Doves for the Rich, Hawks for the Poor?
Distributional Consequences of Monetary Policy

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Abstract

We build a New Keynesian business-cycle model with the novel feature of rich household heterogeneity. Households differ in the amount of their savings, patience, labor productivity, and employment status. Labor-market transitions are subject to matching frictions. We allow for aggregate saving. As a result, household heterogeneity affects the transmission of monetary policy to aggregate consumption, but hardly to GDP. Nevertheless, monetary policy has sizable distributional consequences. A majority of households would prefer monetary policy to be more accommodative than would be optimal absent the heterogeneity and market incompleteness, helping “Main Street” at the expense of “Wall Street.”

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1 Introduction

Monetary policy, not unlike taxes, government expenditures, or social insurance, affects both aggregate economic activity and the distribution and riskiness of all types of income. The latter is important since, in the data, households’ sources of income differ starkly. Wealthier households receive a significant amount of financial and business income, whereas other households rely primarily on labor income or transfers. To be precise, among working-age households in the U.S. the bottom 60 percent of the wealth distribution, “Main Street”, receive virtually none of their income from financial assets, whereas “Wall-Street,” the 5 percent wealthiest households, receive 41 percent of their income from financial assets.\textsuperscript{1} We have little knowledge, however, what the implications of this are for the transmission of monetary policy. Worse, there is little guidance as to what the systematic response of monetary policy to unemployment should look like in an unequal society. Filling these gaps is the goal of the current paper.

The current paper builds a New Keynesian sticky-price business cycle model. Nominal rigidities together with search and matching frictions in the labor market render unemployment fluctuations endogenous to monetary policy. Real wages are privately efficient but assumed to be rigid (as in Hall 2005), giving the monetary authority a reason to deviate from perfect price stability even absent heterogeneity (compare Blanchard and Galí 2010 and Ravenna and Walsh 2012). To this core we add rich household heterogeneity in order to replicate key features of the wealth and income distribution. Financial markets are incomplete, so that households hold precautionary savings against aggregate and idiosyncratic risk. Next to their wealth, households differ in their patience (as in Krusell and Smith 1998 and Carroll et al. 2015), productivity (as in Castañeda et al. 2003 and Nakajima 2012a), and employment status (as in Krusell et al. 2010). We abstract from a portfolio allocation decision and rather assume that households can self-insure only through trading shares in a mutual fund that owns all the firms in the economy. The result is an economy that is neither approximated well with the saver-spender models of the business cycle (that follow Campbell and Mankiw 1989), nor is the representative-agent paradigm a good guide to its policy implications.

To the best of our knowledge our framework is the first that provides a global solution to a New Keynesian model with search and matching frictions, physical capital, self-insurance, and rich household heterogeneity. We find important implications of heterogeneity for monetary policy. This is so, even though we calibrate the model to a tranquil period (1984Q1 to 2008Q3).

We show that heterogeneity has a substantial impact on the business cycle. Relative to a

\textsuperscript{1}These figures are from the 2004 Survey of Consumer Finances. Table 6 in Appendix D provides details.
representative-agent (“RA” henceforth) counterfactual, the volatility of aggregate consumption rises by 14 percent. The volatility of GDP rises by less, namely, 4 percent. Market incompleteness means that a significant mass of households in the heterogeneous-agent (“HA”) model adjust consumption more in response to a recessionary shock than a representative household would, precisely with a view toward adjusting savings by less. They can do so in the aggregate because, in spite of capital adjustment costs, aggregate investment is somewhat elastic. Aggregate investment, therefore, is less volatile in the HA model than the RA counterpart. The changes in transmission are particularly pronounced for monetary policy shocks (shocks to the nominal interest rate), for which the consumption response on impact doubles.

Households disagree with respect to monetary policy. We first illustrate this for a one-time one-standard-deviation monetary shock (with a half-life of three quarters). The shock raises the nominal interest rate by 6.25 basis points (25 basis points annualized), and leads to a contraction in economic activity and an increase in the inequality of earnings, income, wealth, and consumption. Markups increase but the return on capital and employed labor falls. Overall profit income, therefore, falls more than GDP, and more than labor earnings (all in percentage terms). Nevertheless, Main Street suffers more from the shock than Wall Street. The wealthiest are well-insured against an unemployment spell and, by assumption, hold diversified portfolios. They suffer the same welfare cost as the RA model would suggest. The median-wealth households’ welfare costs are 6 times that in the RA model, however, and the wealth-poorest households’ 12 times. For the latter, the shock results in a welfare loss equivalent to 0.12 percent of lifetime consumption. Other dimensions of heterogeneity matter as well. Unemployed households’ welfare costs are four times those of employed households. And, conditioning on wealth, it is the households with the highest earnings capability when employed that lose most from the contraction because a contractionary monetary shock causes a dent in their labor income precisely when they are productive. Some households would be willing to give up a full percentage point of lifetime consumption to avoid the one-time monetary shock.

In light of these results, we next ask how accommodative the different households would like monetary policy to be. Toward this end, we study the transition to a stronger response to unemployment in the central bank’s interest-rate rule while keeping fixed the inflation and unemployment targets in the rule and the response coefficient to inflation. While all households agree that there should be some response of monetary policy to unemployment in the interest-rate rule (as would a representative agent), they disagree about its size. Monetary policy that systematically focuses more on stabilizing the unemployment rate rather than inflation benefits Main Street at the expense of Wall Street. The wealthiest prefer a smaller response to unemployment
than is optimal in the RA version of our model. The median-wealth household, instead, favors
a response to unemployment that is twice as strong as the optimal response in the RA model.
The welfare differences matter quantitatively. For the wealthiest households implementing the
median household’s preferred policy instead of the policy that they prefer means a welfare loss
equivalent to 0.3 percent of lifetime consumption. Falling average dividends and asset prices
induced by rising inflation volatility, more procyclical payouts, and less precautionary saving by
the middle class explain the welfare losses for the wealthy. Generally, welfare losses from more
accommodative monetary policy are largest for those households that would wish to dissave even
if they were employed. For some other households, instead, the consumption-equivalent welfare
gains from more unemployment-focused monetary policy runs as high as 0.7 percent of lifetime
consumption.

Most households in the HA economy have a positive target level of wealth (including impatient
households with high skills). We show that, nevertheless, the median household would not favor
policies that are optimal for a “representative” saver household. Nor is the median household
content with policies aimed at “spenders” only. We show that while a simpler Campbell and
Mankiw (1989)-type saver-scorner model would capture the rough trade-offs, it would fail to
provide clear guidance as to the majorities in favor of the differing policies.

Relation to the Literature

The model economy falls into the New Keynesian class of models of nominal rigidities, which
replicate salient features of the business cycle (Smets and Wouters 2007), particularly if labor-
market search and matching frictions are accounted for (Christiano et al. 2015). The New
Keynesian literature has, to date, either focused on representative households or resorted to the
fiction of a fixed set of households that are representative of their type. Galí et al. (2007) study the
transmission of government spending shocks when some households are spenders/land-to-mouth
consumers (do not participate in asset markets). In Iacoviello (2005) and Curdia and Woodford
(2010), differences in preferences split households into borrowers and savers, the shares of each
group being constant over time. Little work exists that links heterogeneity and labor-market
frictions. An exception is Challe et al. (2013), who assume that all employed households join
representative families. Upon entering unemployment, however, households leave the family with
their share of the family’s wealth. We, instead, do not resort to the fiction of a family.

