Is it is or is it Ain’t my Obligation? Regional Debt in a Fiscal Federation *

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Abstract

This paper studies the repayment of regional debt in a multi-region economy with a central authority: who pays the obligation issued by a region? With commitment, the central government will use its economy-wide taxation power in support of its objective to smooth distortionary taxes and consumption across regions. Absent commitment, this tool of the central government may be exploited to induce it to bailout regional government deficits. We characterize the conditions under which bail-outs occur and their welfare implications. In particular, we show when the gains from a federation may outweigh the welfare costs of a bail-out. We use these insights to comment on actual fiscal relations in three quite different federations: the US, the European Union and Argentina.

1 Introduction

Fiscal constraints, typically in the form of debt limits, on governments within a federation are prevalent. Restrictions on borrowing by states in the U.S. and the Stability and Growth Pact of the European Monetary Union are leading examples. These fiscal constraints have an obvious cost: they limit the ability of a regional government to smooth distortionary taxes over time. These constraints have a more subtle gain: they may

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substitute for the inability of a central government to commit not to bailout and thus finance the debt obligations of a regional government.\footnote{Commitment problems arise from the timing of moves: the central government is induced to respond to the fiscal policy choices of the regional governments. These problems also lie at the heart of other analyses of interactions between levels of governments, including Zarazaga (1995), Chari and Kehoe (1998), Cooper and Kempf (2001), Cooper and Kempf (2004) and Sanguinetti and Tommasi (2004).} By bail-out, we mean taxes and transfers by a central government in excess of those which would be chosen in the commitment solution.

In anticipation of a bail-out, regional governments will run excessive deficits in an attempt to spread the costs of local public goods onto agents in other regions. The resulting inefficiency is a consequence of the commitment problem of the central government. If the central government could commit not to bailout a regional government, excessive deficits would not arise.

Thus we ask: what is the source of the commitment problem and what remedies are there to overcome it? In addressing this question, our focus is on the incentives for a bail-out by a central government more than the exact form of the bail-out.

We highlight two motivations for an ex post bail-out of regional debt by the central government and thus two sources of the commitment problem. First, a central government may bailout the debt of regional government in order to smooth consumption across agents. This consumption smoothing motive reflects the objective of a central government which values equality of consumption across agents in the economy. In the model, the objective function of a central government is to maximize the sum of the welfare of agents in the multiple regions. If the utility functions of the agents are strictly concave, then the objective function of the central government will inherit this property which creates a desire to smooth consumption. Without commitment, the central government can be induced to bailout a regional government if the resulting allocation is more equitable.

The extent of the consumption smoothing motive will depend on the allocation of debt holdings across agents. While individual agents will be, in equilibrium, indifferent regarding the composition of their portfolio, bail-out will occur when a sufficiently large fraction of the debt of a rich region is held by agents in a poorer region. If, instead, debt issued by a region is held solely by agents within that region, then there is no bail-out and each region repays its own obligation with regional taxes.

Second, the central government may have an incentive to smooth distortionary taxes and thus bailout the debt of a regional government. This tax smoothing motive reflects the presence of distortionary labor taxes. The central government has, by assumption, the power to tax all agents at the same rate and thus to smooth the tax burden across agents in different regions. Thus if two regions have different tax burdens, the central government will be induced to use its tax power to smooth distortionary taxes across them. This tax smoothing motive does not rest on the distribution of debt holdings across regions.

Independent of the source of the commitment problem, when a bail-out occurs, it is ex post in the interest of the central government and thus the agents in the economy it represents. However, as in other problems with commitment, bail-outs can create welfare reducing ex-ante effects. That is, if the central government...
had power to commit to its policies *ex ante*, it would not choose these *ex post* bail-outs.\(^2\)

The natural question then is whether the federation should be dissolved in the presence of these commitment problems? The answer depends on the source of the incentive for bail-out.\(^3\) In the consumption smoothing case, the costs of bail-out are severe enough that, absent any gains from federations, members are better off in autarky.

However, if the bail-out reflects a tax smoothing motive, then a federation without commitment power may still be better than autarky. These gains to a federation depend positively on the volatility of the spending shocks, and negatively on their correlation. When spending shocks are sufficiently volatile and negatively correlated across regions, revenue sharing across regions induced by a bail-out is an efficient way to minimize the variability of distortionary taxes in each region. These gains are fully achieved in a federation with commitment and are partially present in one without commitment, though they are marred by regions who take advantage of the inability of the central government to commit.

The policy dilemma, highlighted in our model, is to reap the tax smoothing gains from a fiscal federation while avoiding *ex post* bail-out. Without the power to commit, governments within a federation face the question of limiting *ex post* bail-outs while retaining tax smoothing gains. Institutional features like balanced-budget constitutional provisions are common in many federal unions, as in the U.S. These same issues reappear in the ongoing discussion of fiscal constraints within the European Union. We use our model to comment on some features of these institutions.

### 2 A two region economy

We study allocations in a two-period real economy without money. Before proceeding, it is useful to keep in mind some boundaries of our analysis.

First, as illustrated by the discussion in Section 5, the interaction between regional and central fiscal authorities is widespread and takes many forms. This paper inquires about the implications of such a federation for the management of regional fiscal policy, leaving aside the rationale for its existence. See Wallis and Weingast (2005) for a political economy view of the separation of spending between states and the federal government.\(^4\)

Second, while the federations which motivate this research, such as the US, the EU and Argentina, entail a monetary union, we do not think that monetary interactions are central to the points raised in this paper.

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\(^2\)Though, for the tax smoothing environment some *ex ante* intervention is welfare improving.

\(^3\)Our focus on linking the incentives for bailout with policy remedies distinguishes it from the research on interactions across governments (interest groups) assuming a common budget constraint for financing transfers and expenditures, as in Persson and Tabellini (2000), particularly Chapter 7, Velasco (2000) and the weak central bank case studied in Cooper and Kempf (2004).

\(^4\)A federation entails the sharing of power by multiple levels of government. The political pros and cons of this structure are beautifully summarized in the Federalist Papers, such as the essay by James Madison in Madison (1961). See also the discussion and citations in Oates (1999) and Fllesdal (2007).
Further, by isolating these real interactions we are able to avoid subtle issues of the sources of money demand and money non-neutrality.

This section provides an overview of the model. It then establishes two benchmarks. The first, termed an integrated economy, characterizes the optimal allocation as a solution to the optimization problem of a single planner with commitment power. The second, termed regional autarky, characterizes the optimal allocation when regions act independently. These allocations are used to understand the outcomes under a fiscal federation, studied in the next section.

2.1 Model Overview

A two-period economy allows us to focus on consumption and tax smoothing in a dynamic setting. The economy is composed of two regions. In each period of life, agents supply labor, and consume some of the single good. Each unit of labor input produces 1 unit of this good. After tax, this output can be either consumed or stored. We assume that agents have access to a storage technology with an exogenous return of $R$. Importantly, there is no individual mobility: agents live in one region and cannot avoid being taxed by their local government.

Thus we consider the following optimization problem for a representative agent in one of the regions:

$$\max_{\{n_1, n_2, s\}} u(g + e + n_1(1 - \tau_1) - \frac{n_1^{1+\gamma}}{1+\gamma} - s) + \beta v(n_2(1 - \tau_2) - \frac{n_2^{1+\gamma}}{1+\gamma} + sR)$$  \hspace{1cm} (1)

where $u(\cdot)$ and $v(\cdot)$ are strictly increasing and concave.$^5$ In what follows, we sometimes assume that $v(c_2) = c_2$. This is an important restriction as it allows us to focus on bail-outs associated with the sharing of distortionary taxes rather than bail-outs associated with redistribution of consumption alone. Agents incur a disutility of work, in consumption terms, given by $\frac{n^{1+\gamma}}{1+\gamma}$ where $\gamma > 0$.

Public spending per capita, $g$, enters directly into utility as a perfect substitute for private consumption. The optimization problem is stated here for a given realization of $g$ since policies are contingent on the realized value of this random variable. In the subsequent analysis, $g$ represents the realized value of a random variable, $G$, with a known distribution.$^6$ Calculations for welfare statements will be in terms of expected utility.

There is a first period endowment of $e$ which creates an incentive to save, denoted by $s$. The tax rate on labor income in period $t$ is given by $\tau_t \geq 0$ so labor income, after taxes, is $n_t(1 - \tau_t)$ in period $t$.

The first-order condition for labor in any period is

$$(1 - \tau) \frac{n^{1+\gamma}}{1+\gamma} = n$$ \hspace{1cm} (2)

$^5$Since we are looking at a single region, superscripts identifying the region is suppressed.

$^6$Realizations of spending shocks are a key source of ex post heterogeneity across regions and allow us to isolate the effects of spending on incentives for a bail-out by the central government. Ex ante, regions draw spending shocks from identical marginal distributions. If, as suggested by a referee, there were differences across regions in the distributions of the shocks, then the ex ante benefits of joining a federation would change but the ex post incentives for bail-out, which is the focus of our analysis, would not.
where $\tau$ is the tax rate prevailing in that period. Call this labor supply relationship $n(\tau)$.

Using the labor supply function, define $I(\tau) = \tau n(\tau)$ as the level of tax revenues given a tax rate $\tau$. Throughout our analysis, we study the upward sloping branch of the Laffer curve. So if there are multiple levels of $\tau$ such that $I(\tau) = X$, we select the lowest value of $\tau$.

Let the after-tax total contribution to consumption from work, net of the disutility of work, be

$$Z(\tau) = n(\tau)(1 - \tau) - \frac{n(\tau)^{1+\gamma}}{1+\gamma}.$$  

(3)

Note $Z'(\tau) = -n(\tau)$, when $n(\tau)$ satisfies (2).

Using $Z(\cdot)$, the household optimization problem can be recast as

$$W(\tau_1, \tau_2|g) = \max_s u(g + e + Z(\tau_1) - s) + \beta v(Z(\tau_2) + sR).$$  

(4)

In this problem, saving can either be through non-negative storage or through government debt.\footnote{We assume that the endowment, $e$, is sufficiently large so that the non-negativity constraint on storage is fulfilled, even with positive debt levels. The first-order condition is

$$u'(c_1) = \beta R v'(c_2).$$  

(5)

Let $s(\tau_1, \tau_2)$ be the level of saving satisfying (5). From (4)

$$W_{\tau_1} = u'(c_1)Z'(\tau_1), \quad W_{\tau_2} = \beta v'(c_2)Z'(\tau_2).$$  

(6)

Since $Z'(\tau) = -n(\tau)$, these conditions show the marginal cost of taxation in each of the two periods.

2.2 An Integrated Economy

The multi-region economy to be analyzed in this paper consists of two regions, identical to the one presented above. Throughout the discussion, we use a superscript $i$ to denote region $i = 1, 2$. The public spending shocks, $(G^1, G^2)$, are drawn from a joint distribution with identical marginal distributions. The regional spending shocks may be correlated. In fact, some of our results depend on the magnitude and sign of this correlation.

As a benchmark, we study optimal allocations in this two-region economy. Suppose there is a single government entity setting tax policies for all groups of agents in all periods. As this government internalizes all of the interactions across the regions, we term this the \textit{integrated solution}.\footnote{This might also be termed the Ramsey solution since it provides complete commitment to the single government. As emphasized in Bassetto (2005), even with this extreme power of commitment, implementation may be problematic resulting from the manner in which the government responds to household behavior off the equilibrium path. As our focus here is on comparing the best outcome with complete commitment against the allocations with regional autonomy, we assume the integrated solution obtains under commitment.} The government sets any tax rates after observing the realizations of the public spending shocks.
Let $\tau^i_t$ for $i = 1, 2$ and $t = 1, 2$ denote the period $t$ tax rate set for region $i$. Debt is sold to households to finance the excess of total public spending over tax revenues in period 1.

