^o \) is given by

\[ E_t \sum_{j=0}^{\infty} \left( \beta_j \xi_p^j \frac{\Lambda_{t-j}}{\Lambda_t} Y_{t+j} \left\{ \frac{P^o_t}{P_{t+j}} \frac{1}{\prod_{k=1}^{j} (\pi_{t+k-1}^{\gamma_p} \pi_{t+k})^{-\frac{\lambda_{t+j}^P}{\lambda_{t+j}}} - mc_{t+j}} \right\} \right) = 0. \]

(16)

The final-good price equation (9) can be written as

\[ 1 = (1 - \xi_p) \left( \frac{P^o_t}{P_t} \right)^{-\frac{1}{\lambda_t^P}} + \sum_{j=1}^{\infty} \xi_p^j \left\{ \frac{P^o_{t-j}}{P_{t-j}} \frac{1}{\prod_{k=1}^{j} (\pi_{t-k}^{\gamma_p} \pi_{t-k})^{-\frac{\lambda_{t+j}^P}{\lambda_{t+j}}} - mc_{t+j}} \right\}^{-\frac{1}{\lambda_t^P}} \].

(17)

### 2.3 Central bank

The central bank adjusts the nominal interest rate following a monetary policy rule of the form

\[ R_t^m = M \left( \pi_t, \frac{Y_t}{Z_t}, R_{t-1}^m, z_t^r \right), \]

(18)
where \( z_r \) is a monetary policy shock which captures an unsystematic component of monetary policy.

Although the functional form of \( M(\cdot) \) is not specified at this stage, three assumptions are made regarding this monetary policy rule, as in Benhabib, Schmitt-Grohé, and Uribe (2001). First, the nominal interest rate is nondecreasing in inflation, output, and the past interest rate, i.e., \( \partial M / \partial \pi_t \geq 0 \), \( \partial M / \partial (Y_t/Z_t) \geq 0 \), and \( \partial M / \partial R^n_{t-1} \geq 0 \). Second, there exists the zero lower bound constraint on the nominal interest rate. Thus, \( M > 1 \) for all \( \{\pi_t, Y_t/Z_t, R^n_{t-1}, z_r\} \). Third, around the inflation target, the monetary policy rule satisfies the so-called Taylor principle, that is, the nominal interest rate increases (decreases) more than one percent in response to a one-percent increase (decrease) in the inflation rate.

### 2.4 Steady states and approximated equilibrium conditions

The equations (1)–(8), (10), (12)–(18), are the equilibrium conditions for the model economy. In the model, the variables regarding real economic activities are non-stationary since the level of neutral technology has unit roots with drift as shown in (12). Thus, we rewrite the equilibrium conditions in terms of stationary variables detrended by \( Z_t \), i.e., \( y_t = Y_t/Z_t \), \( c_t = C_t/Z_t \), \( w_t = W_t/Z_t \), \( \lambda_t = \Lambda_t Z^n_t \), \( i_t = I_t/Z_t \), and \( k_t = K_t/Z_t \), so that we can compute the steady states for the detrended variables.

As is argued in Benhabib, Schmitt-Grohé, and Uribe (2001), combining the monetary policy rule (18) described above and the Fisher equation, i.e., \( R^n_t = R_t E_t \pi_{t+1} \), where \( R_t \) is the gross real interest rate, yields two steady states, which we call the targeted-inflation steady state and the deflation steady state. Analogous to Figure 1, in the targeted-inflation steady state, the gross inflation and nominal interest rates are expressed as \( \pi^* > 1 \) and \( R^{n*} = R \pi^* \), respectively. In the deflation steady state, they are denoted by \( \pi^D \) and \( R^{nD} \). Notice that \( \pi^D \) is very close to (but not equal to) \( 1/R < 1 \), and \( R^{nD} \) is very close to (but not equal to) unity.\(^6\) A remarkable feature of our analysis is that the model is approximated around the deflation steady state, whereas the almost all the estimated DSGE models in the existing studies are approximated around the targeted-inflation steady state. Specifically, the monetary policy rule (18) is approximated as

\[
\hat{R}^n_t = \psi_R \hat{R}^n_{t-1} + (1 - \psi_R) (\psi_\pi \hat{\pi}_t + \psi_y \hat{y}_t) + z_r^t, \tag{19}
\]

where \( \psi_R \in [0, 1) \) is the degree of policy rate smoothing, \( \psi_\pi > 0 \) and \( \psi_y > 0 \) are the degrees of policy responses to inflation and output respectively, and the variables with

\(^6\)Hirose (2007) and Aruoba and Schorfheide (2013) consider monetary policy rules which are kinked at zero, and hence their steady-state values are given by \( \pi^D = 1/R \) and \( R^{nD} = 1 \).
\( \xi \) represent log-deviations from steady-state values. This equation appears to be the same as a standard Taylor-type monetary policy rule around the targeted-inflation steady state, but it differs in the policy coefficients. In particular, the degrees of policy responses to inflation and output are very small because there is little room for lowering the nominal interest rate in response to a decrease in inflation and output due to the existence of the zero lower bound. As a consequence, the monetary policy rule approximated around the deflation steady state does not satisfy the Taylor principle.

The monetary policy rule in the present model does not take account of the zero lower bound constraint explicitly in the sense that the nominal interest rate suggested by (19) can take a negative value. However, the effect of ignoring the constraint should be marginal near the deflation steady state, where the slopes of the monetary policy rule with respect to inflation and output are very flat. In such a circumstance, the hypothetical nominal interest rate will not be substantially negative even if large negative shocks against inflation and output occur in the economy.