We build on incomplete-market general equilibrium models with infinitely lived agents and
aggregate uncertainty and flexible prices. Krusell et al. (2010) and Nakajima (2012a) have
introduced search and matching frictions into the framework and explored the effects of unem-
ployment insurance. Doepke and Schneider (2006) and Meh et al. (2010), among others, focus on the wealth redistribution associated with surprise inflation under flexible prices, and Akyol (2004) and Erosa and Ventura (2002) on steady-state inflation. Albanesi (2007) studies the political economy of steady-state inflation. Recently, the literature has branched out to New Keynesian models. Werning (2015) provides analytical results that highlight the importance of countercyclical income risk, which in our model is generated by cyclical unemployment. McKay and Reis (2015) study the role of automatic stabilizers. They keep labor-market risk exogenous. Similarly, Guerrieri and Lorenzoni (2011) do not have a frictional labor market. McKay et al. (2015) study how borrowing constraints alter the efficacy of monetary forward guidance. McKay et al. (2016), Bayer et al. (2015), and Luetticke (2015) study the transmission of monetary policy shocks or other business-cycle shocks when nominal and real assets have different liquidity characteristics. Auclert (2015) finds that the redistribution channel is important for the transmission of monetary policy shocks to consumption. Our paper, instead, abstracts from a portfolio composition decision and from modeling nominal portfolios altogether, building on the cashless limit of Woodford (1998). What sets the current paper apart is accounting for unemployment risk and its endogeneity to systematic monetary policy. We believe that this is of central importance for bringing the current generation of models to speak to the Federal Reserve’s dual mandate.

Another aspect sets the current paper apart: the solution approach. We show how to adapt the approximate-aggregation algorithm in Krusell and Smith (1998) and Reiter (2010) to the current setting. Solving the fully non-linear model allows us to make for welfare comparisons. We document non-linearities in both the RA and HA variants of the model. This is in line with the non-linearities documented by Hairault et al. (2010) and Petrosky-Nadeau et al. (2015) in real model economies. Non-linearities are also central in den Haan et al. (2015), who entertain a heterogeneous-agent economy with labor-market search and matching frictions and money. Nominal wage rigidity amplifies recessions if monetary policy fails to raise the money supply in the face of deflationary pressures. In our model, instead, we assume that monetary policy precludes amplification by following a standard Taylor rule. We show that even within those confines, however, monetary policy does have sizable distributional effects. Last, Ravn and Sterk (2012) study the effect of an exogenous increase in job uncertainty in a New Keynesian model with search and matching frictions and household heterogeneity. Next to the difference in focus, they entertain a much simpler environment without aggregate savings.

The rest of the paper is organized as follows. Section 2 introduces the model. Section 3 highlights the calibration and business-cycle implications of household heterogeneity. Section 4 highlights the effects of shocks on the aggregate economy and inequality. Section 5 discusses the
welfare effects of one-time shocks and of a switch to more accommodative monetary policy. A final section concludes.

2 Model

The model economy is characterized by uninsurable income risk as in Krusell and Smith (1998). We follow Nakajima (2012a) and Krusell et al. (2010) and assume that the job-finding rate of unemployed households is linked to the state of the business cycle through Mortensen and Pissarides (1994) search and matching frictions. In equilibrium a recession will be a period of low job-finding rates. Households face market incompleteness and borrowing constraints. Therefore, they will be impacted differently by economic downturns, depending on their wealth and other characteristics. In addition, we assume nominal rigidities in price setting. This means that systematic monetary policy affects the shape of aggregate fluctuations and income risk.

2.1 States

We define the model in a recursive form. Define \( X \) as the vector of aggregate state variables at the time of production, where \( X = (K, N, \zeta, \mu) \). \( K \) is the aggregate capital stock. \( N \) is aggregate employment. \( \zeta = (Z, \zeta_R, \zeta_F) \) is the vector of aggregate shocks: an aggregate productivity shock, \( Z \), a monetary policy (interest-rate) shock, \( \zeta_R \), and a financial “risk premium” shock, \( \zeta_F \). Smets and Wouters (2007) identify these three shocks as prominent sources of business-cycle fluctuations. Households are heterogeneous and characterized by four time-varying, idiosyncratic states \((e, s, \beta, a)\). \( e \in \{0, 1\} \) denotes employment status: \( e = 1 \) means a household is employed, \( e = 0 \) means it is unemployed. While the job-loss probability will be taken as exogenous, following Shimer (2005), the job-finding rate of unemployed workers is determined endogenously.

We seek to investigate the implications for monetary policy of household income and wealth heterogeneity. We, therefore, have to capture key features of the wealth and income distribution in the U.S. economy. In order to match those, we introduce household heterogeneity in time preferences (as in Krusell and Smith 1998) and skills (as in Castañeda et al. 2003), both of which vary stochastically over time. \( \beta \in B = \{\beta_L, \beta_H\} \) denotes the household’s time-discount factor, where \( 0 < \beta_L < \beta_H < 1 \). For each household, \( \beta \) follows a two-state first-order Markov process with transition matrix \( \pi_B \). The probability of a transition from \( \beta \) to \( \beta' \) is given by \( \pi_B(\beta, \beta') \). Household earnings are governed by the employment status and a household’s skill level. Let \( s \in S \) represent the skill level of a household. The skill follows an S-state first-order Markov process that is independent of the aggregate state. Denoting the transition matrix by \( \pi_S \), the probability of a transition from \( s \) to \( s' \) is \( \pi_S(s, s') \). The skill transitions mean that higher-skilled
households seek to self-insure not only against business-cycle risk and the risk of job loss, but also against a fall in earnings ability. Households would, thus, accumulate precautionary savings even if there were no business-cycle risk. Households can save by investing in a representative mutual fund that owns all firms in the economy. \( a \in A \subseteq \mathbb{R} \) denotes the share holdings of a household. \( \mu(e, s, \beta, a) \) is the type distribution of households, defined as an element of a canonical Borel \( \sigma \)-algebra \( \mathcal{M} \) defined over \( \{0, 1\} \times S \times B \times A \).

### 2.2 Timing

The timing is shown in Figure 1. Aggregate shocks and shocks to households’ skill levels and time preferences are drawn at the beginning of a period and become known immediately. Denote by \( \tilde{\mu} \) and \( \tilde{N} \) the type distribution and aggregate employment, respectively, before labor-market transitions have occurred. Let \( \tilde{X} = (K, \tilde{N}, \zeta, \tilde{\mu}) \) denote the corresponding state of the economy. Employed households lose their jobs with exogenous probability \( \lambda \). Then firms post vacancies. All unemployed households search for jobs. Matching takes place and the aggregate state becomes \( X = (K, N, \zeta, \mu) \). Thereafter, the remaining decisions are made and production takes place.

### 2.3 Households

Preferences are time-separable with a period utility function given by \( u(c) = c^{1-\sigma}/(1 - \sigma) \). We first describe the problem of a household that is employed after the employment transitions have taken place \( (e = 1) \) and has skill level \( s \), time preference \( \beta \), and asset holdings \( a \). The household’s
Bellman equation is given by

\[
W(X, 1, s, \beta, a) = \max_{c, a' \geq 0} \left\{ u(c) + \beta \mathbb{E} \left[ \left( 1 - \lambda + \lambda f(\tilde{X}') \right) W(X', 1, s', \beta', a') + \lambda \left( 1 - f(\tilde{X}') \right) W(X', 0, s', \beta', a') \right] \right\}
\]