The government entity maximizes the welfare of the agents across the two regions, which we measure as the sum of lifetime expected utilities, using population ratios as welfare weights. This could be amended to allow for political pressure different from population ratios to determine the optimal policy.

The optimal tax policy solves

$$\max_{\tau^i_t} \sum_{i=1,2} \Delta^i W(\tau^i_1, \tau^i_2 | g^i)$$

subject to the budget constraint of

$$\sum_{i=1,2} \Delta^i g^i = \sum_{i=1,2} \Delta^i I(\tau^i_1) + \frac{1}{R} [\sum_{i=1,2} \Delta^i I(\tau^i_2)]$$

where $\Delta^i$ is the share of the population in region $i$. Let $W^*$ be the expected value of the objective in the solution to (7) where the expectation is over the joint distribution of $g^i$ for $i = 1, 2$.

**Proposition 1** The solution to (7)-(8) entails tax smoothing over time by each region: $\tau^i_t = \tau^i_t^*$ for $t = 1, 2$. Further, if $g^i$ increases, then $\tau^j_t^*$ increases for $j = 1, 2$. Finally, $\tau^i_t^* = \tau^i_t^*$ for $i = 1, 2$ if: (i) $v(c) = c$, (ii) $g^1 = g^2$ or (iii) $n(\tau)$ is independent of $\tau$.

The optimal taxes, denoted with a superscript $^*$, in periods 1 and 2 are equal for each of the regions, $i = 1, 2$. This tax smoothing reflects the gain to the smoothing of distortionary taxes over time within a region. This form of tax smoothing was also present in the solution of the single region’s problem. But, through integration, the tax rates may be at different levels and thus differ across regions.

The second part of the proposition establishes that tax rates in each region depend on the level of government spending in both regions. This interaction across regions is a key aspect of our model as it highlights the ex ante optimality of revenue sharing. So, for example, an increase in $g^1$ is met by an increase in taxes in both regions 1 and 2. The increase in distortionary taxes needed to finance the increase in region 1 spending is spread across time and across regions.

As stated in the last part of the proposition, complete tax smoothing across regions, defined as $\tau^{1*} = \tau^{2*}$, will occur in two special cases. First, suppose $v(c) = c$. In this case, $\tau^{1*} = \tau^{2*}$ will satisfy the first-order conditions implying $c^1_1 = c^2_1$. Differences in $g^i$ are reflected in different levels of $s^i$ and thus in $c^i_2$. If $g^1 > g^2$, then $s^1 > s^2$ implying $c^1_2 > c^2_2$.

Second, suppose that regional public spending is perfectly correlated, $G^1 = G^2$ with probability one. In this case, the solution with integration will be perfectly symmetric so that tax rates are equal. When the spending levels are perfectly correlated, there are no gains to revenue sharing. In fact, given the symmetry across regions, consumption levels will be equalized as well.

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9 Adding in another tax common to all agents would be redundant.
10 The proofs of the propositions are in the Appendix.
11 A similar argument holds if $u(c) = c$. The case of $v(c) = c$ will be used extensively below.
Finally, assume there are no distortions from taxation: i.e. $\gamma \to \infty$ so that $n(\tau) = \eta$ and $Z(\tau) = \eta(1 - \tau)$. This implies $Z'(\tau) = -\eta$ and $I'(\tau) = \eta$ so $\frac{Z'(\tau)}{I'(\tau)} = -1$. Following the first-order conditions contained in the proof, $c^1_i = c^2_i$ and $c^1_2 = c^2_2$.\footnote{Specifically, (18), developed in the proof of Proposition 1 and contained in the Appendix, implies $c^1_1 = c^2_1$ and, using (5), $c^1_2 = c^2_2$.} Thus when taxes are not distortionary the integrated solution implies consumption smoothing across regions. In fact, this is exactly the same allocation obtained under autarky: the regional tax rate is set to balance the regional budget. In this case, agents’ utility in region $i$ is independent of the realizations $(g^1, g^2)$.

This integrated economy represents a benchmark in two respects. First, there is only a single government entity within the multi-region economy. Second, the government entity has commitment power: it chooses state contingent policies prior to the choices of households.

The analysis is agnostic about the fiscal institutions which underlie the optimization problem stated in (7). From Proposition 1, a single entity with commitment power is sufficient to support an efficient allocation. But is it necessary? Or can other institutional structures with multiple fiscal policymakers, as we see in existing federations, generate the same allocation?

### 2.3 Regional Autarky

To answer this question, we next consider allocations when the regions act in isolation. That is, assume there is a single government in each region, $i = 1, 2$, determining its fiscal policy without any central fiscal authority and thus no bail-outs. Each government is isolated, as there is no exchange nor any spillovers across regions. The resulting allocation, termed regional autarky, provides a benchmark outcome with multiple fiscal players.

To find optimal taxes, the representative government in region $i$ will choose $(\tau^i_1, \tau^i_2)$ to maximize

$$W(\tau^i_1, \tau^i_2 | g^i)$$

given in (4), subject to the present value budget constraint of

$$g^i = \tau^i_1 n(\tau^i_1) + \frac{\tau^i_2 n(\tau^i_2)}{R} = \left( I(\tau^i_1) + \frac{I(\tau^i_2)}{R} \right).$$

The maximal amount of revenue the government can receive in a period is obtained by a tax rate of $\tau = \frac{\gamma}{1 + \gamma}$. We assume that $g^i$ is bounded above by $I(\frac{\gamma}{1 + \gamma}) \frac{1 + R}{R}$ so that (10) can always be satisfied.

If period 1 revenues do not cover $g^i$, then the regional government issues debt to cover the deficit. In equilibrium, this debt pays the same return $R$ as private storage. The optimal tax policy for the isolated regional government is characterized by

**Proposition 2** The optimal tax policy for the isolated region satisfies $\tau^i_t = \tau^{iA}$ for $t = 1, 2$. Further, agent’s utility and consumption levels are decreasing functions of $g^i$.\footnote{Specifically, (18), developed in the proof of Proposition 1 and contained in the Appendix, implies $c^1_1 = c^2_1$ and, using (5), $c^1_2 = c^2_2$.}
Here the optimal allocation for the isolated regions is indicated by a superscript \( A \) and is referred to as the \textit{autarkic equilibrium}. The optimal allocation entails both intertemporal tax smoothing and consumption smoothing. The tax smoothing is seen by the equality of income taxes across periods in Proposition 2. The government uses debt to smooth out distortionary taxes over time. Given the optimal tax policy from Proposition 2 along with the budget constraint, the amount of debt issued by the government is

\[
B_i^A = g_i - \tau_i^A n(\tau_i^A) = \frac{\tau_i^A n(\tau_i^A)}{R}.
\]

Agents use access to capital market to smooth consumption, as in (5).

If labor supply was completely inelastic, then of course taxes would not be distortionary, i.e. \( \gamma \to \infty \) so that \( n(\tau) = \eta \). In this case, the timing of taxes would not matter for consumption and labor allocations. This is the familiar result of Ricardian equivalence applied to this economy.

Clearly, there is no consumption smoothing across regions in autarky. Consumption, \((c_i^A, c_i^A)\), and taxation, \( \tau_i^A \), depend on \( g_i \) but not on \( g_j \) for \( j \neq i \). This is a major difference from the integrated solution. Variations in the exogenous level of \( g_i \) influence the lifetime utility of agents directly through the presence of \( g_i \) in the utility function and also indirectly through the dependence of \( \tau_i^A \) on \( g_i \).

The second part of the proposition shows that utility and consumption levels fall as \( g_i \) increases. Evidently the direct effect of \( g_i \) on utility is more than offset by the taxation effect. To understand this result, if taxes were not distortionary, utility would be independent of \( g_i \) since, from the government budget constraint, total tax revenues would equal the transfer. But, when labor supply is responsive to tax rates, then increases in \( g_i \) are welfare reducing due to the distortions of taxes on labor supply.

### 2.4 Summarizing the Benchmark Solutions

The allocations obtained under regional autarky and the integrated economy can be compared in terms of their predictions for tax policies as well as their welfare implications. The comparison of tax policies comes from Proposition 1 and 2. For both fiscal structures, taxes are constant over time: the regional government can achieve intertemporal tax smoothing in autarky. As the level of regional spending varies, so will the tax rate.

In the integrated economy, the distortionary tax is spread across regions, as highlighted in Proposition 1. In this manner the risk associated with variations in regional spending is shared. As we shall see, this is the source of the welfare gain to an integrated economy.

For the welfare analysis, let \( W^A \) be the expected utility of an agent under the case of regional autarky. As defined earlier, \( W^* \) is the expected value of lifetime expected utility in the integrated economy.\(^{13}\)

**Proposition 3** Comparing expected utility in regional autarky with that obtained in the integrated economy,

(i) \( W^* \geq W^A \), (ii) \( W^* > W^A \) if \( G^1 \neq G^2 \) and labor taxation is distortionary and (iii) \( W^* = W^A \) if taxes are not distortionary or if \( G^1 = G^2 \).

The difference \( W^* - W^A \) is the gain from forming an integrated federation relative to autarky. The gain

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\( ^{13} \)Recall our notation: \( g_i \) is the realized value of the random spending in region \( i \), \( G^i \).
comes from state contingent revenue sharing and thus the smoothing of distortionary taxes across regions. If shocks are perfectly correlated, $G^1 = G^2$, then there is no gain to a federation. Likewise, if taxes are not distortionary, then the integrated solution does not improve upon autarky.

The gain to integration comes from two sources: revenue sharing between regions and the commitment power of the government entity. We now study allocations within a multi-region economy with a central taxation agency to gain insights into the interaction in the fiscal policies of a central government and multiple regional governments.

As we shall see, the incentives which underlie revenue sharing in the integrated federation are also a source of the commitment problem. This highlights a central tension within a federation. Revenue sharing is welfare improving as it facilitates the smoothing of distortionary taxes. But, without commitment by the central government, indebted regions will take advantage of the taxation power of the central government to the detriment of the other region.

3 Decentralized Allocations with Autonomous Regions

Our interest is the allocation from an economy in which regional governments are autonomous and interact with a central government. Key features of our model, matching existing federations, are: (i) a limited taxation capacity of a central government, (ii) a role for fiscally active regional governments and (iii) the prospect of the bail-out of regional debt by the central government through transfers.

In this two-region federation, taxation power is split across a hierarchy of governments: the central government, hereafter (CG), and the government representing the agents in region $i$, hereafter ($RG_i$), are the active fiscal players. The government in region $i$ will select tax rates $\tau^i_t$ for $t = 1, 2$ on labor income. The regional governments can issue debt to cover the excess of regional expenses over regional tax revenues. The central government will set a common tax rate, denoted $\tau_C$, on labor income in period 2.

By the very nature of a federalist structure, a CG does not assume all of the fiscal functions, in contrast to a unitary state. The restrictions we make on the CG reflect advantages to regional over centralized taxation and spending decisions. Our approach is to take this structure as given, characterize the set of equilibria and then compare allocations to the integrated and autarky benchmarks. We can ask: to what extent can a the federalist structure approximate the optimal solution from the integrated economy? Further we are interested in understanding the conditions under which the outcome from the two-region federation dominates the alternative of autarky.