Log-linearizing the other equilibrium conditions represented in terms of the detrended variables and rearranging the resulting equations with the steady-state conditions lead to

\[
\left( 1 - \frac{\beta \gamma}{z^\sigma} \right) \tilde{x}_t = -\frac{\sigma z}{z - \gamma} \left\{ \tilde{c}_t - \frac{\gamma}{z} (\tilde{c}_{t-1} - z^\tau) \right\} + z^b_t \\
+ \frac{\beta \gamma}{z^\sigma} \left\{ \frac{\sigma z}{z - \gamma} \left( E_t \tilde{c}_{t+1} + E_t z^z_{t+1} - \frac{\gamma}{z} \tilde{c}_t \right) - E_t z^b_{t+1} \right\},
\]

(20)

\[
\tilde{x}_t = E_t \tilde{x}_{t+1} - \sigma E_t z^z_{t+1} + \tilde{R}^n_t - E_t \tilde{n}_{t+1},
\]

(21)

\[
\tilde{u}_t = \mu \left( \tilde{R}^k_t - \tilde{g}_t \right),
\]

(22)

\[
\frac{1}{\zeta} \left( \tilde{u}_t - \tilde{u}_{t-1} + z^\tau_t + z^i_t \right) = \tilde{q}_t + \frac{\beta z^{1-\sigma}}{\zeta} \left( E_t \tilde{u}_{t+1} - \tilde{u}_t + E_t z^z_{t+1} + E_t z^i_{t+1} \right),
\]

(23)

\[
\tilde{q}_t = E_t \tilde{x}_{t+1} - \tilde{c}_t - \sigma E_t z^z_{t+1} + \frac{\beta}{z^\sigma} \left( R^k E_t \tilde{R}^k_{t+1} + (1 - \delta) E_t \tilde{q}_{t+1} \right),
\]

(24)

\[
\tilde{u}_t = \frac{1 - \delta}{z} \left( \tilde{u}_{t-1} - z^\tau_t \right) + \frac{R^k}{z} \tilde{u}_t + \left( 1 - \frac{1 - \delta}{z} \right) \tilde{u}_t,
\]

(25)

\[
\tilde{u}_t - \tilde{u}_{t-1} + \tilde{n}_t - \gamma_w \tilde{n}_{t-1} - z^z_t
\]

\[
= \beta z^{1-\sigma} \left( E_t \tilde{u}_{t+1} - \tilde{u}_t + E_t \tilde{n}_{t+1} - \gamma_w \tilde{n}_t + E_t z^z_{t+1} \right)
\]

\[
+ \frac{(1 - \xi_w)(1 - \xi_w \beta z^{1-\sigma})}{\xi_w \chi(1 + \chi w)} \left[ \chi \tilde{u}_t - \tilde{c}_t - \tilde{u}_t + z^b_t \right] + z^w_t,
\]

(26)

\[
\tilde{y}_t = \frac{c}{y} \tilde{c}_t + \frac{i}{y} \tilde{i}_t + \frac{q}{y} z^g_t,
\]

(27)

\[
\tilde{m}_c_t = (1 - \alpha) \tilde{w}_t + \alpha \tilde{R}^k_t,
\]

(28)
\[ \hat{w}_t - \tilde{R}^k_t = \tilde{u}_t + \tilde{k}_{t-1} - \tilde{r}_t - z^*_t, \]  
\[ \tilde{y}_t = (1 + \lambda^p) \left\{ (1 - \alpha) \tilde{r}_t + \alpha \left( \tilde{u}_t + \tilde{k}_{t-1} - z^*_t \right) \right\}, \]  
\[ \tilde{\pi}_t - \gamma_p \tilde{\pi}_{t-1} = \beta z^{1-\sigma} (E_t \tilde{\pi}_{t+1} - \gamma_p \tilde{\pi}_t) + \frac{(1 - \xi_p)(1 - \xi_p \beta z^{1-\sigma})}{\xi_p} \tilde{m}_t + \tilde{z}^p_t, \]  
where \( z^w_t = \frac{(1 - \xi_w)(1 - \gamma_w z^{1-\sigma}) \lambda^w}{1 + \lambda^w} (\tilde{\pi}_t + z^*_t) \) is a composite shock relevant to the labor disutility and the wage markup (hereafter called a wage markup shock), \( \tilde{z}^p_t = \frac{(1 - \xi_p)(1 - \beta_p z^{1-\sigma})}{\xi_p} \tilde{\pi}_t \) is a shock associated with the price markup, and the variables without time subscripts represent their steady-state values. The steady-state relationships are given by

\[ \beta = \frac{z^\sigma}{\tilde{R}}, \]
\[ R^k = R - 1 + \delta, \]
\[ w = (1 - \alpha) \left( \frac{1}{1 + \lambda^p} \right)^{1-\alpha} \left( \frac{R^k}{\alpha} \right)^{-1-\alpha}, \]
\[ k \]
\[ \frac{\alpha z w}{(1 - \alpha) R^k}, \]
\[ \phi = \lambda^p, \]
\[ \frac{k}{y} = (1 + \phi) z^\alpha \left( \frac{k}{l} \right)^{1-\alpha} + \frac{k}{y}, \]
\[ \frac{i}{y} = \left( 1 - \frac{1 - \delta}{z} \right) \frac{k}{y}, \]
\[ \frac{c}{y} = 1 - \frac{i}{y} - \frac{g}{y}. \]

### 2.5 Fundamental shock processes

The model contains seven fundamental shocks, i.e., technology \( z^*_t \), preference \( z^b_t \), investment adjustment cost \( z^i_t \), external demand \( z^g_t \), wage markup \( z^w_t \), price markup \( z^p_t \), and monetary policy \( z^r_t \) shocks. Each of them follows a univariate stationary first-order autoregressive process:

\[ z^x_t = \rho_x z^x_{t-1} + \varepsilon^x_t, \quad \varepsilon^x_t \sim i.i.d. N(0, \sigma^2_x), \]  

where \( \rho_x \in [0, 1) \) and \( x \in \{ z, b, i, g, w, p, r \} \).
3 Model Solution and Econometric Methodology

The equations (19)–(31), together with the stochastic processes of exogenous shocks (32), constitute a linear rational expectation system. It has been well known that sticky price monetary DSGE models have multiple equilibria, often referred to as indeterminacy, if the Taylor principle is not satisfied.\(^7\) As addressed in the preceding section, the monetary policy rule approximated around the deflation steady state does not satisfy the Taylor principle, and hence the present system exhibits equilibrium indeterminacy. In this section, we describe how to solve and estimate the model under indeterminacy.