s.t. \( c + p_a(X)a' = (p_a(X) + d_a(X))a + w(X)s(1 - \tau(X)) \),

The household chooses consumption, \( c \), and the number of shares it wants to carry into the next period, \( a' \geq 0 \), so as to maximize expected lifetime utility subject to its budget constraint. The household takes the job-finding rate, \( f(\tilde{X}') \), the ex-dividend price of shares, \( p_a(X) \), dividends, \( d_a(X) \), the wage, \( w(X) \), and the payroll tax rate, \( \tau(X) \), as given. The expectation operator \( \mathbb{E} \) is conditional on the distribution of aggregate and individual shocks going forward (\( \zeta', s', \beta' \)). The household takes the laws of motion of the aggregate states, \( \tilde{X}' = \tilde{G}(X) \) and \( X' = G(X) \) as given. Next period, the household will keep its current job with exogenous probability \( 1 - \lambda \). If the household loses the job, it searches for employment. Its search intensity is constant. With probability \( f(\tilde{X}') \) the household will find a new job immediately and be employed (at a new firm). Otherwise, the household’s employment status changes to \( e = 0 \) and the household will go through a spell of unemployment. This happens with probability \( \lambda(1 - f(\tilde{X}')) \). Turning to the budget constraint, the household uses the resources it has available for consumption, \( c \), and for purchasing shares that it carries into the next period, \( p_a(X)a' \). The household’s resources (right-hand side of the budget constraint) are composed of the ex-dividend value of the shares that the household owns at the beginning of the period, \( p_a(X)a \), dividends associated with the shares, \( d_a(X)a \), and after-tax labor income, \( w(X)s(1 - \tau(X)) \). Here \( w(X) \) is the real wage per efficiency unit and \( \tau(X) \) is a proportional labor-income tax rate.

If the household is unemployed after the employment transitions have taken place, its Bellman equation is

\[
W(X, 0, s, \beta, a) = \max_{c, a' \geq 0} \left\{ u(c) + \beta \mathbb{E} \left[ f(\tilde{X}') W(X', 1, s', \beta', a') + \left( 1 - f(\tilde{X}') \right) W(X', 0, s', \beta', a') \right] \right\}
\]

s.t. \( c + p_a(X)a' = (p_a(X) + d_a(X))a + b(s) \).

This reflects the fact that next period the unemployed household will move into employment with state-dependent probability \( f(\tilde{X}') \) or will otherwise remain unemployed. Instead of wage income, the unemployed receive unemployment benefits, \( b(s) \). Benefits depend on a household’s
skills in order to introduce in a parsimonious way that the benefits depend on past earnings.

For future reference, denote the resulting optimal decision rules for consumption and share holdings by \( c = g_c(X, e, s, \beta, a) \) and \( a' = g_a(X, e, s, \beta, a) \), respectively.

### 2.4 Mutual Fund

We abstract from a portfolio choice by the individual households. Households delegate financial management to a representative mutual fund, that is, they own all their wealth through equity claims on the fund. There are five types of assets in the economy. Only the mutual fund trades these: equity of producers of final goods, of intermediate goods, of capital services, and of labor services, plus one-period nominal bonds.

Firms are owned by the mutual fund and make decisions so as to maximize their market value. Intermediate goods firms and capital services firms face dynamic decision problems. We follow Favilukis (2013) and assume that the mutual fund prices claims based on the asset-weighted average of its shareholders’ period-to-period valuation. Letting \( u_c(c) \) mark the marginal utility of consumption, in shorthand notation the discount factor the funds apply is

\[
Q(X, X') = \int_{\mathcal{M}} a' \beta \frac{u_c(c')}{u_c(c)} d\mu'.
\]

Appendix A provides the exact formula.\(^2\)

We abstract from modeling public-sector debt or money. Instead, we use the cashless limit assumption (Woodford, 1998) commonly used in New Keynesian models. The central bank controls the rate of return on risk-free nominal private-sector bonds. Since prices are sticky, by setting the nominal interest rate, the central bank influences the expected real rate of return on the nominal bonds and, in effect, the return on all other assets in the economy. Denote by \( \Pi(X) \) the gross rate of inflation. Equilibrium in the bond market requires that all assets be priced according to the mutual fund sector’s discount factor. This, for the bond investment decision, yields a standard Euler equation (for the mutual fund rather than a household)

\[
1 = \mathbb{E} \left[ \frac{Q(X, X') R(X)}{\Pi(X')} \right],
\]

Firms transfer their profits to the mutual fund, which, in turn, distributes profits to house- 

\(^2\) Other papers do not need to specify the discount factor. In Krusell and Smith (1998) households invest directly in capital and firms only have static rental decisions to make. In den Haan et al. (2015) there are only labor firms. They only make one decision (to post a vacancy or not). Neither paper has sticky prices. See Carceles-Poveda and Coen-Pirani (2010) for results regarding investor unanimity with heterogeneous households.
holds in the form of dividends. Since we still need to introduce some notation, we report the
exact expression for dividends per share, \(d_a(X)\), only in equation (9) further below.

### 2.5 Producers of Intermediate Goods

There is a unit mass of firms that produce differentiated intermediate goods. An intermediate
good producer \(j \in [0, 1]\) buys labor and capital services \(l_j\) and \(k_j\) at the competitive rates \(h(X)\)
and \(r(X)\), respectively. The producer sells its output to final goods firms under monopolistic
competition at price \(P_j\). In the following, let \(X_p := (X, \mu_p)\) be state \(X\) augmented by the distri-
bution across firms of last period’s prices, \(P_{j,-1}\). There are nominal rigidities: Price adjustment
is subject to Rotemberg (1982)-type quadratic adjustment costs. The producer’s value is:

\[
J_I(X_p) = \max_{P_j, l_j, k_j} y_j(X, P, P_j) \left( \frac{P_j}{P(X_p)} \right)^{-\epsilon} y(X)
+ \mathbb{E} [\zeta_F Q(X, X') J_I(X', P_j)]
\]

s.t.

\[
y_j(X, P_j, P) = \left( \frac{P_j(X_p)}{P(X_p)} \right)^{-\epsilon} y(X), \tag{1}
\]

\[
y_j(X, P_j, P) = Zk_j^\theta \ell_j^{1-\theta}, \tag{2}
\]

where \(y_j(X, P_j, P)\) is firm \(j\)’s output and \(y(X)\) is production of the final good. Constraint (1)
is the firm’s demand function, \(\epsilon > 1\). \(y(X)\) is the aggregate output of final goods. Equation
(2) is the production function of intermediate good \(j\). \(\Xi \geq 0\) is a fixed cost of production. \(\bar{y}\)
is steady-state output. Parameter \(\psi > 0\) indexes the extent of nominal rigidities. \(\Pi\) takes on a
fixed value.

Letting \(Z\) be steady-state total factor productivity (TFP), TFP itself evolves according to:

\[
\log(Z') = (1 - \rho_Z) \log(Z) + \rho_Z \log(Z) + \epsilon_Z, \text{ where } \epsilon_Z \text{ is i.i.d. } N(0, \sigma_Z^2), \rho_Z \in [0, 1).
\]

The financial shock, \(\zeta_F\), drives a wedge between the firm’s evaluation of future profits and the
mutual fund’s. This shock, which leads to impulse responses similar to the risk-premium shock
of Smets and Wouters (2007), pertains to equity investments only. We assume that

\[
\log(\zeta_F) = \rho_{\zeta_F} \log(\zeta_F) + \epsilon_{\zeta_F}, \text{ where } \epsilon_{\zeta_F} \text{ is i.i.d. } N(0, \sigma_{\zeta_F}^2), \rho_{\zeta_F} \in [0, 1).
\]

If their prices last period are identical, in equilibrium all intermediate good producers will set
the same price this period as well. The optimal behavior of these firms can then be described by
the current aggregate rate of inflation, $\Pi(X)$, and other contemporaneous aggregate variables or
the expectations of each of these. This means past prices, $P_{j,-1}$, are not state variables. Each
producer $j$ faces the same marginal costs and chooses the same amount of labor and capital
inputs, so $k_j = k(X)$ and $l_j = l(X)$. Next, we turn to the production of these inputs.