This multi-regional structure is constructed to provide a simple framework to study the bail-out decision of CG given the debt outstanding of the regions. Given this focus, we do not consider a CG tax in period 1.\textsuperscript{14}

\textsuperscript{14}We could allow period 1 taxation by the CG but assume it is not feasible for the CG to set taxes conditional on the realizations, $(g^1, g^2)$. Requiring CG policy to be independent of $(g^1, g^2)$ provides a role for fiscally active regional governments, using their informational advantage to respond to region specific shocks. So, even if CG was active in period 1, it could only set
The structure lies between the integrated solution in which a single government makes all choices ex ante and regional autarky which lacks any central taxation authority. It has two critical features. First, it allows for the interaction between two levels of governments. Second, it highlights the ability of the regional governments to respond to variations in regional specific shocks.

In addition to controlling different fiscal instruments, these three governments have different objectives. As above, CG maximizes the weighted sum of the utilities of agents across regions while the regional governments are interested only in the welfare of the agents within their region.

3.1 The Game between Governments

Here we describe in detail the interactions between the regional and central governments as well as our equilibrium concept. There are three types of agents: private households, regional governments and the central government.

The order of moves is:

- **Period 1**
  - Nature selects \((g^1, g^2)\).
  - Regional government \(i\) sets \(\tau_i^1\) given \((g^1, g^2)\).
  - Private agents choose period 1 employment and saving given the policies of the regional governments and \((g^1, g^2)\).

- **Period 2**
  - Central government sets \(\tau_C\) and distributes revenues to regions given period 1 household and regional government choices.
  - Regional government \(i\) sets \(\tau_i^2\) given \(\tau_C\), its debt obligations and period 1 household choices. If the regional government does not fully cover its obligations, region \(i\) agents incur a utility loss (default cost) of \(\epsilon\).
  - Private agents choose employment for period 2 given the tax policies of the regional and central government.

The structure emphasizes the key role of CG as being in a weak position of responding to the period 1 fiscal choices of the regional governments. The lack of commitment by CG pertains to the choice of \(\tau_C\) in period 2 relative to period 1 taxes set by the regional governments.\(^{15}\) We do allow CG to choose \(\tau_C\) prior to a common tax across regions. This policy would not be sufficient to replicate the integrated allocation and would leave intact the essence of the interactions between the regional governments and the CG which is at the heart of our analysis.\(^{15}\) Our results do not depend on the details of the timing of the game in period 2. This point is important as it indicates that the results do not rely on a form of commitment to the order of moves in the period 2 game. Cooper, Kempf, and Peled (2004) study the reverse timing in which the regional government moves prior to the central government and characterize Ricardian and bail-out equilibria for the consumption smoothing case.
to the choice of the governments in period 2 and this allows CG to smooth taxes across regions. But CG sets its tax rate given the period 1 taxes chosen by the regional governments. Thus a regional government anticipates a response by CG to variations in regional debt.

Note that embedded in the extensive form game is a default cost of \( \epsilon \), denominated in utility terms. This cost is intended to capture penalties associated with default, such as trade and borrowing restrictions. The penalty is region specific and does not depend on the magnitude of the default. Since government debt is always repaid in equilibrium, this penalty is never incurred along the equilibrium path.

### 3.2 The Players

The essentials of the decentralized economy are the optimization problems and the budget constraints of the households and the governments along with an equilibrium concept. We turn to those details now.

**Households** The households are competitive, taking the actions of the governments as given. As indicated in the order of moves, households make labor supply and saving decisions in period 1 after the regional governments have chosen their taxes for that period. Household’s base their decisions upon beliefs about government policy in period 2. In equilibrium, these beliefs are consistent with the choices of the regional and central governments.

Given the realization of the spending shock, the household optimization problem is

\[
W(\tau_1^i, \tau_2^i, \tau_C^i | g^i) = \max \, s u(g^i + \epsilon + Z(\tau_1^i) - s^i) + \beta v(Z(\tau_2^i + \tau_C^i) + s^i R).
\]  
(11)

The solution of this optimization problem will generate a saving decision, \( s^i(\tau_1^i, \tau_2^i, \tau_C^i, g^i) \). The labor supply decision is incorporated in \( Z(\tau_2^i) \) through \( n(\tau_2^i) \).

**Regional Governments** The intertemporal budget constraint of regional government \( i \) is

\[
g^i = \tau_1^i n(\tau_1^i) + \frac{\tau_2^i n(\tau_2^i + \tau_C^i) + T_i^\Delta}{R}.
\]  
(12)

The regional government’s constraint depends upon the central government’s tax rate (\( \tau_C^i \)) and the per capita transfers, \( \frac{T_i^\Delta}{R} \). The goal of the regional government is to choose \( (\tau_1^i, \tau_2^i) \) to maximize the lifetime utility of the representative agent in region \( i \). It takes as given the savings function and labor supply function of the agents in its region. In period 1, the regional government chooses a tax rate, given \( g^i \). In period 2, its choice of tax rate will depend upon the levels of debt at the start of period 2, \( (B_1^1, B_2^2) \) as well as the choice of the CG, \( \tau_C^i \).

**Central Government** The CG is only active in period 2. Its tax receipts and transfers to the regional governments must balance:

\[
\sum_i T_i = \tau_C \sum_i \Delta_i n(\tau_2^i + \tau_C^i).
\]  
(13)
The central government maximizes the period 2 utility of agents in both regions:

\[
\max_{\tau_C} \sum_{i=1,2} \Delta^i v(c^i_2)
\]  

(14)

CG’s actions in period 2 depend on the period 1 choices of the households and the regional governments. In deciding on the tax rate \(\tau_C\) at the start of the second period, the central government anticipates the response of the regional governments and the private agents in that period. The period 1 tax rates and hence debt levels are determined before the CG moves.

### 3.3 Equilibrium Concept

The equilibrium concept allows for the interactions of competitive and strategic players, as in Chari and Kehoe (1990). The regional governments and the central government are strategic players while the households are competitive. Following the order of moves, when a regional government determines its fiscal policy in period 1, it anticipates the effect of its policy on the subsequent actions of the central government and the households. Likewise, when the central government decides upon its common tax in period 2, it considers the response of the regional governments to its actions, taking as given the period 1 fiscal policy of the regional governments. We look for subgame perfect Nash equilibria of the extensive form game played by the competitive players. As indicated by our proofs, we solve for these equilibria using backward induction.

For a given realization \((g^1, g^2)\), an equilibrium is

- a pair of period 1 taxes rates for each region, \((\tau^1_1, \tau^2_1)\),
- a tax rate of the CG depending on the levels and holding distribution of regional debts, \(\tau^C(B^1, B^2, \theta^1, \theta^2)\), where \(\theta^i\) denotes the fraction of the debt of region \(i\) held by region \(i\) households,
- and period 2 tax rates for each region, \((\tau^2_1(B^1, B^2, \theta^1, \theta^2, \tau^C), \tau^2_2(B^1, B^2, \theta^1, \theta^2, \tau^C))\).

The basis for including \((\theta^1, \theta^2)\) is discussed at length below. In equilibrium, these functions solve the maximization problems given above.

### 4 Equilibria

Using the results of the integrated economy, summarized in Proposition 1, as a benchmark, our goal is to understand when that allocation can be decentralized in the two-period game between governments outlined above. Our results indicate that only in special circumstances can the integrated solution be decentralized.
4.1 A Ricardian Equilibrium

We begin with a special case in which the integrated solution can be decentralized in the multi-region economy. The key to this result is the assumption that taxes are not distortionary: $\gamma \rightarrow \infty$, implying $n(\tau) \equiv \eta$.

Let $\theta^i$ denote the fraction of region’s $i$’s debt held by its own members, and $B^i$ the amount of period 1 debt issued by the region $i$ government per region $i$ agent. To construct this equilibrium, assume $\theta^i = 1$ for $i = 1, 2$. In equilibrium, agents will be indifferent regarding the composition of their portfolio since, with no default, government debt and storage will yield the same return. Yet, the distribution of debt holdings will matter for the equilibrium outcome. The assumption $\theta^i = 1$ for $i = 1, 2$ is thus innocuous at the level of the individual but is instrumental in the construction of the equilibrium.

In this equilibrium, the period 2 debt obligations of region $i$ will be paid by tax revenues from region $i$ taxation. Agents in both regions will anticipate this taxation in deciding upon their savings in period 1.

Since each region holds its own debt, there are no links across the regions in equilibrium and thus a Ricardian outcome should occur as long as $CG$ has no incentive to intervene. As we shall see, in the Ricardian equilibrium, $CG$ has no incentive to intervene since the consumption levels of the agents are equal across the two regions.

**Proposition 4** When taxes are not distortionary: (i) there exists a Ricardian equilibrium if $\theta^i = 1$, (ii) for any $\theta^i < 1$ there exists no Ricardian equilibrium for sufficiently small default cost.

We call this a Ricardian equilibrium because consumption levels and thus welfare are independent of the level of $g^i$ and the timing of taxes needed to pay for the public good. In equilibrium each regional government pays off its own debt, and recognizes this responsibility when deciding on the tax rate in the first period. Consumption levels are smoothed over time and are equal across regions.\(^\text{16}\)

We compare this allocation to that obtained under autarky and the integrated federation. There is complete consumption smoothing across regions in all cases and no interaction across regions. Thus there are no gains from integration when taxes are not distortionary but, at least in the Ricardian equilibrium, no welfare costs either. Formally, letting $W^R$ be expected utility in the Ricardian equilibrium, we find

**Proposition 5** When taxes are not distortionary, the Ricardian equilibrium allocation is the same as that obtained under autarky and integrated federation implying $W^R = W^A = W^\ast$.

Though $CG$ is not committed to $\tau^C = 0$, ex post it has no incentive to bailout the regional governments. There is no incentive to bailout because: (i) in the Ricardian equilibrium, period 2 consumption allocations are equal across regions and (ii) there is no gain to the smoothing of taxes since labor supply is totally inelastic. We now study how incentives for bail-out arise.

\(^{16}\)The assumption of perfect substitutability of private and public consumption is necessary for the construction of this Ricardian equilibrium.
4.2 Bail-out

The construction of the Ricardian equilibrium rests on two assumptions. First, debt of region $i$ is held only by region $i$ agents. Second, the tax on labor income is not distortionary. Thus the only incentive for intervention by $CG$ is to smooth consumption but, in equilibrium, there was no need for further consumption smoothing.

We now argue that if these assumptions are relaxed, then $CG$ will have an incentive to bailout the regional governments in period 2. This bail-out will destroy the Ricardian equilibrium and lead to socially inefficient allocations: the resulting allocations will differ from the integrated solution.

The discussion is organized around two rationales for a $CG$ bail-out.\textsuperscript{17} The first concerns the gains from ex post consumption smoothing across regions when taxes are not distortionary. The second comes from the desire of $CG$ to smooth distortionary taxes across regions even when there are no gains to consumption smoothing.

4.2.1 Consumption Smoothing

Under the assumption that taxes are not distortionary, we characterize a bail-out equilibrium driven by the desire of $CG$ to smooth ex post consumption across regions in period 2. $CG$’s optimization problem of maximizing $\sum_i \Delta^i v(c^i_2)$, given available resources in period 2, results in equalization (smoothing) of consumption levels across regions due to the strict concavity of $v(\cdot)$.

We analyze the outcome in period 2 given the levels of debt issued by each region, $B^i$, and the fraction of this debt held in total by all agents in region $i$, $\theta^i$, for $i = 1, 2$. After solving for the period 2 outcome, we characterize the choice of period 1 taxes by the regional governments.