3.1 Solution under indeterminacy

In solving the rational expectations system under indeterminacy, we follow the approach of Lubik and Schorfheide (2003), who provide a full set of non-unique solutions in linear rational expectations models by extending the solution algorithm developed by Sims (2002).\(^8\) In their approach, the log-linearized system is written in the following canonical form:

\[
\Gamma_0 (\theta) s_t = \Gamma_1 (\theta) s_{t-1} + \Psi_0 (\theta) \varepsilon_t + \Pi_0 (\theta) \eta_t, \tag{33}
\]

where \(\Gamma_0 (\theta), \Gamma_1 (\theta), \Psi_0 (\theta)\) and \(\Pi_0 (\theta)\) are the conformable matrices of coefficients that depend on the structural parameters \(\theta\), \(s_t\) is a vector of endogenous variables including those expected at \(t\), and \(\varepsilon_t\) is a vector of disturbances to fundamental shocks. \(\eta_t\) is a vector of endogenous forecast errors, defined as:

\[
\eta_t = s_t^E - E_{t-1} s_t^E ,
\]

where \(s_t^E\) is a subvector of \(s_t\) that contains expected variables. In the present model, \(s_t^E = [\tilde{c}_t, \tilde{\lambda}_t, \tilde{\pi}_t, \tilde{\nu}_t, \tilde{R}_t^k, \tilde{q}_t, \tilde{w}_t, z_t^c, z_t^b, z_t^i]'\).

According to Lubik and Schorfheide (2003), the full set of rational expectations solutions is of the form:

\[
s_t = \Phi_1 (\theta) s_{t-1} + \Phi_\varepsilon (\theta, \tilde{M}) \varepsilon_t + \Phi_\zeta (\theta) \zeta_t, \tag{34}
\]

where \(\Phi_1 (\theta), \Phi_\varepsilon (\theta, \tilde{M})\), and \(\Phi_\zeta (\theta)\) are the coefficient matrices, \(\tilde{M}\) is an arbitrary matrix, and \(\zeta_t\) is a reduced-form sunspot shock,\(^9\) which is a non-fundamental stochastic distur-

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\(^7\)See, for instance, Bullard and Mitra (2002) or Woodford (2003).

\(^8\)Sims’ solution method generalizes the technique in Blanchard and Kahn (1980) and characterizes one particular solution in the case of indeterminacy.

\(^9\)Lubik and Schorfheide (2003) originally express the last term in (34) as \(\Phi_\zeta (\theta, M_\zeta) \zeta_t\), where \(M_\zeta\) is another arbitrary matrix, and \(\zeta_t\) is a vector of sunspot shocks. For identification, Lubik and Schorfheide
bance.\textsuperscript{10} For estimation, it is assumed that $\zeta_t \sim i.i.d. N(0, \sigma^2_{\zeta})$. The solution (34) has two important features under indeterminacy. First, business cycle fluctuations are generated not only by fundamental shocks but also by sunspot shocks. Second, the equilibrium representation cannot be unique due to the arbitrary matrix $\tilde{M}$, i.e., the model has multiple solutions, and different solutions may exhibit different propagation of shocks. Therefore, in order to specify the law of motion for the endogenous variables, we need to pin down $\tilde{M}$; otherwise, any path can be considered as the equilibrium of the model.

In this paper, following Lubik and Schorfheide (2004), the components of the arbitrary matrix $\tilde{M}$ are estimated using Bayesian methods. To this end, we construct a prior distribution that is centered on a particular solution $M^*(\theta)$. Specifically, we replace $\tilde{M}$ with $M^*(\theta) + M$ and set the prior mean for $M$ equal to zero. In the subsequent empirical analysis, two particular solutions are considered. One is the particular solution employed in Lubik and Schorfheide (2004), referred to as the “continuity solution,” where $M^*(\theta)$ is chosen such that the contemporaneous impulse responses of endogenous variables to fundamental shocks, i.e., $\partial s_t / \partial \varepsilon_t$, are continuous at the boundary between the determinacy and indeterminacy region. While Lubik and Schorfheide (2004) analytically characterize the continuity solution for a prototypical New Keynesian model, we numerically find the same solution for our medium-scale DSGE model. The other particular solution is obtained by setting $M^*(\theta) = 0$, called the “orthogonality solution,” where the contributions of fundamental shocks $\varepsilon_t$ and sunspot shocks $\zeta_t$ to the forecast errors $\eta_t$ are orthogonal. This particular solution is often used in the literature because it can be directly obtained with the algorithm described in Sims (2002).\textsuperscript{11} In what follows, this paper conducts Bayesian model comparison to investigate which particular solution is well fitted to the data.

### 3.2 Bayesian inference

The model is estimated using Bayesian methods. Seven quarterly time series of Japan’s economy are used as observable variables: the log difference of real GDP, real consumption, real investment and real wage, the log of hours worked, the log difference of the GDP deflator, and the overnight call rate. Real GDP, real consumption, and real investment are on a per capita basis, divided by the population over 15 years old. The real series

\textsuperscript{10}If the equilibrium is determinate, the solution (34) is reduced to $s_t = \Phi^D(\theta) s_{t-1} + \Phi^D(\theta) \varepsilon_t$.

\textsuperscript{11}Hirose (2007, 2008, 2013) estimates indeterminate DSGE models based on the orthogonality solution.
of consumption and investment are respectively obtained by dividing the nominal private consumption expenditures and gross private domestic investment series by the GDP deflator. The series of real wage and hours worked are constructed following Sugo and Ueda (2008).

The sample period is from 1999:1Q to 2013:1Q, when the Bank of Japan conducted the virtually zero interest rate policy, with the exception of August 2000–March 2001 and July 2006–December 2008, and the inflation rate was almost always negative. Thus, equilibrium dynamics around the deflation steady state can well replicate Japan’s macroeconomic fluctuations during the period.

The data are related to model-implied variables by the following measurement equations:

\[
\begin{bmatrix}
100\Delta \log Y_t \\
100\Delta \log C_t \\
100\Delta \log I_t \\
100\Delta \log W_t \\
100\log I_t \\
100\Delta \log P_t \\
100\log R^n_t \\
\end{bmatrix}
= 
\begin{bmatrix}
\tilde{z} \\
\tilde{z} \\
\tilde{z} \\
\tilde{z} \\
\tilde{l} \\
\tilde{\pi} \\
\tilde{r} + \tilde{\pi} \\
\end{bmatrix}
+ 
\begin{bmatrix}
\tilde{y}_t - \tilde{y}_{t-1} + \tilde{z}_t \\
\tilde{c}_t - \tilde{c}_{t-1} + \tilde{z}_t \\
\tilde{\pi}_t - \tilde{\pi}_{t-1} + \tilde{z}_t \\
\tilde{w}_t - \tilde{w}_{t-1} + \tilde{z}_t \\
\tilde{l}_t \\
\tilde{\pi}_t \\
\tilde{R}_t \\
\end{bmatrix},
\]

where \(\tilde{z} = 100\log z, \tilde{l} = 100\log l, \tilde{\pi} = 100\log \pi,\) and \(\tilde{r} = 100\log R.\)