2.6 Producers of Capital Services

There is a representative capital-producing firm, the value of which is

$$J_K(X, K) = \max_{v, i, K} \left\{ r(X)Kv - i + \mathbb{E} [ \zeta_F Q(X, X') J_K(X', K') ] \right\}$$

s.t. $K' = (1 - \delta(v))K + \zeta \left( \frac{i}{K} \right)K$. \hfill (3)

The capital-producing firm produces homogeneous “capital services.” It sells these to the in-
termediate goods sector at the competitive rate $r(X)$. Capital services are the product of the
capital stock, $K$, and the utilization rate of capital, $v$. The rate of depreciation of capital, $\delta(v)$,
increases with utilization. The functional form follows Greenwood et al. (1988), namely,

$$\delta(v) = \delta_0 v^{\delta_1}, \; \delta_0 > 0, \; \delta_1 > 1.$$  

Capital producers decide how much to invest in next period’s capital stock, $K'$. Due to capital
adjustment costs, capital investment ($i$) does not translate one-to-one into new capital. We
follow Jermann (1998) by specifying the functional form of these adjustment costs as

$$\zeta \left( \frac{i}{K} \right) = \zeta_0 \left( \frac{i}{K} \right)^{1-\zeta_1} + \zeta_2.$$  

The problem of the capital-producing firm characterizes aggregate investment $i(X)$, the aggregate
utilization rate $v(X)$, and the aggregate capital stock next period $K'(X)$.

2.7 Producers of Labor Services

Labor agencies produce homogeneous “labor services.” Labor agencies may be matched with
exactly one household or they are not matched. The value of a matched labor agency is

$$J_L(X, s) = (h(X) - w(X))s + \mathbb{E} [ \zeta_F Q(X, X')(1 - \lambda)J_L(X', s')]$$

The labor agency produces an amount $s$ of labor services, where $s$ is the skill level of the household
it employs. Labor services are sold at competitive rate $h(X)$. Wage payments are indexed by the
skill level. Last, the continuation value reflects the fact that only with probability \((1 - \lambda)\) will the match between a labor agency and its household be producing in the next period. Otherwise, the match will be dissolved. The exposition here anticipates that due to free entry, the value of a labor agency not matched to a household is zero.

Employment is subject to search and matching frictions. As a result, there is a wide range of wages that are bilaterally efficient and can, thus, arise in equilibrium. In line with assumptions found elsewhere in the New Keynesian literature, for example, in Blanchard and Galí (2010), we postulate that the wage evolves according to

\[
\log w(X) - \log \bar{w} = \epsilon_w \cdot \left[ \log \left( \frac{GDP(X)}{N(X)} \right) - \log \left( \frac{GDP}{N} \right) \right].
\]

Above, \(\bar{w}\) is the steady-state wage level. The wage reacts to labor productivity, \(GDP(X)/N(X)\). Here gross domestic product, \(GDP(X)\), is defined as production, \(y(X)\), net of price adjustment costs and fixed costs, both of which we see as intermediate inputs. That is,

\[
GDP(X) := y(X) - \Xi - \frac{\psi}{2}(\Pi(X) - \Pi)^2y(X).
\]

Parameter \(\epsilon_w \in [0, 1]\) in (5) represents the elasticity of the wage with respect to measured labor productivity. Values of \(\epsilon_w < 1\) can be interpreted as reflecting “wage stickiness,” which serves to amplify labor-market fluctuations, as in Shimer (2004) and Hall (2005).\(^3\)

Labor agencies not yet matched to a household can post a vacancy at cost \(\kappa\). Labor firms cannot target their vacancies at households with specific individual characteristics. The following free-entry condition governs the number of vacancies in equilibrium:

\[
\kappa = \frac{M(\tilde{X}, V)}{V} \int_M J_L \left( \hat{G}(\tilde{X}), s \right) d\mu.
\]

Here \(X = \hat{G}(\tilde{X})\) characterizes the law of motion for \(\tilde{X}\). Vacancies will be created up to the point where the cost of creating a vacancy (left-hand side) just balances the expected gain (right-hand side). The latter is determined by the product of the expected value of a match to the firm, and the probability that an individual vacancy will be filled. The job-filling probability is the ratio of the aggregate mass of new matches, \(M\), to the aggregate mass of vacancies, \(V\). Matches form

\(^3\) We have verified that during all simulations of the model and at all grid points used in the model solution neither the household nor the labor agency would prefer to terminate a match.
according to the following matching function:

\[ M(\tilde{X}, V) = \left( \frac{\left( U(\tilde{X}) + \lambda N(\tilde{X}) \right)V}{\left( \left( U(\tilde{X}) + \lambda N(\tilde{X}) \right)^{\alpha} + (V)^{\alpha} \right)^{\frac{1}{\alpha}}} \right)^{\frac{1}{\alpha}}, \quad \alpha > 0. \]  

(6)

Here \( U(\tilde{X}) + \lambda N(\tilde{X}) \) is the measure of households searching for a job (remember the timing assumption discussed in Section 2.2). Matching function (6), taken from den Haan et al. (2000), ensures that job finding and job filling rates are always well-defined probabilities. All households without a job have the same job finding rate, namely,

\[ f(\tilde{X}) = \frac{M(\tilde{X}, V(\tilde{X}))}{U(\tilde{X}) + \lambda N(\tilde{X})}. \]

### 2.8 Producers of the Final Good

Final goods can be used for consuming, investing, facilitating price adjustment, and creating vacancies. There is a representative competitive final good firm. It transforms the differentiated intermediate goods \( y_j, j \in [0, 1] \), into a homogeneous output good, taking the input prices \( P_j(X_p) \) as given. The problem of the representative final good producer is

\[
\max_{y, (y_j)_{j \in [0, 1]}} P(X_p)y - \int_0^1 P_j(X_p)y_jdj \quad \text{s.t.} \quad y = \left( \int_0^1 \frac{y_j}{\epsilon - 1} dj \right)^{\frac{1}{\epsilon - 1}}.
\]

The optimal decision translates into the demand function anticipated in (1).

### 2.9 Central Bank and Fiscal Authority

The central bank sets the gross nominal interest rate, \( R \), according to the following Taylor rule

\[
\log \left( \frac{R(X)}{R_{CB}} \right) = \phi_\Pi \log \left( \frac{\Pi(X)}{\Pi_{CB}} \right) - \phi_u \left( U(X) - \bar{U}^{CB} \right) + \zeta_R,
\]

(7)

where \( U(X) = 1 - N(X) \) is the unemployment rate at the end of the period. All else equal, the central bank thus raises the nominal rate above \( R_{CB} \) whenever inflation exceeds the inflation target of \( \bar{\Pi}^{CB} \) (parameter \( \phi_\Pi > 1 \)) and when the unemployment rate is lower than its target value \( \bar{U}^{CB} \) (parameter \( \phi_u \geq 0 \)). The superscripts “CB” here signal that these are target values given to the central bank. There are persistent monetary policy shocks, \( \zeta_R \), that capture deviations
from typical behavior, with

\[ \log(\zeta_R') = \rho \zeta_R \log(\zeta_R) + \epsilon \zeta_R , \quad \text{where } \epsilon \zeta_R \text{ is i.i.d. } N(0, \sigma^2_{\epsilon z_R}) , \quad \rho \zeta_R \in [0, 1). \]

The fiscal authority runs a balanced-budget policy so that

\[ \int_{\mathcal{M}} 1_{\epsilon=0} b(s) \, d\mu = \tau(X) \int_{\mathcal{M}} 1_{\epsilon=1} w(X)s \, d\mu. \] (8)

The government pays unemployment benefits \( b(s) \) for an unemployed household of skill \( s \) (\( 1 \) marks the indicator function). This is financed by a proportional tax \( \tau(X) \) on the labor income of employed households.\(^4\)

### 2.10 Aggregate Laws of Motion

Next, we discuss how to construct the aggregate law of motion. For expositional purposes, we use two sets of aggregate state vectors, \( \tilde{X} \) and \( X \), that differ in time of measurement (at the beginning of the period and at the end of the period, respectively); compare Section 2.2. We therefore have three types of laws of motion, \( X' = G(X) \), \( \tilde{X}' = \tilde{G}(X) \), and \( X = \tilde{G}(\tilde{X}) \). Let us focus on one element of \( X \) at a time.