The following proposition states the conditions for bail-out in the case of $g^1 > g^2$. This is without loss of generality since the only difference across regions is in the realized value of their spending shock.

**Proposition 6** For $g^1 > g^2$, if default costs are small enough then $CG$ fully bails-out the obligation of both regional governments and regional governments choose not to tax their citizens in either period, iff $\theta^1 < \Delta^1$ and $\theta^2 > \Delta^2$.

When $g^1 > g^2$ and taxes are common to all agents, then period 1 income is higher for region 1 agents. Thus, from the household optimization condition, (5), $s^1 > s^2$.\textsuperscript{18} So region 1 households would have higher consumption in period 2 if the $CG$ did not intervene.

$CG$ prefers consumption be equal across agents in the different regions. This is an immediate consequence of $v''(\cdot) < 0$. Thus $CG$ will intervene and bailout if that action redistributes in favor of region 2. Such a redistribution requires $\theta^1 < \Delta^1$ and $\theta^2 > \Delta^2$.

\textsuperscript{17}We are grateful to Marco Bassetto for discussions which led to the enhanced development of this section.

\textsuperscript{18}This ordering of saving levels does not require that public and private spending be perfect substitutes. If first period utility was $u(\xi g^1 + e - s^1)$ in the bail-out equilibrium, then $g^1 > g^2$ implies $s^1 > s^2$ as long as $\xi > 0$. Thus the assumption of perfect substitutability, $\xi = 1$, is needed to support the Ricardian equilibrium but is not necessary for the bail-out result.
The incentive for a bail-out of region 1 debt is illustrated in Figure 1 which indicates, given \( g^1 > g^2 \), the consumption levels of the old agents in both regions under different outcomes. This graph takes as given the savings decisions of the agents and thus total resources available for consumption are fixed in the second period, as indicated by the negatively sloped resource constraint. The allocation under a bail-out is labeled \( B \), the allocation under default is labeled \( D \) and the one under full regional taxation is labeled \( R \). As seen in this figure, the bail-out by \( CG \) redistributes from region 1 to region 2 agents relative to the default allocation. This occurs because the holding of debt of agents, as in Proposition 6, implies that default would increase inequality in \textit{ex post} consumption relative to the bail-out allocation.

The point labeled \( R \) indicates the outcome in which the regional government repays its debt obligations. The level of consumption for region 1 agents is lower at point \( R \) than at the default point, \( D \), because the repayment of debt obligation entails a large redistribution of region 1 to region 2 agents.\(^\text{19}\) Thus the region

\(^{19}\text{The allocation labeled } R \text{ is not the Ricardian allocation since the debt of region 1 is not held entirely by agents in that region. This point is above the 45 degrees line, because the bulk of region 1 debt is held by region 2 agents. Consequently, the redistribution generated by regional tax financing of debt repayment implies higher second period consumption for region 2 agents.}\)
1 government prefers bail-out to the repayment of its debt obligations by a regional tax.

As total consumption in the second period is fixed, given endowments and storage decisions, the redistribution by \( CG \) increases the consumption of region 2 agents and reduces the consumption of region 1 agents relative to the default allocation. Thus the redistribution per se is not favorable to region 1 agents. Still, their consumption is higher under a bail-out than if they paid the entire tax bill. In effect, the region 1 agents are able to take advantage of \( CG \)'s desire to redistribute consumption away from them and in equilibrium end up better off than under repayment of their debt.

This bail-out result contrasts with the Ricardian equilibrium characterized in Proposition 4. Comparing these results, the key difference between bail-out and regional taxation is in the holding of the regional debt. In the construction of the Ricardian equilibrium, it was crucial that \( \theta_i = 1 \) for \( i = 1, 2 \). Else, the regional government would choose to default. If the bulk of the debt is held by the region with the lower level of saving, then \( CG \) will have an incentive to bailout the regional government and thus effect a more equitable distribution of consumption.

Comparing these propositions highlights an important element of the model without distortionary taxes: there are multiple equilibria. If we fix \( g^1 > g^2 \), there will be an isolated Ricardian equilibrium in which \( \theta^i = 1 \) for \( i = 1, 2 \) and a set of bail-out equilibria satisfying \( \theta^1 < \Delta^1 \) and \( \theta^2 > \Delta^2 \). For other values of \( \theta^i \), there will be default and thus these values are not consistent with equilibrium. This multiplicity is reflected in the distribution of debt holdings across the economy: in the Ricardian equilibrium debt is held within the issuing region while debt holdings are more widespread in the bail-out equilibrium. Of course, at the individual level, agents remain indifferent about the composition of their portfolios, given the perfect substitutability between bonds and storage.

There are other implications of the bail-out. First, the regional governments set taxes in period 2 equal to zero, given the bail-out by \( CG \). Further, the regional governments in period 1 will anticipate a bail-out by the government. Consequently each regional government will alter the pattern of intertemporal taxation by reducing the tax in period 1 to zero as well and thus use the \( CG \) bail-out of its debt obligation to share the full burden of taxation with the other region.

Letting \( \tilde{W} \) be expected life-time utility in a bail-out equilibrium, we find

**Proposition 7** The expected utility in a bail-out equilibrium is less than the expected utility under integration and autarky: \( \tilde{W} < W^A = W^* \).

As indicated in Figure 1, the equilibrium allocation with bail-out is different from the Ricardian allocation. Since the Ricardian equilibrium entailed the same allocation of consumption as in the integrated solution, the equilibrium characterized in Proposition 6 yields lower expected utility than the integrated solution. Further, since the Ricardian equilibrium is the same outcome as autarky, expected utility under bail-out must be less than expected utility under autarky.

The source of the utility loss in the bail-out equilibrium comes from the dependence of period 2 consumption on the realizations of public spending shocks, \((g^1, g^2)\). This dependence is induced by the bail-out.
of CG. So, looking at Figure 1, the allocation under autarky would be on the 45° line while under bail-out the allocation depends on \((g^1, g^2)\), introducing uncertainty into second period consumption.\(^{20}\) Since the realizations \((g^1, g^2)\) do not expand the resources available, lifetime expected utility must be lower in the bail-out equilibrium.

### 4.2.2 Tax Smoothing

Here we explore a second basis for a bail-out by CG. If the amount of debt outstanding is not equal across regions, i.e. \(B^1 \neq B^2\), then, in the absence of default, a common tax will minimize the distortion associated with the repayment of the regional debt. Thus a bail-out will occur in the second period.

To make this argument clear, we make three assumptions. First, assume labor taxes are distortionary as employment depends on the tax rate. Second, we assume \(v''(\cdot) = 0\) throughout this section so that the consumption smoothing motive is eliminated from the analysis. From the discussion of the integrated solution, \(v''(\cdot) = 0\) implies tax smoothing across time and across regions is optimal. Third, a sufficiently large \(\varepsilon\) is needed to support this repayment. Else, the regional governments will avoid distortionary taxes by defaulting on outstanding debt and CG will not intervene.

In equilibrium, there is full bail-out of regional obligations in period 2. In anticipation of this bail-out, the regions run excessive deficits and this creates a socially inefficient distortion in the intertemporal pattern of labor taxes.

**Proposition 8** There exists a bail-out equilibrium in which \(\tau_2^i = 0\), CG sets a tax rate \(\tau\) such that \(I(\tau) = R \sum_{i=1,2} \Delta^i B^i\) and the regional governments set period 1 tax rates lower than \(\tau\).

The second period outcome, given \(B^i\), is for CG to bailout the regional debt and set a tax rate of \(\tau\). In response, the regional governments set \(\tau_2^i = 0\) for \(i = 1, 2\). In addition, the regional governments will reduce period 1 tax rates to take advantage of CG bail-out in period 2. Hence, \(\tau_1^i < \tau\) and this is an intertemporal tax distortion. However, the tax rate in period 1 need not fall to zero since there is a utility loss to agents from having taxes excessively distorted across periods.

There are two differences between the two bail-out cases. First, the rationale for bail-out is different. In the consumption smoothing case, the CG intervenes to smooth consumption across agents in different regions. The gains to intervention come from the strict concavity of the utility function over second period consumption. In the tax smoothing case, the intervention comes from the gain of spreading distortionary taxes over the regions. Second, the role of default costs is different. For the consumption smoothing case, a bail-out equilibrium exists even if \(\varepsilon\) is small. But, for the tax smoothing case, \(\varepsilon\) must be large enough that the governments are willing to impose the distortionary taxes rather than default.\(^{21}\)

\(^{20}\)Under autarky, second period resources and hence consumption levels, do not depend on the first period shock, as in the Ricardian equilibrium.

\(^{21}\)Even if \(\varepsilon\) is large, there still exist bail-out equilibria based upon consumption smoothing.
Turning to the welfare properties of this bail-out equilibrium, we prove it is not always welfare reducing relative to the autarky equilibrium. In this discussion we assume that both regions are of the same size: $\Delta = \frac{1}{2}$. Letting $\bar{W}$ be the expected utility under bail-out for the tax smoothing case, we find

**Proposition 9** The expected utility in a bail-out equilibrium is smaller than the expected utility under integration. If the correlation of the spending shocks is large (near 1), then a bail-out equilibrium is worse than autarky. The difference between the expected utility in a bail-out equilibrium and autarky increases with the variability of spending shocks when the correlation of the spending shocks is sufficiently small (near $-1$).

Comparing the integrated and bail-out outcomes, there is smoothing of distortionary taxes across the regions in both cases. In the integrated solution, taxes were also smoothed over time. For the equilibrium characterized in Proposition 8, in contrast, the anticipated bail-out of the central government induces the regional governments to run excessive deficits. As a consequence, distortionary taxes are not smoothed over time. This inefficiency is a consequence of the inability of $CG$ to commit not to bailout the debt of the regional governments. Thus the equilibrium is not the same as that from the integrated economy and expected utility is lower than in the integrated solution.

The welfare cost of the bail-out equilibrium relative to autarky is that agents in one region are led to finance the excessive deficits of the other region. When the correlation of the spending shocks, $\text{corr}(G^1, G^2)$, is near 1, the gains to tax smoothing across regions are small. In this case, the costs of the bail-out due to tax distortions are large enough that autarky yields expected utility in excess of expected utility under a bail-out.

Comparing the expected utility under bail-out to autarky, there are two important factors highlighted in Proposition 9: the correlation and variability of the spending shocks. Relative to autarky, there are gains to tax smoothing across regions in the bail-out equilibrium. These gains are larger when the $\text{corr}(G^1, G^2)$ is near -1. Further, these gains are larger if the variability of spending is large.

Following the logic underlying Proposition 9, it is possible to construct parametric examples in which expected utility in a bail-out equilibrium exceeds that under autarky. To illustrate, suppose $\gamma = 1, \beta = R = 1$, $\Delta = \frac{1}{2}$, $u(c) = v(c) = c, G^i = \bar{G} \pm \zeta$ with equal probability, and $\text{corr}(G^1, G^2) = -1$. We hold $\bar{G}$ constant, and compute the equilibria under the different regimes for different values of $\zeta$.

The construction of the bail-out equilibrium uses the first-order condition of a regional government, (32) in the Appendix, along with the government budget constraint, (37) in the Appendix, to solve for the two tax rates $\bar{\tau}_1$ and $\bar{\tau}_2$ given the fixed aggregate spending by the two regions, $\bar{G}$. Total regional government spending is constant due to the assumption that $\text{corr}(G^1, G^2) = -1$. Likewise, tax rates in the integrated economy equilibrium are also independent of $\zeta$. In contrast, taxes in the autarkic equilibrium vary with $\zeta$.