Before estimation, some parameters are fixed to avoid identification issues. Following Sugo and Ueda (2008), we set the steady-state depreciation rate at \(\delta = 0.06/4,\) the capital elasticity of output at \(\alpha = 0.37,\) and the steady-state wage markup at \(\lambda^w = 0.2.\)

The steady-state ratios of external demand to output are set at the sample mean, i.e., \(g/y = 0.248.\)

Table 1 summarizes the prior distributions of parameters. Most of the priors for parameters that characterize the private-sector behavior \((\sigma, \gamma, \chi, 1/\zeta, \mu, \gamma^w, \xi^w, \gamma^p, \xi^p, \lambda^p)\) follow from Justiniano, Primiceri, and Tambalotti (2010), except for the parameters that determine the relative risk aversion \(\sigma\) and the inverse elasticity of the utilization adjustment cost \(\mu.\) The priors for \(\sigma\) and \(\mu\) are set according to Smets and Wouters (2007) and Sugo and Ueda (2008), respectively.

The prior distributions of the monetary policy parameters \((\psi^r, \psi^m, \psi^y)\) are distinctive in our analysis. During the sample period, the adjustment of the nominal interest rate was severely constrained by the zero lower bound, and the Bank of Japan kept the nominal interest rate almost zero for most of the period. Thus, a very small value 0.2 is assigned to the prior means for the policy response parameters \(\psi^m, \psi^y\) whereas a large value 0.9 is set for the policy smoothing parameter \(\psi^r.\)
Priors for the steady state values for the balanced growth rate, hours worked, the inflation rate, and the real interest rate ($\bar{\bar{z}}, \bar{\bar{l}}, \bar{\pi}, \bar{r}$) are set to be the normal distribution with a mean based on the sample average of the corresponding data. For these parameters, relatively tight priors are imposed to ensure the stationarity of the system. Notice that the prior mean for $\bar{\pi}$ is negative, which is consistent with the deflation steady state considered in the present model.

Priors for the shock persistence parameters ($\rho_x, x \in \{z, b, i, g, w, p, r\}$) are set to be the Beta distribution with a mean of 0.5 and a standard deviation of 0.15, and priors for the standard deviations of the shock innovations ($\sigma_x, x \in \{z, b, i, g, w, p, r, \zeta\}$) are set to be the Inverse Gamma distribution with a mean of 0.5 and a standard deviation of an infinity. Fairly wide intervals are set for the components of the arbitrary matrix $M$. For each component, we assign the normal distribution with mean zero and a standard deviation of 0.5.

The likelihood function is evaluated using the Kalman filter. Draws from the posterior distribution of model parameters are generated with the Metropolis–Hastings algorithm. Based on the posterior draws, we make inference on the parameters, impulse response functions, and variance decompositions.

4 Empirical Results

This section presents the estimation results. Based on the estimates of parameters, impulse response functions, and shock decompositions, we find the remarkable features of the Japanese economy during the zero interest rate period.

4.1 Model selection

For the post-1999 sample, the model is estimated based on two particular solutions, i.e., the continuity solution and the orthogonality solution, depending on the choice of $M^*(\theta)$. We investigate which particular solution is empirically more plausible by computing marginal data densities.

Let $\mathcal{M}_c$ and $\mathcal{M}_o$ denote the model based on the continuity solution and the orthogonality solution, respectively. The resulting log marginal data densities are

\[ \log p(Y^T|\mathcal{M}_c) = \]

---

12 In the estimation procedure, 200,000 draws are generated and the first 20,000 draws are discarded. The scale factor for the jumping distribution in the Metropolis-Hastings algorithm is adjusted so that the acceptance rate is about 25 percent.
\[-371.8 \text{ and } \log p(Y^T|\mathcal{M}_o) = -373.4, \text{ where } Y^T \text{ is the sample of observations.}\] Then, the Bayes factor is given by

\[
\frac{p(Y^T|\mathcal{M}_c)}{p(Y^T|\mathcal{M}_o)} = 4.648.
\]

According to Jeffreys (1961), this value is interpreted as “substantial” evidence in favor of the continuity solution. Therefore, in the subsequent analysis, the results based on the continuity solution are considered as baseline estimates for the Japanese economy during the post-1999 period.

### 4.2 Parameter estimates

Table 2 reports the posterior means and 90-percent credible intervals for the model parameters in three cases. The second to fifth columns compare the parameter estimates based on the continuity solution with those based on the orthogonality solution. Both estimates are by and large in line with each other. Below, we focus on the former estimates as a baseline.

For comparison, the last two columns in the table show the posterior estimates when a similar model is estimated for the period from 1983:2Q to 1998:4Q. During the period, the economy was in a normal state in the sense that the Bank of Japan was able to adjust the nominal interest rate to achieve an implicit inflation target. Taking account of such a situation, the model in this case differs from the baseline in two respects. First, the model is approximated around the targeted-inflation steady state, as is standard in the literature. Thus, we set the prior mean for steady-state inflation at \( \tilde{\pi} = 0.214 \), which is a sample mean for the pre-1999 period.\(^{14}\) Second, the monetary policy rule satisfies the Taylor principle. Then, the equilibrium is determinate, and hence the sunspot shock \( \zeta_t \) and the arbitrary matrix \( M \) no longer affect the equilibrium dynamics. Thus, priors for the policy response parameters, \( \psi_x \) and \( \psi_y \), are the Gamma distributions with means of 1.5 and 0.125, and standard deviations of 0.15 and 0.1, respectively. For the policy smoothing parameter \( \psi_r \), the Beta distribution is set with a mean of 0.5 and a standard deviation of 0.15.

The households’ preference parameters for the post-1999 sample are substantially different from those for the pre-1999 sample. The estimates regarding relative risk aversion \( \sigma \) and labor supply elasticity \( \chi \), which are still controversial in the literature, are much

\(^{13}\)The log marginal data densities are approximated using the harmonic mean estimator proposed by Geweke (1999).

\(^{14}\)The prior means for the steady-state balanced growth rate, hours worked, and the real interest rate \((\bar{z}, \bar{I}, \bar{R})\) are also set at their corresponding sample means, i.e., 0.476, 1.172, 0.778, respectively.
smaller; in particular $\sigma$ is less than one for the post-1999 sample, even concerning its 90 percent interval. The habit persistence parameter $\gamma$ is also smaller, implying less internal persistence in consumption dynamics during the period.