First, installed capital \( K \) does not change during a period. It therefore does not differ between \( \tilde{X} \) and \( X \), so we only need one law of motion for \( K \); compare equation (4):

\[ K'(X) = \left[ 1 - \delta(v(X)) \right] K + \zeta \left( \frac{i(X)}{K} \right) K \]

where \( v(X) \) and \( i(X) \) are obtained from the optimization problem of the capital-producing sector.

Next, the law of motion for employment during the production stage of this period is given by

\[ N(X) = (1 - \lambda)N(\tilde{X}) + M(\tilde{X}, V(\tilde{X})). \]

Since there are no labor-market transitions at the end of the period, employment at the beginning of the next period coincides with employment at the end of the current period: \( N'(\tilde{X}') = N(X) \).

Last, we need to keep track of the type distribution of households. Remember that, at the beginning of a period, the type distribution is \( \tilde{\mu}(\tilde{e}, s, \beta, a) \), where \( \tilde{e} \) is the employment status before the separations and hiring occur. The type distribution during the period (after the

\(^4\) The proportional tax reduces the labor income of employed households, yet does not distort their labor supply decisions (search effort is assumed to be exogenous) nor firms’ hiring decisions (because of the wage rule).
transitions in employment status) is \( \mu(e, s, \beta, a) \). \( \tilde{\mu} \) and \( \mu \) are linked as follows:

\[
\mu(1, s, \beta, a) = f(\tilde{X})\tilde{\mu}(0, s, \beta, a) + [1 - \lambda + \lambda f(\tilde{X})] \tilde{\mu}(1, s, \beta, a),
\]

\[
\mu(0, s, \beta, a) = [1 - f(\tilde{X})]\tilde{\mu}(0, s, \beta, a) + \lambda[1 - f(\tilde{X})] \tilde{\mu}(1, s, \beta, a).
\]

Notice that, from \( \tilde{\mu} \) and \( \mu \), only the employment status changes. The transition between the type distribution at the end of the period, \( \mu \), and the type distribution at the beginning of the next, \( \tilde{\mu}' \), is characterized by

\[
\tilde{\mu}'(e, s, \beta, A) = \sum_{s \in S} \pi_S(s, \bar{s}) \sum_{\beta \in B} \pi_B(\beta, \bar{\beta}) \int_{\mathcal{M}} \mathbb{1}_{e=e} \mathbb{1}_{s} \mathbb{1}_{\beta} g_a(X, e, s, \beta, a) \in A d \mu(e, s, \beta, a),
\]

with \( A \in A \) being a subset of the space of the share holdings and \( \pi, \bar{s}, \bar{\beta} \) being individual states in \( \{0, 1\} \), \( S \), and \( B \) respectively. \( \pi_S(s, \bar{s}) \) marks the probability to transit from skill state \( s \) to state \( \bar{s} \) at the end of the period. \( \pi_B(\beta, \bar{\beta}) \) is the corresponding transition probability for a household’s time-preference parameter.

### 2.11 Market Clearing and Equilibrium

Six types of markets open in the model: markets for final goods, intermediate goods, labor services, capital services, shares of the mutual fund, and financial markets on which the mutual fund trades. The final goods market clears if

\[
y(X) = \int_{\mathcal{M}} g_e(X, e, s, \beta, a) \, d\mu + i(X) + \kappa V(\tilde{X}) + \int_0^1 \left[ \Xi + \frac{\psi}{2} \left( \frac{P_j}{P_{j-1}} - \bar{P} \right)^2 \bar{y} \right] d_j,
\]

where the terms on the right-hand side are aggregate consumption, investment, the vacancy posting costs and price adjustment costs, respectively. The markets for all intermediate goods clear whenever \( y(X) = Zk^j l_j^{1-\theta}, \forall j \in [0, 1] \),

The market for labor services clears if \( \int_{\mathcal{M}} s \mathbb{1}_{e=1} d\mu = \int_0^1 \ell_j \, dj \); The market for capital services clears if \( v(X)K = \int_0^1 k_j \, dj \); the market for shares of the mutual funds if \( \int_{\mathcal{M}} g_a(X, e, s, \beta, a) \, d\mu = 1 \).

Dividends are given by

\[
d_a(X) = \int_0^1 \left[ y_j(X) \frac{P_j(X)}{P(X)} - r(X)k_j(X) - h(X)\ell_j(X) - \Xi - \frac{\psi}{2} \left( \frac{P_j}{P_{j-1}} - \bar{P} \right)^2 \bar{y} \right] d_j + r(X)K(X)v(X) - i(X) + \int_{\mathcal{M}} (h(X) - w(X)) s \, d\mu - \kappa V(\tilde{X}).
\]

The first line is the profits in the intermediate goods sector, the second line profits in the capital
services and labor services sectors net of investment in capital and vacancies, respectively.

Last, the financial markets on which the mutual fund trades clear when all firms are held by
the mutual fund and inside bonds are in zero net supply. For a formal definition of recursive
equilibrium, see Appendix B.

3 Calibration

The model is solved numerically using a solution method that adapts Krusell and Smith (1998)
and Reiter (2010). Our solution algorithm, described in detail in Appendix C, takes non-
linearities and uncertainty in both aggregate and idiosyncratic dynamics into account.

We calibrate the model to the U.S. economy, one period being a quarter. The calibration
sample is 1984Q1 to 2008Q3. Several parameters are set such that the steady state in the model
without aggregate shocks (“steady state” henceforth) matches long-run averages in the data.
Other parameters are set with a view to targeting second moments in the data. The latter set of
parameters is chosen to match the second moments in the representative-agent (“RA” henceforth)
counterpart of our heterogeneous-agent model (“HA” henceforth). Given the numerical burden
of the solution algorithm used to solve the HA model, we do not use the simulated method of
moments to directly match the heterogeneous-agent model to long-run moments. In this context,
we choose the systematic response of monetary policy to unemployment ($\phi_u$), the investment
adjustment cost parameter ($\zeta_1$), and the volatilities of the financial and TFP shocks to jointly
match the volatility of HP-filtered log inflation, unemployment, output, and consumption. We
focus on consumption volatility, a target directly relevant for welfare. Tables 1 and 2 summarize
the values we choose for the calibrated parameters. We will discuss these choices next.