For $\bar{G} = 0.4$ there is a bail-out equilibrium in which $\bar{\tau}_1 = 0.205, \bar{\tau}_2 = 0.3861$, and the tax rate for the integrated economy equilibrium is $\tau^* = 0.2764$. Since there is no aggregate uncertainty, there is no variability in these allocations. The welfare gain from integration is about 2.2% of expected lifetime utility.
The outcome under autarky is constructed for each of the two possible realizations of \( G^i \) using the budget constraint, (10), to compute the tax rate given \( \zeta \). With \( \zeta = 0 \), the expected lifetime utility under autarky exceeds that under bailout since there is no uncertainty in taxes under either regime but an intertemporal distortion in taxes under bailout. As the variability of \( \tau^A \) increases with \( \zeta \), the expected utility in autarky will decrease.

The welfare calculations are summarized in Table 1 for three levels of uncertainty in taxes under autarky, represented by \( \zeta \). The second column is the difference between expected utility in bail-out and autarky, \( W - W^A \). If the variability of spending, (and thus of \( \tau^A \)), is low enough, autarky dominates bail-out. But, as the variability of tax rates increases, the bail-out equilibrium generates higher expected utility than autarky.

<table>
<thead>
<tr>
<th>( \zeta )</th>
<th>welfare gain to bailout, ( (W - W^A) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.01</td>
<td>-0.0192</td>
</tr>
<tr>
<td>0.05</td>
<td>-0.0115</td>
</tr>
<tr>
<td>0.1</td>
<td>0.0463</td>
</tr>
</tbody>
</table>

Table 1: Welfare Comparison: Autarky vs. Bailout

There is an interesting contrast between this result and that obtained for the welfare implications of monetary union with a centralized monetary authority. Proposition 3 of Cooper and Kempf (2004) argues that there are welfare gains to a monetary union when shocks are highly correlated across countries. The argument, following Mundell, is that the delegation of a policy instrument to a central authority is not too costly when shocks are sufficiently positively correlated. Yet, as indicated by Proposition 9, when shocks are highly correlated, then there are little gains to tax smoothing and autarky may dominate a federation without commitment.22 Thus, in contrast to monetary unions, federations created to take advantage of tax smoothing are more likely to be welfare improving if shocks are not too positively correlated.

4.3 The Value of Commitment

The conclusion from this analysis is that the combination of autonomous regional governments and a central government leads to equilibria which will generally differ from the integrated (first-best) equilibrium, except under very stringent conditions. This returns us to the question raised earlier about the source of this sub-optimality: is it the interaction of multiple fiscal players or the lack of commitment by the central government?

The answer to this question is lack of commitment. However, the precise form of policy commitment needed to support the integrated allocation depends on the rationale for the bailout.

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22 The key to the point is not the source of the shocks per se but rather whether they induce positive correlation in distortionary taxes in autarky.
First, suppose that labor supply is totally inelastic so that the desire for consumption smoothing by $CG$ is at the heart of the bail-out. Suppose $CG$ chooses period 2 fiscal and redistributive policy at the beginning of period 1, before the regional governments set fiscal policies for periods 1 and 2. It is now the regional governments which respond to $CG$. 

Specifically, assume that $CG$ is able to credibly announce to the $RGs$ and the agents in the whole economy both the central tax rate $\tau^C$ and the fraction of $CG$’s total fiscal income that each region will receive, i.e. the commitment to a revenue sharing scheme between regions. For this policy:

**Proposition 10** Under commitment of the central government, when taxes are non-distortionary, the integrated solution is the unique equilibrium.

The policy which supports this integrated equilibrium allocation is quite simple: the $CG$ sets $\tau^C = 0$.\(^{23}\) With an inactive central government, there is no room for bail-out and the regional governments are forced to bear the burden of regional fiscal policies.

However, when the bail-out is due to tax smoothing, matters are not as transparent. A policy in which the $CG$ does not intervene, i.e. $\tau^C = 0$, is sub-optimal. The reason is that in the integrated solution, the common tax played an important role in facilitating the smoothing of distortionary taxes across the regions. Thus an inactive $CG$, which was essential to eliminate the bail-out equilibrium in Proposition 10, is insufficient once taxes are distortionary.

Nonetheless, a more elaborate policy by the $CG$ is sufficient to support the integrated solution with multiple fiscal agents in the case of linear utility.\(^{24}\) Recall from the discussion of the integrated solution in Section 2.2, the optimal policy was a common tax, $\tau^*$, across time and across regions when $v''(\cdot) = 0$. The optimal tax rate did depend on the realization of $G^i$ so the tax collected from the region with the low value of $G^i$ subsidized the region with the high realization of $G^j$.

Suppose that the game between regions is modified to provide commitment to the $CG$. To be precise, suppose $CG$ can commit to: (i) a tax rate $\tau^C = \tau^*$ in period 2 and (ii) transfer $T^i$ to region $i$, for $i = 1, 2$. This commitment occurs in period 1 after the realization of $(G^1, G^2)$ so that these choices are state dependent.\(^{25}\) $CG$ applies this tax and transfer policy in period 2 if and only if the regional governments imposes tax rates $t^*_i = \tau^*$ for $i = 1, 2$ in period 1. That is, if the regional governments imposed the optimal tax rates from the integrated solution in period 1, then $CG$ would follow with the optimal common tax in period 2 along with transfers to the regional governments to cover their deficits. Those transfers are given by: $T^i = \Delta'R(g^i - \tau^* n(\tau^*))$. Else, $CG$ would neither tax nor transfer in period 2.

Each regional government could choose to enter the fiscal federation under this policy or not. This policy clearly succeeds in supporting the integrated solution. That is, there exists a Nash equilibrium in which

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\(^{23}\)Note that this policy does not require the $CG$ to know the realized values of the spending shocks.

\(^{24}\)There may be weaker forms of a policy with commitment which could also support the integrated allocation.

\(^{25}\)Thus if the rationale for the federation was an informational advantage of regional governments, this policy may not be feasible. In that case, the sub-optimality of the equilibria in the federation relative to the integrated solution would reflect both a commitment problem and the informational asymmetry.
each regional government agrees to the terms proposed by the $CG$. Else, the outcome would be the autarky which is worse that the integrated solution due to the foregone gains of tax smoothing.

This discussion, along with Proposition 10, illustrates that commitment by $CG$ lies at the heart of the bail-out problem. Interestingly, the nature of the commitment differs dramatically across the rationales for bail-out. If consumption smoothing is the source of the incentive to bail-out, the best action is for $CG$ to commit not to intervene. But, if tax smoothing is the rationale for bail-out, then $CG$ must commit to a substantial economic role in order to reap the gains from revenue sharing and, at the same time, prevent the regional governments from abusing this intervention. Absent this commitment power, the bail-out allocation is the likely outcome.

5 Coping with Fiscal Pressures: Lessons from Existing Federations

Our analysis has shown the benefits of smoothing distortionary taxes across regions. These gains to fiscal integration are attainable if the central government can commit to its fiscal policy. But, without commitment power, these gains may be offset by the choices of regional governments to borrow excessively in the first period. As a consequence, autarky may be preferable to joining the federation.

It is instructive to use the model to interpret how various federations have attempted to deal with this commitment problem through various forms of fiscal constraints. Our goal here is not to explain the differences across these federations but rather to study them through the perspective of our model.

Of course, these federations differ from our model economy in a fundamental way: they all have a common currency which is absent from our model. Still, the sources of bail-outs highlighted in the real economy will be present in a monetary setting as well. Thus we use the lessons of our model to focus on the fiscal interactions within these federations. As discussed in the conclusion, there are surely additional interactions within these federations created by the presence of a common currency and thus a single central bank.

5.1 Argentina

Argentina is one of the most decentralized federations and has repeatedly struggled with bail-outs at the federal level. The sub-national “provincial” governments are responsible for almost 50% of the total consolidated public expenditures (more than 2/3 when pensions are excluded). However, most taxation powers are delegated to the central government. Hence, Argentina is characterized by large vertical transfers; in 1997, 56% of provincial expenditures were financed through transfers from the central government. Nine provinces finance less than 20% of their expenditures with their own resources.

The intergovernmental transfer scheme appears particularly complex, Saiegh and Tommasi (1999) call it the “federal fiscal labyrinth” of Argentina. Altogether in the 90’s, both the central and the provincial
governments have run large deficits. As a result, the provincial debts soared, growing on average from 40% of total revenues in 1994 to 55% in 1999.

Provinces facing unsustainable deficits have regularly been bailed-out by the central government, through various schemes, as discussed in Tommasi, Saiegh, and Sanguinetti (2001). These bail-outs have represented a significant proportion of the financial needs of provinces: in 1994, they amounted to almost 90% of the total provincial deficits. In particular, the pension regimes of 11 provinces have been nationalized between 1994 and 1996.

Altogether the central government seems unable to monitor the provincial fiscal policies, both in terms of taxation and public expenditures. According to Nicolini, Posadas, Sanguinetti, Sanguinetti, and Tommasi (2002), this has created “a general moral hazard problem that undermines the incentives of lower units to behave in fiscally responsive ways”.

The mixture of large intergovernmental transfers, low control of local public expenditures and recurrent bail-outs are clearly indicative of a weak central government, lacking commitment power, and a sub-optimal situation, characterized by large distortions due to the tax system. From the perspective of our model, these allocations appear inconsistent with the integrated solution. Instead, these transfers and bail-outs reflect the commitment problems highlighted in Propositions 6 and 8. Further, the incentive for bail-out may also reflect political pressure by regional governments in Argentina instead of population weights in the objective function of $CG$.

Various institutional reforms such as the currency board system put in place in the 1990s and the ensuing discussions of dollarization were attempts to reinforce the power of the central government to resist pressures from the provincial fiscal authorities. However these reforms have been incomplete as they were not addressing the core of the problem: the federal fiscal labyrinth that left the central government vulnerable to appeals for bailing out by regions with large debts. The task confronting Argentine public authorities is to directly tackle the issue of distorted fiscal federalism with the aim of stricter fiscal responsibilities for the provincial governments. A survey of the various plans aiming at modifying the intergovernmental fiscal relations in Argentina that have been discussed and enforced in the recent years. Cuevas (2003) concludes that while useful they do not represent a major break from the past.

5.2 U.S.

The U.S. is a federation of a different nature than Argentina. Relative to Argentina, the central government in the U.S. is largely immune to fiscal pressures of the regional (state) governments. There are automatic interregional transfers from the Federal budget to the states. For example, the

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26 Cooper and Kempf (2001) discuss these policies using a model of regional pressures within a federation.

27 As noted in Velasco (2000), studies of the U.S. support the hypothesis of tax smoothing. Still, the history of the U.S. is replete with incidents in which states defaulted on their debt, as discussed in Wallis, Sylla, and Ginath (2004). Further, as discussed in Wallis and Weingast (2005), many of the large expenditures in the early history of the U.S. were financed by state governments rather than the federal government despite gains from spreading these costs.
Welfare Reform Act of 1996 replaced a system of federal transfers automatically linked to welfare expenditures decided by state authorities by a system of lump-sum federal transfers with the states being fully responsible for any additional dollar spent on welfare. This is a clear evidence of the inability of states to pressure the Federal government for more aid.