The elasticity parameter regarding investment adjustment cost $1/\zeta$ is almost the same while the elasticity of capital utilization adjustment costs $\mu$ is smaller for the post-1999 sample. The parameters related to wage and price setting behavior ($\gamma_w, \xi_w, \gamma_p, \xi_p, \lambda^p$) are not much different, although the wage and price indexation parameters slightly decrease.

As for the monetary policy parameters, the policy smoothing parameter $\psi_r$ is large, and the policy reaction parameters ($\psi_{\pi}, \psi_{y}$) are very small, reflecting the fact that the Bank of Japan kept the nominal interest rate virtually zero for most of the post-1999 period.

The mean estimates for the steady state balanced growth rate, hours worked, the inflation rate, and the real interest rate ($\bar{z}, \bar{I}, \bar{\pi}, \bar{r}$) are almost the same as the prior means because the tight priors are set on these parameters.

Some of the shocks’ persistence parameters ($\rho_b, \rho_i, \rho_g, \rho_p$) are lower for the post-1999 sample than for the pre-1999 sample. A straightforward explanation for this result is that the observed data have exhibited less persistent dynamics during the zero interest rate period. Another possible explanation is that, as addressed in Lubik and Schorfheide (2003, 2004), Farmer and Buyer (2007), and Fujiwara and Hirose (2014), the model under indeterminacy can internally generate more persistent dynamics observed in the data without relying on the persistency of exogenous shocks.

The standard deviations of the shocks ($\sigma_x, x \in \{z, b, i, g, w, p, r, \zeta\}$) are not much different for both periods, except for that of the preference shock $\sigma_b$. The small standard deviation of the preference shock implies that the consumption Euler equation in the present model can well capture the consumption dynamics without any wedges for the post-1999 sample. On the contrary, the large standard deviation for the pre-1999 sample can be explained by the increased volatility in consumption due to the consumption tax increases in 1989 and 1997.

As for the components of the arbitrary matrix $M$, some of them ($M_z, M_w, M_p$) are far different from zero, indicating that the propagation of the fundamental shocks can be altered compared with the one in the continuity solution. With the posterior estimates of $M$, we can specify a set of particular solutions and characterize equilibrium dynamics under indeterminacy.
4.3 Impulse responses

Figure 2–9 present the Bayesian impulse responses of output, consumption, investment, wage, hours worked, inflation, and the nominal interest rate to one-standard-deviation shocks about technology, preference, investment adjustment cost, external demand, wage markup, price markup, monetary policy, and sunspot. In each panel, the solid line and dashed lines respectively show the posterior mean and 90-percent credible interval for the estimated responses to each one-standard-deviation shock, in terms of percentage deviation from the steady state. As in the previous subsection, each figure compares the responses estimated for the post-1999 sample under indeterminacy with those for the pre-1999 sample under determinacy.

Remarkable changes are found in the estimated impulse responses to the shocks about preference (Figure 3), investment adjustment cost (Figure 4), and external demand (Figure 5). A common feature of these shocks is that they directly affect the demand side of the economy. According to the pre-1999 estimates, these shocks have an inflationary effect, as in the same type of DSGE models in the existing studies. For the post-1999 sample, however, the effect on inflation is ambiguous, that is, the 90-percent intervals are quite large and contain both positive and negative values. This result comes from the estimated arbitrary matrix $M$ and its parameter uncertainty, which is an inherent feature under indeterminacy. Why inflation can both decrease and increase in response to the positive demand shocks? Initially, the demand shocks would have a positive effect on inflation. In response to an increase in inflation, the central bank would raise the nominal interest rate following a monetary policy rule. Around the deflation steady state, however, the degree of policy response to inflation is very small due to the existence of the zero lower bound, and hence the monetary policy rule does not satisfy the Taylor principle. Consequently, the real interest rate would decrease. The low real interest rate would stimulate demand for goods and lead to higher inflation. Such a loop can make the inflation trajectory explosive, which cannot be an equilibrium. Therefore, inflation must decrease in this case. If the initial inflationary effect is moderate, the loop does not necessarily give rise to an explosive path. In such a case, an increase in inflation can be an equilibrium.

This finding about the inflation responses to the demand shocks for the post-1999 period provides a novel view about inflation dynamics in Japan. In the literature, Nishizaki and Watanabe (2000) show that Japan’s short-run Phillips curve became flatter as the inflation rate approached zero. De Veirman (2009) also provides evidence of gradual flattening of the Phillips curve since the late 1990s and argues the reason why large negative
output gap did not accelerate deflation during the period. While their analyses are based on the estimates of reduced-form equations, our analysis provides a structural interpretation for their arguments. According to the Phillips curve (31) that relates inflation to real marginal cost, its slope is expressed as \((1 - \xi_p)(1 - \xi_p \beta z^{1-\sigma})/\xi_p\). Evaluating this slope based on the posterior mean estimates in Table 2 gives 0.028 for the post-1999 sample and 0.017 for the pre-1999 sample. Thus, the slope itself did not become flat. Rather, the ambiguity of the inflation responses to demand shocks leads to a weak comovement between inflation and output, which can be identified as flattening of the Phillips curve in the estimation of reduced-form equations.

As for the propagation of technology (Figure 2), wage markup (Figure 6), price markup (Figure 7), and monetary policy shocks (Figure 8), the impulse responses estimated for the post-1999 sample are similar to those for the pre-1999 sample. The responses to these shocks are qualitatively the same as those in Christiano, Eichenbaum, and Evans (2005), Smets and Wouters (2003, 2007), and Justiniano, Primiceri, and Tambalotti (2010) since our model shares a lot of similarities with theirs.

The sunspot shock affects equilibrium dynamics only for the post-1999 period. The identified sunspot shock has positive effects on all the observables presented in Figure 9. The sunspot shock in the present model is constructed in a reduced-form manner, following Lubik and Schorfheide (2004), and hence has positive effects on expectational variables irrelevant to fundamentals. Such non-fundamental beliefs are self-fulfilling under indeterminacy and affect the actual variables in the same direction.

### 4.4 Variance decompositions

Table 3 presents the mean estimates of the asymptotic forecast error variance decompositions of output growth, consumption growth, investment growth, wage growth, hours worked, inflation, and the nominal interest rate, for the post- and pre-1999 samples.\(^{15}\) Each number shows the relative contribution of technology, preference, investment adjustment cost, external demand, wage markup, price markup, monetary policy, and sunspot shocks, in percentage terms.