3.1 Households

We set the coefficient of relative risk aversion to $\sigma = 1.5$, as, for example, in Castañeda et al.
(2003). Heterogeneity in skills and time preferences allows us to match key moments related
to earnings risk and the wealth distribution. We assume that the time-preference parameter
$\beta$ follows a two-state ($\beta_L$ and $\beta_H$) first-order Markov process with values $\beta_L$ and $\beta_H$. Skills $s$
follow a four-state first-order Markov process. Time preferences and skills are independent of the
business cycle. We parameterize the time-discount factor and the skill process by ensuring that
in the steady state the model meets the following targets: (i) the Gini index of wealth is 0.81;
(ii) the wealth-poorest 30 percent of households have a total net worth of 0; (iii) the standard
deviceation of residual earnings of continuously employed workers is 0.19; (iv) the autocorrelation

---

5 The RA model has $\beta = 0.99$ so as to match the same steady-state real rate as in the HA model.
Table 1: Calibrated Parameters

| Parameters | Households | Capital services | \( \Xi \) | \( \overline{R}^{CB} \) | \( \sigma \) | \( \zeta_0 \) | \( \gamma_p \) | \( \phi_u \) | \( \phi_{\Pi} \) | \( \beta_L \) | \( \zeta_2 \) | \( \lambda \) | \( \phi \) | \( \beta_H \) | \( \delta_0 \) | \( \alpha \) | \( \rho_R \) | \( \pi_Ss, s' \) | \( \delta_1 \) | \( \overline{w} \) | \( \sigma_R \times 100 \) | \( \sigma_F \times 100 \) | \( \theta \) | \( \epsilon \) | \( \kappa \) | \( \rho_Z \) | \( \psi \) | \( \Pi \) | \( \Pi^{CB} \) | \( \sigma_Z \times 100 \) | \( \zeta \) | \( \Psi \) | \( \sigma^2 \) | \( \sigma^2_F \) | \( \sigma^2_Z \) | \( \sigma_{\Psi} \) | \( \Pi_{\Psi} \) | \( \sigma_{\Psi}^2 \) | \( \Pi_{\Psi}^{CB} \) | \( \sigma_{\Psi}^{CB} \) | \( \Pi_{\Psi}^{CB} \) | \( \sigma_{\Psi}^{CB} \) | \( \Pi_{\Psi}^{CB} \) | \( \sigma_{\Psi}^{CB} \) | \( \Pi_{\Psi}^{CB} \) | \( \sigma_{\Psi}^{CB} \) |
|-----------|------------|-----------------|---|------------------|-----------|-----------|-------------|--------|--------|-------------|-----------|-----------|-------------|--------|--------|-------------|--------|--------|-------------|--------|--------|-------------|--------|--------|-------------|--------|--------|-------------|--------|--------|-------------|
|          | 0.303      | 1.015           | 3.021 | 0.0017           | 0.721     | 0.104     | 0.670      | 0.849 | 0.993 | 0.015       | 0.150     | 0.721     | 0.015       | 0.97    | 0.0017 | 0.670      | 0.849 | 0.993 | 0.015       | 0.150 | 0.721 | 0.015       | 0.849 | 0.993 | 0.015       | 0.150 | 0.721 | 0.015       | 0.849 | 0.993 | 0.015       |
| \( \pi_B \beta, \beta' \) | Table 2 | \( \beta_L \) | \( \beta_H \) | \( \pi_Ss, s' \) | \( s_1 \) | \( s_2 \) | \( s_3 \) | \( s_4 \) | \( \Pi \) | \( \Pi^{CB} \) | \( \Pi_{\Psi}^{CB} \) | \( \Pi_{\Psi}^{CB} \) | \( \Pi_{\Psi}^{CB} \) | \( \Pi_{\Psi}^{CB} \) | \( \Pi_{\Psi}^{CB} \) | \( \Pi_{\Psi}^{CB} \) | \( \Pi_{\Psi}^{CB} \) | \( \Pi_{\Psi}^{CB} \) | \( \Pi_{\Psi}^{CB} \) | \( \Pi_{\Psi}^{CB} \) | \( \Pi_{\Psi}^{CB} \) | \( \Pi_{\Psi}^{CB} \) | \( \Pi_{\Psi}^{CB} \) | \( \Pi_{\Psi}^{CB} \) | \( \Pi_{\Psi}^{CB} \) | \( \Pi_{\Psi}^{CB} \) | \( \Pi_{\Psi}^{CB} \) | \( \Pi_{\Psi}^{CB} \) | \( \Pi_{\Psi}^{CB} \) | \( \Pi_{\Psi}^{CB} \) | \( \Pi_{\Psi}^{CB} \) | \( \Pi_{\Psi}^{CB} \) | \( \Pi_{\Psi}^{CB} \) | \( \Pi_{\Psi}^{CB} \) | \( \Pi_{\Psi}^{CB} \) | \( \Pi_{\Psi}^{CB} \) | \( \Pi_{\Psi}^{CB} \) | \( \Pi_{\Psi}^{CB} \) | \( \Pi_{\Psi}^{CB} \) |

Notes: The table shows the calibrated parameters. The main text explains the calibration targets.

of residual earnings of continuously-employed workers is 0.95; (v) 1 percent of the households are much more skilled than the rest (“super-skilled,” see below); and (vi) the probability of remaining a super-skilled worker is 0.97. Targets (i) and (ii) are derived using the 2007 Survey of Consumer Finances. The values of targets (iii) and (iv) are taken from Krueger et al. (2015) and capture persistent idiosyncratic shocks to earnings conditional on staying employed. Targets (v) and (vi) are based on the discussion in Nakajima (2012b) on how to calibrate an income process with super-skilled households.

For the time-discount factor, we impose two additional targets: a real rate of return of 4 percent in the steady state and we calibrate the transition matrix such that each household redraws its discount factor on average every 40 years and has an equal chance of drawing each of the two in this event. The idea follows Krusell and Smith (1998) and aims to capture intergenerational changes in the saving behavior of dynasties. This results in values \( \beta_L = 0.849 \) and \( \beta_H = 0.993 \) and the transition matrix shown in the left matrix of Table 2. For the income process we use four discrete skill levels. \( s_1 \) is the lowest skill level, \( s_2 \) a medium skill level, and \( s_3 \) a high skill level. The fourth skill level, \( s_4 \), is used to capture vastly more productive households, the “super-skilled.” We parameterize the skill transitions as follows. Skill transitions are independent of the business cycle (and of employment). The process of transitions between the lower three skill levels is assumed to be governed by a discretized AR(1) process for the log of individual productivity with mean zero, persistence \( \rho_s \) and variance of the innovation \( \sigma^2_s \). We discretize using the algorithm described in Rouwenhorst (1995). For the transitions to or from the super-skilled households, we use an AR(1) process for residual earnings after removing age, education, and time effects.

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6 The value is taken from Krueger et al. (2015), who estimate an AR(1) process for residual labor earnings after removing age, education, and time effects.
Table 2: Transition Probabilities

<table>
<thead>
<tr>
<th>Time-discount factor, $\pi_B(\beta, \beta')$</th>
<th>Skills, $\pi_S(s, s')$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>tomorrow</td>
</tr>
<tr>
<td>today</td>
<td>$\beta_L$</td>
</tr>
<tr>
<td>today</td>
<td>$\beta_H$</td>
</tr>
</tbody>
</table>

| |                 | $s_1$ | $s_2$ | $s_3$ | $s_4$ |
| today | 0.9733 | 0.0260 | 0.0000 | 0.0007 |
| today | 0.0260 | 0.9474 | 0.0260 | 0.0007 |
| today | 0.0000 | 0.0260 | 0.9733 | 0.0007 |
| today | 0.0223 | 0.0223 | 0.0223 | 0.9331 |

Notes: Transition probabilities per quarter. Left: $\pi_B(\beta, \beta')$. Right matrix: $\pi_S(s, s')$. $s_1$: lowest skill group, $s_4$: highest skill group. Rounding for the table means rows may not sum to 1.

state, we assume that the probability of becoming super-skilled is the same for each normal skill level. Similarly, a household that loses its super skills is equally likely to transition into either of the three “normal” skill levels. With these assumptions, there are three sets of parameters associated with the super-skilled state: the probability of staying super-skilled, $\pi_{s_4, s_4}$, the probability that a “normal” household becomes super-skilled, $\pi_{s_1, s_4} = \pi_{s_2, s_4} = \pi_{s_3, s_4}$, and the productivity of the super-skilled, $s_4$. Table 2 reports the resulting transition probabilities per quarter.