All states but one (Vermont) have a fiscal constraint on their ability to borrow. Their stringency is not uniform and they do not forbid creative accounting measures. As a consequence, the effectiveness of these constraints is disputed. Whereas Poterba (1996) found that they have been effective in limiting state fiscal profligacy, a recent study by Canova and Pappa (2004) claims that these effects have been overemphasized: the differences between states with restrictive limits and states with looser ones are statistically and often economically unimportant.

From the perspective of our model, the federal transfer system can be characterized as an \textit{ex ante} revenue sharing scheme, with a large commitment power given to the federal level. This is close to the integrated allocation characterized in Proposition 1. Excessive deficits at the state level financed by federal bail-outs are unlikely in the U.S. federation. Hence, there is no wonder that the fiscal restraints on states’ fiscal policies are found ineffective. If anything, one might argue that these limits are too severe in that state governments are unable to smooth taxes over time.

5.3 E.U.

The E.U. does not qualify as a substantial fiscal federation. The budget of the European commission is small and represents at most 1.30\% of the E.U.’s GDP. Moreover the European commission has hardly any autonomous source of income and depends partially on contributions made by member countries. In the constitutional treaty recently discussed, the devolution of taxing powers to the E.U. itself is possible but will require unanimous approval by the member countries: hence it remains an unlikely event at best.\footnote{As far as the European Central Bank is concerned, a provision in the Maastricht Treaty explicitly prohibits any contribution from any of its entities to any public administration.}

In response to a fear of an increase in national public deficits, E.U. countries set up the Stability and Growth Pact. Importantly this treaty concerns all E.U. members, not just those belonging to the EMU. In our view, this is evidence that the real interactions stressed in our model are central to institutional provisions on debt limits in federations.

The critical provisions of the pact are threefold: a) each country would have to balance its public deficit over the medium term, b) national annual public deficits (including all public administrations and agencies) would be limited to 3\% of GDP, except under severely depressing conditions, c) in case of breach of this constraint, the faulty country would be liable to pecuniary penalties.\footnote{For a complete discussion, see www.ec.europa.eu/economy_finance/about/activities/sgp/sgp_en.htm.}

In fact, the restrictions in the Stability and Growth Pact have proven to be unenforceable. Following the November 2003 decision by the European council of Finance ministers not to apply these penalties to France and Germany despite their public deficits above 3\%, the European council issued a revision of the pact in
October 2005. In effect, this version considerably weakens the fiscal discipline and its enforcement in the E.U. The various limits set on national fiscal policies act more as guidelines than as effective constraints on the discretionary powers of national Treasuries.

From the perspective of our model, two inconsistencies appear in the E.U. situation. First, if there is no bail-out possibility because of the absence of any “federal” taxation, then there is no need to limit deficit spending. The importance of the Stability and Growth Pact as a way of curbing “excessive deficits” disappears when there is no central government that can be induced to make transfers to the national treasuries.

Second, these limits on deficit spending by national governments, if enforced, are likely to be harmful as they limit tax smoothing. Therefore it is no surprise that lately member countries have decided to loosen them.

Proposition 9 provides further insights into the E.U. situation. As argued in that proposition, federations can yield higher expected utility than autarky even if bail-outs occur in equilibrium. Thus, fiscal restrictions may be unnecessary to obtain welfare benefits from a federation. However, note that the E.U., following the arguments of Mundell, was constructed with the idea that countries with positively correlated shocks were natural partners. While this may indeed be true from the perspective of a monetary union, Proposition 9 tells us that in this case the gains to fiscal integration may be relatively small since there is little room for tax smoothing across countries.

The absence of fiscal federalism in Europe is delicate to assess. According to our theory, a fiscal federalism scheme in Europe is desirable to smooth taxes across regions. But this requires a strong commitment technology such that the central/federal authority is not vulnerable to the bail-out of national authorities running deficits. From this perspective, rather than issuing unenforceable strict fiscal limitations or harmless guidelines, the European countries should rather think about the establishment of a credible fiscal federalism scheme.

6 Conclusion

The goal of this paper was to study the factors which determine who pays the obligation incurred by a region within a federation. Within our framework, the best policy is one of ex ante fiscal federalism. It is efficient relative to autarky because it spreads distortionary taxes across large groups of agents.

But the central government may not have the commitment power needed to support this allocation. Ex post the central government may be induced to bailout the regional government in order to smooth consumption and distortionary taxes across the regions. In anticipation, regional governments will adjust their tax profiles to take advantage of the central government.\footnote{Though we focus on the response in terms of tax schedules, more generally regional governments will also have an incentive to increase spending given the sharing of these expenditures induced by the bail-out.}
We characterize the conditions under which bail-outs occur and their welfare implications. We associate the occurrence of bail-outs with the inability of the central government to commit.

With regards to the consumption smoothing motive, two types of equilibria are possible: a Ricardian equilibrium without bail-out and a bail-out equilibrium. Their occurrence depends on the distribution of debt holdings across agents in the federation.

Focusing on the tax smoothing motive, a fiscal federalism scheme with a weak central government may be better than autarky when the correlation between regional public expenditures shocks is low and their variance is sufficiently large. But, when the shocks across regions are highly positively correlated, then the adverse effects of the bail-outs dominate and there is a welfare loss from the federation relative to autarky. We discuss some observations on inter-governmental fiscal relations in the U.S., E.U. and Argentina, in light of these insights.

One important element in the interaction between regions and the central government missing from our analysis is active monetary policy. In the case of Argentina, the creation of regional monetary instruments during the currency period, and more generally the monetization of regional debt, played a key role in the economic crisis of 2001. For the E.U., even though there is no federal income tax, there is a central monetary authority and thus pressure from individual countries may still be exerted upon that entity.

As an example, Cooper, Kempf, and Peled (2004) analyzes a monetary economy in which a central bank, induced by a consumption smoothing motive, uses an inflation tax to bailout a region’s debt. That analysis may extend to a tax smoothing motive as well. This analysis naturally leads to the question of whether certain policy rules could provide the basis for commitment of a central bank. This question is part of ongoing research.

A second element for future consideration is the determination of the default costs. These are taken as given here and do play a role in supporting bail-out equilibria with tax smoothing. The source of these costs and, more interestingly, how they are determined within a federation, are of interest for future study.
7 Appendix

Proof of Proposition 1

Proof. Taking derivatives of (7) with respect to $\tau_i^1$ and $\tau_i^2$ implies

\[ Z'(\tau_i^1)u'(c_i^1) = -\lambda I'(\tau_i^1) \]  \hspace{1cm} (15)

\[ \beta Z'(\tau_i^2)u'(c_i^2) = -\frac{\lambda}{R} I'(\tau_i^2) \]  \hspace{1cm} (16)

for $i = 1, 2$ where $\lambda$ is the multiplier associated with (8). Using the household’s first order condition of $u'(c_i^1) = \beta R v'(c_i^2)$, (15) and (16) imply

\[ \frac{Z'(\tau_i^1)}{I'(\tau_i^1)} = \frac{Z'(\tau_i^2)}{I'(\tau_i^2)}. \]  \hspace{1cm} (17)

Let $\xi(\tau)$ be the elasticity of labor supply with respect to the labor tax. From (2), $\xi(\tau) = -\frac{\tau}{(1-\tau)}$ so $\xi(\tau)$ is monotonically decreasing in $\tau$. By definition, $\frac{Z'(\tau)}{I'(\tau)} = -\frac{1}{1+\xi(\tau)}$ which is monotone in $\tau$. Thus $\tau_i^t = \tau_i^{*t}$ for $t = 1, 2$ and region $i = 1, 2$.

Across regions, the first-order conditions imply

\[ \frac{u'(c_1^i)Z'(\tau_1^{*1})}{I'(\tau_1^{*1})} = \frac{u'(c_2^i)Z'(\tau_2^{*2})}{I'(\tau_2^{*2})} \]  \hspace{1cm} (18)

for $i = 1, 2$. The marginal “cost-benefit” ratios of tax distortions, measured by the ratio of marginal utility loss to marginal tax revenue gain, are equalized across regions. This condition can be rewritten as

\[ \frac{u'(c_1^i)}{u'(c_2^i)} = \frac{1 + \xi(\tau_1^{*1})}{1 + \xi(\tau_2^{*2})} \]  \hspace{1cm} (19)

To see that both tax rates increase in $g^1$, suppose without loss of generality that $g^1$ increases. Suppose $\tau_2^{*2}$ does not change so that, from the household budget constraint, $c_2^2$ will not change. We consider other possibilities below.

In order for the single government’s budget constraint to hold, $\tau_1^{*1}$ has to increase. Given the household budget constraint, $c_1 + c_2/R = g + e + Z(\tau^1) + Z(\tau^2)/R$, $c_1^1$ decreases as $\tau_1^{*1} = \tau_1^1 = \tau_1^2$. Therefore the LHS of (19) increases as $\tau_1^{*1}$ increases. The RHS decreases as $\tau_1^{*1}$ increases because $\xi(\tau)$ is a decreasing function of $\tau$. There is an inconsistency so that $\tau_2^{*2}$ must change when $g^1$ and $\tau_1^{*1}$ increase. If, as $g^1$ increases, $\tau_2^{*2}$ decreases while $\tau_1^{*1}$ increases, applying a similar argument also leads to an inconsistency. So, $\tau_2^{*2}$ must increase if both $g^1$ and $\tau_1^{*1}$ increase in order for (19) to hold.

\[ ^{31}\text{While the region 1 household has more resources when } g^1 \text{ increases, it also has to bear the burden of a distortionary tax. If there was no distortion, then the increase in } g^1 \text{ and the increase in } \tau_1^{*1} \text{ would just offset and household resources would not change. But, due to the distortionary taxes, as } g^1 \text{ increases, consumption falls.} \]
Suppose instead that $\tau^*_1$ is constant or decreases while $\tau^*_2$ increases to finance the increase in $g^1$. By a similar argument, there is an inconsistency with (19). Hence, for (19) to hold, the increase in $g^1$ must be met by an increase in $\tau^*_i$ for $i = 1, 2$.

Complete tax smoothing across regions, defined as $\tau^*_1 = \tau^*_2$, will occur in two special cases. If $v(c) = c$, then the household’s intertemporal optimization condition implies $u'(c^1_i) = u'(c^2_i)$. Hence $\tau^*_1 = \tau^*_2$ will satisfy (18). Second, if $G^1 = G^2$ with probability one, the solution with integration will be perfectly symmetric so that tax rates are equal.

Proof of Proposition 2

Proof. Taking derivatives of (9) with respect to $\tau^*_i$ and $\tau^*_j$ implies

$$Z'(\tau^*_i)u'(c^1_i) = -\lambda I'(\tau^*_i)$$

(20)

$$\beta Z'(\tau^*_j)v'(c^2_j) = -\frac{\lambda}{R} I'(\tau^*_j)$$

(21)

for $i = 1, 2$ where $\lambda$ is the multiplier associated with (10).

Using the household’s first order condition of $u'(c^1_i) = \beta R v'(c^2_j)$, (20) and (21) imply

$$\frac{Z'(\tau^*_i)}{I'(\tau^*_i)} = \frac{Z'(\tau^*_j)}{I'(\tau^*_j)}.$$  

(22)

Since, as argued in the proof of Proposition 1, $Z'(\tau)/I'(\tau)$ is monotone, $\tau^*_i = \tau^*_j \equiv \tau^*$. Let $V(g^i) \equiv W(\tau^A(g^i), \tau^A(g^j))$ where $\tau^A(g^i)$ is given in Proposition 2 along with the regional government’s budget constraint of $G = I(\tau^A \frac{(1+R)}{R})$. Taking a derivative of $V(g^i)$ with respect to $g^i$ and using (5) implies

$$V'(g^i) = u'(c^1_i)(1 - \frac{1}{1 + \xi(\tau^A(g^i))}).$$

(23)

As $\xi(\tau) = \frac{-\tau}{\tau(1-\gamma)}$, $\frac{1}{1+\xi} > 1$ as long as $\tau \in (0, \frac{1}{1-\gamma})$. This restriction on $\tau$ is justified by observing that $\tau = \frac{\gamma}{1+\gamma}$ is the tax rate which maximizes the government tax revenue, $\tau n(\tau)$. As $\gamma > 0$, $\xi(\tau^A(g^i)) \in (-1, 0)$ and so $V'(g^i) < 0$.