Overall business cycle implications during the zero interest rate period are analyzed by focusing on the decomposition of output growth for the post-1999 sample. The primary source of output fluctuations is the technology shock, which accounts for almost a half of its volatility. This finding is consistent with the conventional wisdom in the business cycle

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\(^{15}\)The asymptotic variances are computed by solving a discrete Lyapunov equation for the system of log-linearized equilibrium conditions.
research for the U.S. economy (e.g., King and Rebelo, 1999) and the results in the existing studies on Japan’s business cycles in the 1980s and 1990s (e.g., Hayashi and Prescott, 2002; Sugo and Ueda, 2008; Hirose and Kurozumi, 2012). The second largest contribution to output fluctuations is the investment adjustment cost shock. The same result has been obtained by Hirose and Kurozumi (2012) for the Japanese economy before 1999. The external demand shock also plays a substantial role in explaining output volatility, which is compatible with a common view that Japanese economic expansion in the mid-2000s was largely dependent on export demand. A distinctive feature in our analysis is that we can assess to what extent the sunspot shock affect the macroeconomic fluctuations. However, the estimated contribution of the sunspot shock to output growth turns out to be very small. Thus, Japan’s output fluctuations in the zero interest period are mainly driven by fundamental shocks rather than non-fundamental changes in expectations.

These findings are confirmed by the historical decomposition. Figure 10 shows the historical decomposition of the output growth rate in terms of percentage deviation from the steady state, evaluated at the posterior mean estimates of parameters. As is consistent with the results in the variance decomposition, the shocks about technology, investment adjustment cost, and external demand are the main driving force of output fluctuations. In particular, the technology shock contributes to the ups and downs of output growth in the same direction for most of the sample period. In contrast, the sunspot shock plays an offsetting role in explaining output fluctuations, that is, it contributes in the opposite direction to output movements. Therefore, the sunspot shock contributes to macroeconomic stability during the period, although an economy in the deflation equilibrium could be unexpectedly volatile due to sunspot fluctuations.

As for the variance decompositions of the other observed variables, the price markup shock has sizeable contributions in addition to the technology shock. This finding is interpreted as follows. Even if the deflation steady state is taken into account, inflation fluctuations are largely explained by the exogenous movements rather than endogenous feedback mechanism in the model. The effects of this shock are broadly transmitted to the other macroeconomic variables because, around the deflation steady state, the monetary policy is not able to sufficiently react to the movements of inflation and output due to the zero lower bound constraint on the nominal interest rate.

The pre-1999 estimates show that the preference shock exhibits larger contributions to all the observed variables, compared with those for the post-1999 sample. This result is explained by the change in the estimate of its standard deviation, as mentioned in Section 4.2. The contributions of the price markup shock become smaller for most of the variables for the pre-1999 sample. Regarding the last finding, the opposite explanation to the one
in the previous paragraph applies in that the central bank’s adjustment of the nominal interest rate is not constrained around the targeted-inflation steady state.

5 Concluding Remarks

In this paper, we have estimated a medium-scale DSGE model with the deflation steady state for the Japanese economy during the zero interest rate period. Although the equilibrium of the model is indeterminate due to the zero lower bound constraint, a specific equilibrium path is selected by extending the Bayesian methods developed by Lubik and Schorfheide (2004). The estimated model differs from a standard model with the targeted-inflation steady state in that demand shocks do not necessarily have an inflationary effect. This finding provides a structural interpretation about flattening of the short-run Phillips curve observed in Japan. According to the estimates of variance decompositions, Japan’s business cycle fluctuations are mainly driven by the shocks about technology, investment adjustment costs, and external demand. In contrast, the effect of the sunspot shock to macroeconomic volatilities turns out to be very small. Rather, the sunspot shock contributes to stabilize the economy during the period.

Our analysis assumes that the Japanese economy has been stuck in the deflation equilibrium since the Bank of Japan adopted the zero interest rate policy in 1999. However, there is much uncertainty about the timing of the regime shifts between the targeted-inflation and deflation steady state. In order to identify the dates of the steady-state changes, a regime switching between the two steady states, as in Aruoba and Schorfheide (2013), must be incorporated into the present model. Estimating such a regime switching DSGE model with possibly indeterminate equilibria is not an easy task but will be a fruitful avenue for future research.\footnote{Bianchi (2013) estimates a DSGE model switching between determinacy and indeterminacy regimes using a particular solution proposed by Farmer, Waggoner, and Zha (2011). Farmer, Waggoner, and Zha (2009) provide a sunspot solution for indeterminate equilibria in Markov switching rational expectations models.}
References


Table 1: Prior distributions of parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Distribution</th>
<th>Mean</th>
<th>S.D.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma$</td>
<td>Relative risk aversion</td>
<td>Gamma</td>
<td>1.500</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Habit persistence</td>
<td>Beta</td>
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<td>Beta</td>
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</tr>
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<td>$\xi_w$</td>
<td>Wage stickiness</td>
<td>Beta</td>
<td>0.660</td>
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<tr>
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<td>Price indexation</td>
<td>Beta</td>
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<tr>
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<td>Price stickiness</td>
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<td>$\rho_b$</td>
<td>Persistence of preference shock</td>
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<td>Beta</td>
<td>0.500</td>
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<tr>
<td>$\rho_g$</td>
<td>Persistence of external demand shock</td>
<td>Beta</td>
<td>0.500</td>
</tr>
<tr>
<td>$\rho_w$</td>
<td>Persistence of wage markup shock</td>
<td>Beta</td>
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</tr>
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<td>$\rho_p$</td>
<td>Persistence of price markup shock</td>
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<td>0.500</td>
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<tr>
<td>$\rho_r$</td>
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<td>Beta</td>
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<tr>
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<td>$\sigma_b$</td>
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<td>Inverse gamma</td>
<td>0.500</td>
</tr>
<tr>
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<td>Inverse gamma</td>
<td>0.500</td>
</tr>
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<td>$\sigma_g$</td>
<td>Standard deviation of external demand shock</td>
<td>Inverse gamma</td>
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<tr>
<td>$\sigma_w$</td>
<td>Standard deviation of wage markup shock</td>
<td>Inverse gamma</td>
<td>0.500</td>
</tr>
<tr>
<td>$\sigma_p$</td>
<td>Standard deviation of price markup shock</td>
<td>Inverse gamma</td>
<td>0.500</td>
</tr>
<tr>
<td>$\sigma_r$</td>
<td>Standard deviation of monetary policy shock</td>
<td>Inverse gamma</td>
<td>0.500</td>
</tr>
<tr>
<td>$\sigma_\zeta$</td>
<td>Standard deviation of sunspot shock</td>
<td>Inverse gamma</td>
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<tr>
<td>$M_z$</td>
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<td>0.000</td>
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<tr>
<td>$M_b$</td>
<td>Arbitrary parameter on preference shock</td>
<td>Normal</td>
<td>0.000</td>
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<td>$M_i$</td>
<td>Arbitrary parameter on investment adj. cost shock</td>
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<tr>
<td>$M_g$</td>
<td>Arbitrary parameter on external demand shock</td>
<td>Normal</td>
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<tr>
<td>$M_w$</td>
<td>Arbitrary parameter on wage markup shock</td>
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<tr>
<td>$M_p$</td>
<td>Arbitrary parameter on price markup shock</td>
<td>Normal</td>
<td>0.000</td>
</tr>
<tr>
<td>$M_r$</td>
<td>Arbitrary parameter on monetary policy shock</td>
<td>Normal</td>
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</table>