Figure 12 in Appendix D shows that the calibrated model closely matches the wealth distribution in the U.S. economy. In the same appendix, Table 6 shows that the model matches the main feature originating from this observation, namely, that “Wall Street” (the 5 percent wealthiest households) derive a large share of income from financial wealth, whereas “Main Street” (the remaining 95 percent) does not.

3.2 Producers of Capital Services

We set the curvature parameter of capital adjustment costs to $\zeta_1 = 0.104$. We require the steady state with adjustment costs to be the same as the one without such costs. This determines the remaining parameters for capital adjustment. The resulting values are shown under “Capital services” in Table 1. Furthermore, we require that the utilization rate of capital be unity in the steady state and that the steady-state depreciation rate be 6 percent per year. This pins down parameters $\delta_0$ and $\delta_1$.

3.3 Producers of Intermediate Goods

We set the fixed costs $\Xi$ so as to generate a steady-state profit share in GDP of intermediate good producers of 3 percent. $\theta$, the exponent of capital in the production function, is calibrated to deliver a quarterly capital to GDP ratio of 8. The parameter $\psi$ governs the price adjustment cost. We set it to 38.08. If we were to linearize the Phillips curve (we do not), the slope of the Phillips...
curve thus implied would be equal to that of a Calvo-Yun-type New Keynesian Phillips curve (without strategic complementarities) when prices lasted for 5 quarters on average given our choice of the elasticity of substitution. This matches the slope of the Phillips curve as commonly estimated, for example, in Galí and Gertler (1999). We set the reference level of inflation to $\Pi = 1.005$, meaning firms index their prices to an inflation rate of 2 percent per year. We set the elasticity of substitution across intermediate goods to $\epsilon = 3$, implying a steady-state markup of 50 percent. A low value for the elasticity allows us to generate persistent deviations of inflation from target while calibrating a relatively small $\psi$. This choice makes the model numerically more stable in a wider range of parameters of the monetary policy rule as a smaller share of output is used to cover the adjustment costs in states with large price changes.

3.4 Labor Market

The value for the separation rate $\lambda = 0.10$ is consistent with the JOLTS data. We set the elasticity of the matching function with respect to the number of searchers so as to have a steady-state unemployment rate of 5.7 percent. This results in a value of $\alpha = 1.716$. Following den Haan et al. (2000), in the model’s steady state we assume a quarterly vacancy-filling rate of 0.71. Using the steady-state free-entry condition, this yields a vacancy posting cost of $\kappa = 0.249$. Any wage that leaves both the matched labor-services firm and the household with a positive surplus from continuing the match will be an equilibrium wage. We parameterize wage equation (5) as follows. Following Hagedorn and Manovskii (2008), we target an elasticity of wages with respect to productivity of $\epsilon_w = 0.45$. We set the steady-state wage per efficiency unit of labor to $\bar{w} = 0.670$. This generates a labor share of 63 percent. The wage stays in the bargaining set in all the simulations reported in the paper.

3.5 Central Bank

The inflation target ($\Pi^{CB}$) is set such that the model implies a steady-state inflation rate of 2 percent annualized, in line with the Federal Reserve System’s inflation objective. The rate $R^{CB}$ used in the Taylor rule (7) is chosen to deliver a target for the steady-state real rate of return of 4 percent. The response of the policy rate to inflation in the Taylor rule is set at $\phi_\Pi = 1.5$ as in Taylor (1993). The response parameter to unemployment is set to $\phi_u = 0.107$. The target level for the unemployment rate, $U^{CB}$, is set to the steady-state level of unemployment (0.057).

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7 The range of values for this parameter used in the literature is fairly wide. See, for example, Kuester (2010) and Midrigan (2011) for references.
3.6 Fiscal Authority

The unemployment benefit system mimics the system in place in the U.S. in that the replacement rate is assumed to be 40 percent of the steady-state wage (as in Shimer 2005) for the lower skill groups. Benefits are capped at 40 percent of the mean earnings in the economy. That is, $b(s) = \min(b \cdot s \cdot \bar{w}, b \cdot \text{economy-wide average earnings})$. The payroll tax rate is set so as to balance the budget on a period-by-period basis. The choices above imply a steady-state payroll tax rate of 2.42 percent.

3.7 Shocks

The steady-state level of the TFP shock, $\bar{Z}$, is chosen so as to normalize steady-state GDP to unity. The persistence of the TFP shock is $\rho_Z = 0.95$. The standard deviation of the TFP shock ($\sigma_Z = 0.0058$) was calibrated such that GDP in the RA version of the model (once HP-filtered) has the same standard deviation as HP-filtered GDP in the data. The persistence of the monetary shock, $\rho_D = 0.8$, was chosen so as to have persistence of the real rate after a monetary shock. Its standard deviation, $\sigma_D$, is calibrated to 0.000625, so that the annualized size of a typical monetary shock is 6.25 basis points (25 basis points annualized), the typical size of monetary shocks in VAR studies over our calibration sample. See, for example, Altig et al. (2011). Finally, we calibrate the financial shock to have the same persistence as the monetary shock, and a standard deviation such that the model meets the volatility targets specified at the beginning of this section.

3.8 Business-Cycle Statistics

In this section we show that the calibrated model matches the business-cycle facts well. In addition, we show that household heterogeneity changes the business cycle. In the HA model consumption is almost 15 percent more volatile than in the RA counterpart. At the same time investment is 10 percent less volatile. GDP on net is 4 percent more volatile in the HA model than in the RA counterpart.

Table 3 compares second moments implied by the model and the data (based on HP-filtered series with smoothing parameter 1,600). The data are described in Appendix D. The first three columns report second moments in the baseline model with heterogeneous agents (the HA model). The second block of three columns reports the second moments of the representative-agent version of the model (the RA model). These moments are included here for two reasons. First, because we use the RA model to calibrate the model to second moments in the data. Second, it shows how much the heterogeneity influences aggregate fluctuations. The final set of columns reports
### Table 3: Model vs. Data – Second Moments

<table>
<thead>
<tr>
<th>Output and components</th>
<th>Model</th>
<th>Data 1984Q1-2008Q3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>HA: heterog. hh.</td>
<td>RA: represent. hh.</td>
</tr>
<tr>
<td></td>
<td>Std Corr AR(1)</td>
<td>Std Corr AR(1)</td>
</tr>
<tr>
<td>GDP (GDP)</td>
<td>1.69 1.00 0.63</td>
<td>1.62 1.00 0.64</td>
</tr>
<tr>
<td>Consumption (c)</td>
<td>1.02 0.99 0.69</td>
<td>0.89 0.87 0.71</td>
</tr>
<tr>
<td>Investment (i)</td>
<td>5.28 0.98 0.73</td>
<td>5.09 0.96 0.89</td>
</tr>
<tr>
<td>Capacity utilization (v)</td>
<td>0.96 0.78 0.24</td>
<td>2.21 0.84 0.94</td>
</tr>
<tr>
<td>Labor market</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Employment N(X)</td>
<td>0.65 0.90 0.64</td>
<td>0.65 0.86 0.96</td>
</tr>
<tr>
<td>Unemployment U(X)</td>
<td>10.9 -0.90 0.65</td>
<td>10.2 -0.86 0.95</td>
</tr>
<tr>
<td>Vacancies (V)</td>
<td>8.94 0.75 0.07</td>
<td>11.1 0.91 0.93</td>
</tr>
<tr>
<td>Job finding rate (f)</td>
<td>5.37 0.88 0.38</td>
<td>5.13 0.80 0.83</td>
</tr>
<tr>
<td>Productivity and Prices</td>
<td>GDP(N(X))</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1.14 0.97 0.62</td>
<td>1.07 0.87 0.88</td>
</tr>
<tr>
<td>Wage W(X)</td>
<td>0.51 0.97 0.62</td>
<td>0.95 0.41 0.84</td>
</tr>
<tr>
<td>Inflation Π[1]</td>
<td>0.67 -0.32 0.62</td>
<td>0.67 0.27 0.27</td>
</tr>
<tr>
<td>Nominal rate R[1]</td>
<td>0.97 -0.14 0.58</td>
<td>1.24 0.61 0.92</td>
</tr>
</tbody>
</table>

Notes: The table compares moments of the data and two variants of the model (heterogeneous households, representative households). The model moments are based on 1,000 repeated simulations of the model. Each simulation is initialized with 500 periods of simulations that are dropped for the computation of the moments. The next 139 periods are kept. In each case, we take the natural log of the data and compute the cyclical component of the data multiplied by 100 so as to have percentage deviations from trend. The trend is an H-P-trend with weight 1,600. We then drop the first 20 and last 20 observations and compute moments of interest. Finally, we average across the simulations. The left block shows the model's moments, the block on the right the moments in the data. The model fits the data well.