In order for utility to fall as $g^i$ increases, the consumption levels in at least one period must fall. From (5), the consumption levels move together. Thus as $g^i$ increases, consumption falls in each period.

Proof of Proposition 3

Proof. (i) $W^* < W^A$ is not possible since autarky is feasible in the integrated setting, implying $W^* \geq W^A$.

(ii) From the proof of Proposition 1, the tax rates under the two institutional structures are different whenever $g^1 \neq g^2$. If these tax differences are distortionary, i.e. $\gamma < \infty$, then $W^* > W^A$. 

27
(iii) If the shocks are perfectly positively correlated so that \( g^1 = g^2 \) in all states of nature, then the regions are identical and thus expected utility is the same under integration and autarky. From (18), if \( g^1 = g^2 \), then by symmetry \( \tau^1 = \tau^2 \). As a consequence, the budget constraint holds for each region. This is exactly the same level of taxation which would occur in autarky so that \( W^* = W^A \).

If taxes are not distortionary, \( \gamma \to \infty \), then \( Z'(\tau)I' = -1 \) so that, using (15) and (16), taxes and thus consumption allocations are the same across regions in the integrated solution. As a consequence, the budget constraint holds for each region. This is also true for the allocation under autarky so that \( W^* = W^A \).

Proof of Proposition 4

Proof. First we show that if \( \theta^i = 1, \forall i = 1, 2 \) then \( RG_i \) is indifferent between tax and default and thus will choose to tax given any positive default cost. Then we argue that \( CG \) will set \( \tau^C = 0 \).

For this discussion, let \( b^{ij} \) be the amount of region \( j \)'s debt held by a region \( i \) agent. Thus \( b_i = b_{ii} + b^{ij} = \theta^i B_i \Delta_i^i + (1 - \theta^i) B_j \Delta_i^j \).

To check the incentives of \( RG_i \), suppose \( CG \) sets \( \tau^C = 0 \) and thus does not bail-out in period 2. What are \( RG_i \)'s options given the (anticipated) \( CG \)'s policy? If it taxes, private agents in region \( i \) have utility of \( v(\eta + R(s^i - b^i)) \). If it defaults the consumption is the same since only region \( i \) agents hold region \( i \) debt.

With a negligible default cost, the region will choose to raise taxes to pay-off its debt: \( \tau^i_2 \eta = Rb^i \).

Anticipating this taxation and \( \tau^C = 0 \), private agent’s savings in period 1 will solve

\[
W(\tau^i_1, \tau^i_2 \mid g^i) = \max_{s^i} u(e + g^i + Z(\tau^i_1) - s^i) + \beta v(Z(\tau^i_2) + s^i R).
\]

As \( Z(\tau) = (1 - \tau) \eta \) when taxes are not distortionary, the first order condition is \( u'(e + g^i + (1 - \tau^i_1) \eta - s^i) = \beta R v'(R (\eta + (s^i - b^i)) R) \). Using \( \tau^i_2 \eta = Rb^i \) and \( g^i - \tau^i_1 \eta = b^i \), the first-order condition can be written as

\[
u'(e + \eta - (s^i - b^i)) = \beta R v'(R (\eta + (s^i - b^i)).
\]

Thus, the difference \( (s^i - b^i) \) is the same for all regions regardless of the period 1 realization of \( g^i \). This means that along the path of the Ricardian equilibrium, period 2 consumption is equal across agents in the two regions. That is, anticipating tax financed debt service in period 2, agents save the excess regional transfer they get in period 1, so that \( c^1_2 = c^2_2 \).

To check the incentives of \( CG \), note that by choosing a value of \( \tau^C \), it can transfer consumption across regions to maximize its objective of

\[
\sum_i \Delta^i u(c^i_2)
\]

given a resource constraint of:

\[
\sum_i \Delta^i c^i_2 = \eta + R \sum_i \Delta^i (s^i - b^i)
\]
Given that $v(\cdot)$ is strictly concave, $CG$ would choose a consumption allocation with complete equality. But this is precisely the consumption allocation in the Ricardian equilibrium. Thus $CG$ is unable to intervene to further smooth consumption and it will choose $\tau_C = 0$.

For $\theta^i < 1$, $RG_i$ strictly prefers default over regional tax for $\epsilon$ equal to zero, even when it anticipates that $CG$ will set $\tau_C = 0$. If the regional government taxes agents in its region, some of the revenue is used to pay off agents in the other region. If the regional government defaults, consumption of agents in its region will thus be higher. So, for any $\theta^i < 1$, there will exist a sufficiently small default cost such that no Ricardian equilibrium exists.

**Proof of Proposition 5**

Proof. In the Ricardian equilibrium, $RG_i$ chooses $(\tau_1^i, \tau_2^i)$ to maximize $W(\tau_1^i, \tau_2^i | g^i)$, defined in (24) after $g^i$ has been realized, with $\tau_C = 0$ subject to the constraint of $g^i = \left(I(\tau_1^i) + I(\tau_2^i)\right)$.

This is exactly the same problem a regional government solves in autarky, as in Proposition 2. As the first-order conditions are the same for these two problems, so will be the allocations and consequently the levels of expected utility: $W^R = W^A$.

From Proposition 3, consumption allocations and thus welfare are the same in the integrated federation as under autarky. Thus, $W^R = W^* = W^A$. ■

**Proof of Proposition 6**

Proof. Moving first at the start of period 2, $CG$ selects a common tax and the distribution of the tax revenue across regions, given the outstanding debt obligations of the two regions. Specifically, it solves:

$$
\max_{T^1, T^2} \sum \Delta iv(\eta + Rk^i + (\frac{\theta^i}{\Delta})T^i + (\frac{1 - \theta^i}{\Delta})T^j - (T^1 + T^2)) - \Delta^i \epsilon \Upsilon(\Delta^i B^i R - T^i)
$$

where $T^i \in [0, \Delta^i B^i R]$ is $CG$’s transfers to $RG_i$, $T^1 + T^2$ is the common economy-wide tax (per agent) needed to finance such payments, and $\Upsilon(x) = 1$ if $x > 0$ and zero otherwise. Full bail-out by $CG$ is denoted by $T^i = \Delta^i B^i R$ for $i = 1, 2$. Here $\eta$ is the tax-independent labor supply.

If $CG$ chooses less than full bail-out for any region $i$, then $RG_i$ will default on the remaining debt. As shown in the proof of Proposition 4, $RG_i$ is indifferent between default and taxation when $\theta^i = 1$ and $\epsilon = 0$. For any $\theta^i < 1$, $RG_i$ will strictly prefer default for $\epsilon = 0$ and, by continuity, for $\epsilon$ near zero. Thus the only equilibrium if $\theta^i \neq 1$ is full bail-out.

Social welfare under a full bail-out can be written as:

$$
\sum \Delta^i v(\eta + Rk^i + D^i_1 + D^i_2)
$$

where $D^i_1 = (\frac{\theta^i}{\Delta} - 1)\Delta^i B^i R$ and $D^i_2 = (\frac{1 - \theta^i}{\Delta} - 1)\Delta^i B^i R$. $D^i_1$ is the net transfer to an agent in region $i$ from bailing-out region $j$’s debt, and $D^i = \sum_j D^i_j$ is the net transfer to such an agent from a full bail-out of all regional debts. The $D^i_j$ variables are key as they measure the direction and the extent of the redistribution.
from each of the two assets. As total consumption must equal total resources, \( \sum \Delta_i c_i^2 = \sum \Delta_i (\eta + Rk^i) \), we have \( \sum \Delta_i D_i = 0 \).

CG will prefer to bailout the regional governments iff \( \theta^1 < \Delta^1 \) and \( \theta^2 > \Delta^2 \). In this case, \( D_1^1 < 0, D_1^2 > 0, D_2^1 > 0, D_2^2 > 0 \) and consequently, \( D^2 > 0 > D^1 \).

With this pattern of debt holding, the bail-out of the debt of both regions will redistribute resources away from region 1 and towards region 2. Such a redistribution in favor of region 2 will lead to a more equitable distribution of consumption given the assumption \( g^1 > g^2 \) which implies \( s^1 > s^2 \). Hence, \( D^2 > 0 > D^1 \) implies that the repayment of the debt through a common tax will raise the consumption of region 2 agents relative to region 1. Since this allocation is more equitable than the allocation under default, CG prefers it to the default allocation.

If either \( \theta^1 \geq \Delta^1 \) or \( \theta^2 \leq \Delta^2 \), then the CG will not have an incentive to bail-out the debt of both regions. With sufficiently small default costs, the regional government will choose to default on period 2 debt. This is not an equilibrium outcome.

In anticipation of a bail-out, \( RGi \) solves the following problem, recognizing the impact of its own debt on the tax to be set by CG:

\[
\max_{\tau_i^1} u(g^i + e + \eta(1 - \tau_i^1) - s^i) + \beta v(\eta(1 - \tau) + s^i R).
\]  
(30)

s.t.:

\[
B^i = g^i - I(\tau_i^1),
\]

\[
\tau = R \eta \sum \Delta_i B^i,
\]

\[
u'(c_i^1) = \beta R v'(c_i^2).
\]

The first constraint defines the debt issued by region \( i \) given its period 1 tax and the realized \( g^i \). The second reflects the tax rate set by the central government in period 2 in a full bail-out, which satisfies \( \tau \eta = R(\Delta B^1 + (1 - \Delta)B^2) \). The third constraint reflects agents’ optimal saving.

The derivative of \( RGi \)'s objective with respect to \( \tau_i^1 \), taking into account the constraints, is:

\[
-\eta u'(c_i^1) + \beta v'(c_i^2) \eta \Delta^i R.
\]  
(31)

This is always negative given the saving first-order condition of the representative agent. Hence \( \tau_i^1 = 0 \).

Proof of Proposition 7

**Proof.** Immediate from Proposition 3 and the fact that the bail out allocation differs from the integrated allocation. ■

Proof of Proposition 8

**Proof.** At the start of period 2, the debt levels (per regional agent) in each of the two regions are given as \( (B^1, B^2) \) where \( B^i = g^i - I(\tau_i^1) \). CG moves first and selects a common tax, \( \tau \), for period 2 and transfers
revenues across regions. This tax generates revenue of \( \tau \left( \Delta n (\tau + \tau_1^2) + (1 - \Delta) n (\tau + \tau_2^2) \right) \). The remainder of the region’s debt obligation is financed by a region \( i \) specific tax, \( \tau_i^1 \).

Suppose, contrary to the actual game, that at the start of period 2, \( CG \) selects both regional taxes, \( \tau_i^2 \) for \( i = 1, 2 \). It solves

\[
\max_{\tau_1^2, \tau_2^2} \sum_i (\Delta_i \left[ Z(\tau_2^i) + R s^i \right])
\]

subject to:

\[
R(\sum_i \Delta_i B^i) = \sum_i \Delta_i I(\tau_2^i).
\]

Here \( B^i \) is the amount of debt per region-head issued by \( RG_i \). Hence, this corresponds to an amount of debt per federation-head equal to \( \Delta_i B^i \).