Note: The Inverse Gamma priors are of the form $p(\sigma|\nu, s) \propto \sigma^{\nu-1}e^{-\nu \sigma^2/2s^2}$, where $\nu = 2$ and $s = 0.282$. 27
Table 2: Posterior distributions of parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Post-1999 (Continuity)</th>
<th>Post-1999 (Orthogonality)</th>
<th>Pre-1999</th>
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<tbody>
<tr>
<td></td>
<td>Mean 90% interval</td>
<td>Mean 90% interval</td>
<td>Mean 90% interval</td>
</tr>
<tr>
<td>( \sigma )</td>
<td>0.736 [0.528, 0.940]</td>
<td>0.853 [0.650, 1.050]</td>
<td>1.833 [1.232, 2.410]</td>
</tr>
<tr>
<td>( \gamma )</td>
<td>0.351 [0.244, 0.461]</td>
<td>0.360 [0.255, 0.468]</td>
<td>0.620 [0.494, 0.752]</td>
</tr>
<tr>
<td>( \chi )</td>
<td>1.889 [0.790, 2.923]</td>
<td>2.191 [0.911, 3.327]</td>
<td>3.006 [1.743, 4.249]</td>
</tr>
<tr>
<td>( \mu )</td>
<td>2.430 [1.227, 3.544]</td>
<td>2.448 [1.340, 3.508]</td>
<td>1.599 [0.997, 2.223]</td>
</tr>
<tr>
<td>( \gamma_w )</td>
<td>0.286 [0.136, 0.430]</td>
<td>0.297 [0.146, 0.444]</td>
<td>0.327 [0.168, 0.487]</td>
</tr>
<tr>
<td>( \xi_w )</td>
<td>0.351 [0.244, 0.461]</td>
<td>0.360 [0.255, 0.468]</td>
<td>0.620 [0.494, 0.752]</td>
</tr>
<tr>
<td>( \xi_p )</td>
<td>1.889 [0.790, 2.923]</td>
<td>2.191 [0.911, 3.327]</td>
<td>3.006 [1.743, 4.249]</td>
</tr>
<tr>
<td>( \lambda_p )</td>
<td>0.204 [0.098, 0.304]</td>
<td>0.213 [0.101, 0.320]</td>
<td>0.165 [0.089, 0.245]</td>
</tr>
<tr>
<td>( \psi_r )</td>
<td>0.846 [0.783, 0.910]</td>
<td>0.851 [0.790, 0.914]</td>
<td>0.881 [0.817, 0.946]</td>
</tr>
<tr>
<td>( \psi_\pi )</td>
<td>0.089 [0.020, 0.155]</td>
<td>0.164 [0.039, 0.285]</td>
<td>1.298 [1.075, 1.514]</td>
</tr>
<tr>
<td>( \psi_y )</td>
<td>0.136 [0.097, 0.175]</td>
<td>0.136 [0.096, 0.175]</td>
<td>0.462 [0.424, 0.500]</td>
</tr>
<tr>
<td>( \tau )</td>
<td>-0.004 [-0.084, 0.079]</td>
<td>-0.004 [-0.086, 0.074]</td>
<td>1.168 [1.085, 1.249]</td>
</tr>
<tr>
<td>( \pi )</td>
<td>-0.312 [-0.386, -0.237]</td>
<td>-0.332 [-0.412, -0.249]</td>
<td>0.195 [0.118, 0.270]</td>
</tr>
<tr>
<td>( \rho_z )</td>
<td>0.359 [0.229, 0.483]</td>
<td>0.417 [0.254, 0.570]</td>
<td>0.321 [0.182, 0.459]</td>
</tr>
<tr>
<td>( \rho_b )</td>
<td>0.448 [0.217, 0.683]</td>
<td>0.468 [0.218, 0.709]</td>
<td>0.576 [0.378, 0.773]</td>
</tr>
<tr>
<td>( \rho_i )</td>
<td>0.368 [0.205, 0.525]</td>
<td>0.291 [0.143, 0.426]</td>
<td>0.507 [0.398, 0.613]</td>
</tr>
<tr>
<td>( \rho_g )</td>
<td>0.856 [0.794, 0.921]</td>
<td>0.893 [0.838, 0.951]</td>
<td>0.937 [0.908, 0.969]</td>
</tr>
<tr>
<td>( \rho_w )</td>
<td>0.228 [0.083, 0.359]</td>
<td>0.249 [0.097, 0.396]</td>
<td>0.169 [0.058, 0.276]</td>
</tr>
<tr>
<td>( \rho_p )</td>
<td>0.294 [0.111, 0.468]</td>
<td>0.217 [0.074, 0.350]</td>
<td>0.470 [0.243, 0.702]</td>
</tr>
<tr>
<td>( \rho_r )</td>
<td>0.393 [0.195, 0.585]</td>
<td>0.458 [0.236, 0.682]</td>
<td>0.326 [0.178, 0.479]</td>
</tr>
<tr>
<td>( \sigma_z )</td>
<td>1.662 [1.346, 1.948]</td>
<td>1.604 [1.294, 1.901]</td>
<td>1.805 [1.459, 2.164]</td>
</tr>
<tr>
<td>( \sigma_b )</td>
<td>0.339 [0.157, 0.528]</td>
<td>0.454 [0.142, 0.744]</td>
<td>5.977 [3.673, 8.251]</td>
</tr>
<tr>
<td>( \sigma_w )</td>
<td>0.333 [0.267, 0.400]</td>
<td>0.335 [0.266, 0.406]</td>
<td>0.397 [0.329, 0.467]</td>
</tr>
<tr>
<td>( \sigma_p )</td>
<td>0.434 [0.325, 0.543]</td>
<td>0.479 [0.363, 0.595]</td>
<td>0.345 [0.220, 0.472]</td>
</tr>
<tr>
<td>( \sigma_r )</td>
<td>0.057 [0.047, 0.065]</td>
<td>0.055 [0.047, 0.063]</td>
<td>0.120 [0.100, 0.141]</td>
</tr>
<tr>
<td>( \sigma_\zeta )</td>
<td>0.403 [0.165, 0.628]</td>
<td>0.434 [0.159, 0.691]</td>
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</tr>
<tr>
<td>( M_z )</td>
<td>-0.665 [-1.085, -0.200]</td>
<td>-0.570 [-0.776, -0.377]</td>
<td>- -</td>
</tr>
<tr>
<td>( M_b )</td>
<td>0.012 [-0.685, 0.720]</td>
<td>0.048 [-0.544, 0.743]</td>
<td>- -</td>
</tr>
<tr>
<td>( M_i )</td>
<td>0.011 [-0.122, 0.141]</td>
<td>-0.019 [-0.152, 0.114]</td>
<td>- -</td>
</tr>
<tr>
<td>( M_g )</td>
<td>-0.077 [-0.156, 0.006]</td>
<td>-0.188 [-0.298, -0.075]</td>
<td>- -</td>
</tr>
<tr>
<td>( M_w )</td>
<td>-0.546 [-1.025, -0.078]</td>
<td>-0.329 [-0.812, 0.177]</td>
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</tr>
<tr>
<td>( M_p )</td>
<td>-0.599 [-1.042, -0.118]</td>
<td>-1.698 [-2.137, -1.252]</td>
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<tr>
<td>( M_r )</td>
<td>0.032 [-0.775, 0.866]</td>
<td>-0.041 [-0.839, 0.808]</td>
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</table>
Table 3: Variance decompositions