With \( \lambda \) as the multiplier on the budget constraint, the first-order conditions are:

\[
Z' \left( \tau_2^i \right) = \lambda I' \left( \tau_2^i \right) \quad i = 1, 2.
\]

Following previous arguments, this implies \( \xi \left( \tau_2^1 \right) = \xi \left( \tau_2^2 \right) \) so that \( CG \) would choose \( \tau_1^1 = \tau_2^2 \).

Thus, given debt levels \( CG \) would like the regions to set equal tax rates in the second period. This can be achieved by \( CG \) if it sets the common period 2 tax rate, \( \tau \), high enough to finance the entire debt obligation of the regional governments and distributes the revenues to them as needed to finance their debt.

This tax rate solves \( I(\tau) = R(\sum_i \Delta_i B^i) \). Denote the (smallest) value of \( \tau \) satisfying this requirement as \( \sigma (B^1, B^2) \) where \( \sigma_i (B^1, B^2) = \frac{\partial \sigma}{\partial \tau_i} = \frac{\Delta_i B_i}{\tau_i} \).

Thus the regional governments know that the central government will have an incentive to fully finance the debt obligations of the regions. Accordingly, \( \tau_1^i = 0 \) for \( i = 1, 2 \). Given this, in period 1 each regional government solves:

\[
\max_{\tau_1^i} u \left( g^i + e + Z(\tau_1^i) - s^i \right) + \beta \left( Z(\sigma (B^1, B^2)) + R s^i \right).
\]

Using \( B^i = g^i - I(\tau_1^i) \), the first order condition can be written as

\[
u' \left( c_1^i \right) \frac{Z' (\tau_1^i)}{T'(\tau_1^i)} = \beta Z'(\tau) \sigma_i (B^1, B^2) I'(\tau_1^i).
\]

Using \( \sigma_i (B^1, B^2) \) from above and \( u' (c_1^i) = \beta R \), we can rewrite this as

\[
\frac{Z' (\tau_1^i)}{T'(\tau_1^i)} = \frac{Z'(\tau)}{T'(\tau)} > \frac{Z'(\tau)}{T'(\tau)}.
\]

The inequality arises from \( \Delta_i < 1 \) and \( Z' (\cdot) < 0 \). Using the definitions of \( Z(\tau) \) and \( I(\tau) \), this condition can be rewritten as

\[
\frac{1}{1 + \xi (\tau_1^i)} < \frac{1}{1 + \xi (\tau)}
\]

implying \( \tau_1^i < \tau \) for \( i = 1, 2 \).
Proof of Proposition 9

Proof. Throughout this proof, the following assumptions are maintained:

- \( v''(\cdot) = 0 \)
- Distorting labor taxes, \((\gamma < \infty)\)

Tax revenue from a tax rate \(\tau\) is \(I(\tau) = \tau n(\tau)\), where \(n(\tau) = (1 - \tau)^{1/\gamma}\). We only consider tax rates such that \(\tau \in [0, \frac{\gamma}{1+\gamma}]\). On this interval, \(I(\cdot)\) is strictly increasing and strictly concave.\(^{32}\)

Since \(v(\cdot)\) is linear and \(u(\cdot)\) is strictly concave, \(c_1^i\) is a constant determined by the saving first order condition. Thus \(c_1^i = c\) for any realization of the random public spending shocks, \((g^1, g^2)\) in all three allocations under consideration: autarky, integrated economy, and the bail-out equilibrium characterized in Proposition 8.

Expected life-time utility of an agent is thus given by:

\[
W = u(c) + \beta E\{ (Rs(\tau_1) + Z(\tau_2)) \}
\]

where first and second period taxes, and hence labor income and savings, may depend on the realization of \((g^1, g^2)\). From the agents’ first period budget constraint:

\[
s(\tau_1) = g + e + Z(\tau_1) - c,
\]

so that

\[
W = u(c) + \beta RE\{ G + e + Z(\tau_1) + \frac{Z(\tau_2)}{R} - c \}.
\]

The determination of \((\tau_1, \tau_2)\) will depend on the institution.

**Autarky** From Proposition 2, for every realization of \(G\), a common tax rate is set for both periods, \(\tau_1 = \tau_2 = \tau^A(g)\), which is implicitly defined by:

\[
g = \left(1 + \frac{1}{R}\right) I(\tau^A(g)). \tag{34}
\]

Expected utility under autarky is then:

\[
W^A = u(c) + \beta R \left[ e - c + E\{ G + \left(1 + \frac{1}{R}\right) Z(\tau^A(G))\} \right], \tag{35}
\]

where the expectation is taken with respect to the marginal distribution of \(G\).

\(^{32}\)This can be directly verified from the first and second derivatives of \(I(\cdot)\) for \(\tau \in [0, \frac{\gamma}{1+\gamma}]\).
**Integrated Economy**  From Proposition 1 and the discussion following it for the linear $v(\cdot)$ case, there is a single tax rate for any realized pair of regional spending shocks, applied in all regions in both periods, which satisfies:

$$\sum_i \Delta^i g^i = \left(1 + \frac{1}{R}\right) I\left(\tau^*(g^1, g^2)\right).$$

The corresponding expected lifetime utility for the integrated economy is therefore:

$$W^* = u(c) + \beta R \left[ e - c + E\{G^i + \left(1 + \frac{1}{R}\right) Z(\tau^*(G^1, G^2))\} \right].$$

**(36)**

**Tax-smoothing Bail-out**  From Proposition 8 we have a first period tax set by the regional government, $\tau^i_1$, and a common second period tax set by $CG$, $\bar{\tau}$, which depend on the realizations of public spending shocks in both regions. These tax rates must satisfy the budget constraint

$$\sum_i \Delta^i (g^i - I(\tau^i_1)) = \frac{1}{R} I(\bar{\tau}).$$

**(37)**

The expected utility of a region $i$ agent is given by:

$$W = u(c) + \beta R \left[ e - c + E\{G^i + Z(\tau^i_1) + \frac{1}{R} Z(\bar{\tau})\} \right].$$

**(38)**

**Welfare Comparison: Integration versus Bail-out**  $W^* \geq W$ is clearly true since the equilibrium under bail-out was feasible in the integrated solution. Moreover, as in Proposition 8, the tax rates in the bail-out equilibrium are not the same as those obtained under integration. Thus $W^* > W$.

**Welfare Comparison: Bail-out versus Autarky**  We first argue that if the correlation between $g^1$ and $g^2$ is near 1, then $W < W^A$. Comparing the welfare measures under autarky and bail-out, we can write $(W - W^A) = (W - W^*) + (W^* - W^A)$. As argued above, $(W - W^*)$ is negative. By Proposition 3, $(W^* - W^A)$ is positive due to the ability to share tax revenues and thus smooth regional taxes. However, when $corr(G^1, G^2) = 1$, there is no scope for tax smoothing over the regions. Therefore $W^* = W^A$ and $W - W^A$ is consequentially negative. By continuity, this is true for $corr(G^1, G^2)$ sufficiently close to 1.

We now argue that $W - W^A$ is increased if the correlation between $G^1$ and $G^2$ is near -1 and the variance of $G^i$ is sufficiently large. For this argument, we also assume $\Delta = \frac{1}{2}$.

When $corr(G^1, G^2) = -1$ and $\Delta = \frac{1}{2}$, the average of the $g^i$ is always equal to the $\bar{G} = \frac{1}{2}(g^1 + g^2)$. Using (32) and (37), $\tau$ and $\tau^i_1$ are constant in the bail-out equilibria. Therefore $W$ does not depend on the variance of public expenditures.

But this is not the case under autarky, where a regional government must bear the risk of variability in $G$. From Proposition 2, the regional government will set a tax rate constant over the two periods which depends on the realized value of $G$. With this in mind, the lifetime utility of an agent in an autarkic allocation is

$$W^A(\tau) = \max_s u\left(\frac{1 + R}{R} I(\tau) + e + Z(\tau) - s + \beta Z(\tau) + sR\right)$$

**(39)**

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where we used the government budget constraint \( g = \frac{1+R}{\tau}I(\tau(g)) \). Thus, instead of thinking about the variability of \( G \), we will focus instead on the variability of \( \tau \).

Using \( n(\tau) = (1-\tau)\frac{\tau}{\gamma} \) and the definition of \( Z(\tau) \) from (3), \( W^A(\tau) \) is strictly concave. The first derivative of \( W^A(\tau) \) with respect to \( \tau \) is

\[
W^A'(\tau) = -\frac{\tau}{\gamma}(1-\tau)^{\frac{1-\gamma}{\gamma}}
\]

and the second derivative is

\[
W^A''(\tau) = -\frac{(1-\tau)^{\frac{1-\gamma}{\gamma}}}{\gamma} \frac{\gamma - \tau}{\gamma}.
\]

Under the restriction that \( \tau \in [0, \frac{\gamma}{1+\gamma}], \tau < \gamma \) so that \( W^A''(\tau) < 0 \).

Increased variability of \( G \) around its mean implies increased variability of taxation under autarky. A mean preserving spread in \( G \) implies an increase in the mean of tax rates given the concavity of the tax revenue function \( I(\tau) \). Since \( W^A(\tau) \) is concave in \( \tau \), it follows that the expectation of \( W^A(\tau) \) with respect to \( \tau \) decreases, and \( \bar{W} - W^A \) increases, as the variability of \( G \) increases.

**Proof of Proposition 10**

**Proof.** We define commitment as the fiscal policy decided by \( CG \) at the beginning of period 1 and applied in period 2, even before the regional governments decide and implement their own fiscal policies. The realizations of \( g^i \) for \( i = 1, 2 \) are known by \( CG \) when it chooses its tax.

Consider the special case where \( \gamma \to \infty \) and \( n^i_t = \eta, \forall i, t \). This implies \( I'(\tau) = \eta = -Z'(\tau) \).

\( CG \) decides \( \tau^C \) and the two transfers \( T^1 \) and \( T^2 \) given to the \( RGs \) in period 2. The budget constraint for the regional government \( i \) is:

\[
g^i = \tau^i_1 \eta + R^{-1} \left[ \tau^i_2 \eta + T^i \right].
\]

The tax policy of \( RGi \) solves:

\[
\max_{\tau^i_1, \tau^i_2} u(g^i + e + Z(\tau^i_1) - s^i) + \beta v(Z(\tau^i_2 + \tau^C) + s^i R)
\]

subject to (42). The FOCs are:

\[
u'(c^i_1) = \lambda^i
\]

\[
\beta R v'(c^i_2) = \lambda^i
\]

where \( \lambda^i \) is the multiplier associated with the budget constraint of \( RGi \).

The maximization problem of \( CG \) is:

\[
\max_{\tau^C, T^1, T^2} \sum \Delta^i \left[ u(g^i + e + Z(\tau^i_1) - s^i) + \beta v(Z(\tau^i_2 + \tau^C) + s^i R) \right]
\]

s.t.

\[
u'(c^i_1) = \beta R v'(c^i_2), \forall i.
\]

\[
T^1 + T^2 = \tau^C \eta.
\]
When $\tau^C = 0 = T^1 = T^2$, each RG acts in isolation and we obtain the Ricardian solution. Since, using Proposition 3, this is the same allocation as the integrated solution, the policy of $\tau^C = 0 = T^1 = T^2$ is the solution to (45).

$\blacksquare$
References