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<th>Δ log C_t</th>
<th>Δ log I_t</th>
<th>Δ log W_t</th>
<th>log l_t</th>
<th>Δ log P_t</th>
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<td>Technology</td>
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<td>0.3</td>
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<td>Investment adj. cost</td>
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<td>0.3</td>
<td>14.5</td>
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<td>16.3</td>
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<td>1.0</td>
<td>0.1</td>
<td>3.8</td>
<td>0.9</td>
<td>2.1</td>
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<td>Wage markup</td>
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<td>0.4</td>
<td>0.3</td>
<td>14.6</td>
<td>3.5</td>
<td>1.3</td>
<td>3.6</td>
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<tr>
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<td>33.8</td>
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<td>3.2</td>
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<tr>
<td>Technology</td>
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<td>16.9</td>
<td>35.8</td>
<td>79.2</td>
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<td>1.8</td>
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<td>0.2</td>
<td>8.3</td>
<td>1.6</td>
<td>24.4</td>
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<td>0.1</td>
<td>4.1</td>
<td>1.4</td>
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<td>22.6</td>
<td>0.6</td>
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<td>2.1</td>
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<td>11.3</td>
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<td>Monetary policy</td>
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<td>0.6</td>
<td>1.5</td>
<td>0.0</td>
<td>3.6</td>
<td>0.1</td>
<td>11.5</td>
</tr>
</tbody>
</table>

Notes: The table shows the mean estimates of the asymptotic forecast error variance decompositions of output growth, consumption growth, investment growth, wage growth, hours worked, inflation, and the nominal interest rate.
Figure 1: Interest rate and inflation in Japan

Notes: This figure plots the overnight call rate and the percent change in the GDP deflator from one year earlier for the sample from 1981Q1 to 2013Q1. The thick solid line is a nonlinear monetary policy rule fitted to the data. The dotted and dashed lines represent long-run Fisher relations with the real interest rates fixed at 3.30 and 1.42, respectively.
Figure 2: Impulse responses to technology shock

Notes: Each panel depicts the posterior mean (solid line) and 90-percent credible interval (dashed lines) for the impulse response to a one-standard-deviation technology shock in terms of percentage deviation from the steady state.
Figure 3: Impulse responses to preference shock

Notes: Each panel depicts the posterior mean (solid line) and 90-percent credible interval (dashed lines) for the impulse response to a one-standard-deviation preference shock in terms of percentage deviation from the steady state.
(i) Post-1999

Figure 4: Impulse responses to investment adjustment cost shock

Notes: Each panel depicts the posterior mean (solid line) and 90-percent credible interval (dashed lines) for the impulse response to a one-standard-deviation investment adjustment cost shock in terms of percentage deviation from the steady state.
Figure 5: Impulse responses to external demand shock

Notes: Each panel depicts the posterior mean (solid line) and 90-percent credible interval (dashed lines) for the impulse response to a one-standard-deviation external demand shock in terms of percentage deviation from the steady state.
Figure 6: Impulse responses to wage markup shock

Notes: Each panel depicts the posterior mean (solid line) and 90-percent credible interval (dashed lines) for the impulse response to a one-standard-deviation wage markup shock in terms of percentage deviation from the steady state.
Figure 7: Impulse responses to price markup shock

Notes: Each panel depicts the posterior mean (solid line) and 90-percent credible interval (dashed lines) for the impulse response to a one-standard-deviation price markup shock in terms of percentage deviation from the steady state.
Figure 8: Impulse responses to monetary policy shock

Notes: Each panel depicts the posterior mean (solid line) and 90-percent credible interval (dashed lines) for the impulse response to a one-standard-deviation monetary policy shock in terms of percentage deviation from the steady state.
Figure 9: Impulse responses to sunspot shock

Notes: Each panel depicts the posterior mean (solid line) and 90-percent credible interval (dashed lines) for the impulse response to a one-standard-deviation sunspot shock in terms of percentage deviation from the steady state.
Notes: The figure depicts the output growth rate in terms of percentage deviation from the steady-state and the contribution of each shock, evaluated at the posterior mean estimates of parameters.