A result of economic substance is that allowing for heterogeneity implies notable changes to the business cycle. The model with heterogeneous households generates much more volatility in aggregate consumption than we observe under representative households. It is important to bear in mind, however, that households in the model can save and that the supply of aggregate savings (capital and employment relationships) is endogenous to their desire to save. Therefore, the higher volatility of consumption does not translate one-to-one into more volatile aggregate demand. Rather, the volatility of output rises only by 4 percent. The reason is that the volatility of investment falls by 11 percent relative to the RA environment. Table 7 in the appendix shows that simple saver-spender models (introduced in more detail in Section 5.2) capture the change
in business-cycle dynamics only if calibrated to a large share of spenders.

We split the presentation of the results that follow into two blocks. In the next section, we discuss the transmission of shocks in the HA model and contrast this with the RA variant. This will be conditional on the baseline calibration of the Taylor rule $\phi_\pi = 1.5, \phi_u = 0.107$. Thereafter, in Section 5, we will discuss the implications for the design of the monetary rule.

4 Transmission of TFP and Monetary Policy Shocks

This section discusses in detail how heterogeneity affects the transmission of shocks. This section has a central result: Allowing for household heterogeneity changes the transmission of shocks, in particular monetary policy shocks. For these, the consumption response on impact is about twice as large in the HA model as in the RA model. The impact response of GDP is larger as well, but less so than for consumption.

4.1 Transmission of a Technology Shock

We first discuss the response of the economy to a TFP shock, which is the main driver of the business cycle in the model. The starting point is the “stochastic steady state.” Figure 2 shows the effect of a one-standard-deviation positive TFP shock for the aggregate economy. The solid line shows the response in the baseline model (the model with heterogeneous households). The dashed line shows the response in the model with a representative household. Allowing for household heterogeneity leads to a slightly stronger response of output on impact. GDP rises by about 1.2 percent. The composition of the increase in output differs considerably from the representative-household case, though. In particular, consumption rises by about 15 percent more in the model economy with heterogeneous households. In response to the TFP shock, demand for labor services increases and so does the price of labor services. Labor services firms, therefore, post more vacancies. The job-finding rate rises markedly, by about 2 percentage points on impact. The unemployment rate falls, namely, by about 0.3 percentage point. The wage rises.

Figure 3 shows the consumption response on impact depending on a household’s wealth, its time-preferences, skills, and employment status. The figure highlights the reasons for the stronger consumption response. On the one hand, the shock shifts workers from unemployment

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8 The stochastic steady state is defined as follows. We initialize the economy at the non-stochastic steady state. Then we simulate 500 periods of the stochastic economy, assuming that in each period unexpectedly the innovations to the three shocks are zero. The resulting state in period 500 is the “stochastic steady state.”

9 The response of (un)employment is consistent with the responses to permanent TFP shocks identified by Ravn and Simonelli (2008) and Altig et al. (2011) by means of long-run restrictions, and the responses identified by sign restrictions to persistent but possibly transitory TFP shocks in Dedola and Neri (2007).
Notes: Impulse response to a one-standard-deviation TFP shock, $Z$. Solid line: the model economy with heterogeneous households. Dashed line: same economy but with a representative household. For most of the variables, the y axis shows percent deviations from the no-shock path (y-label “percent”). “pp.” refers to the deviation from the no-shock path in percentage points. The x-axis shows time since the shock in quarters.

to employment. If they were close to the borrowing constraint, moving to employment would increase their consumption notably. On the other hand, even the unemployed’s consumption will be affected by the TFP shock. The unemployed do not directly participate in the higher TFP level through their income since benefits are constant over the cycle. Therefore, if they do not hold wealth, their consumption does not react to the shock at all. The unemployed do benefit indirectly, however, if they have wealth. First, their future unemployment risk falls (since the job-finding rate rises persistently). Second, if they have savings, the value of these savings increases. Third, future earnings increase. All of this leads all but the wealth-poorest unemployed households to dissave faster and consume more. Consumption rises most for unemployed households with saving around the median household savings. Apart from very low wealth levels, for impatient households consumption of the unemployed rises by more than consumption of the employed. Figure 14 in the appendix shows the steady-state savings policies for each of the types. From this it is apparent that unemployed households generally dissave. The same figure shows that matters are more complicated for employed households. Employed households of the patient type ($\beta_H$) save whenever they are of a skill level of at least $s_2$. Low-skilled households with $\beta = \beta_H$ will only accumulate savings when their wealth is below about one-tenth of annual income. Among the impatient households, the lower two skill groups will generally dissave, even
Figure 3: Consumption Response to a TFP shock

Notes: Consumption response (on impact) to a one-standard-deviation positive TFP shock by household type ($\beta$, $e$, $a$, $s$). For each decile of the wealth distribution, skills are ordered from lowest to highest (“$s_1$” to “$s_4$”). No bar means that, for that type, consumption on impact does not respond to the TFP shock. y-axis: percentage increase in consumption relative to no-shock baseline.

if employed. The two highest skill groups ($s_3$ and $s_4$) will accumulate savings, instead, until their wealth level hits, respectively, about a tenth and a half of annual income.

The mirror image of a stronger response of consumption in the HA model than in the RA economy is that investment does not rise by as much. The consumption response alone would suggest that GDP in the HA model reacts very differently from the RA counterpart. The compensating weaker response of investment shows the importance of modeling an economy in which aggregate saving can respond to shocks. The effect of reduced aggregate investment manifests itself over time. Ten years after the shock, the capital stock has risen by 0.1 percentage point less than in the representative-agent economy; see the right panel of Figure 4. The mutual fund’s investment policy translates into differences in asset price and dividend dynamics. The fund decides whether to invest cash flow or to pay dividends. The mirror image of persistently lower investment in the HA model is that dividend payments are persistently higher in the early
quarters. Eventually they fall below the RA economy’s level, however (beyond quarter 20 after the shock). Turning to the share price, the generally lower response of the share price in later quarters is in line with the lower investment. What is interesting to note, however, is that the initial response of the share price in the two economies differs by almost 10 percent. Reduced demand for precautionary savings in an expansion and lower investment means that the share price does not rise as steeply as in the RA model.

We showed impulse responses from the stochastic steady state. Appendix E documents that there is considerable state dependence. For example, the same-size innovation to the TFP shock reduces both inflation and the unemployment rate by about twice as much if the shock happens in a deep recession rather than in a strong boom. The state dependence is present already in the RA model. Allowing for household heterogeneity amplifies this state dependence only slightly.

4.2 Transmission of a Monetary Policy Shock

Next, we analyze the effect of a contractionary one-standard-deviation monetary shock. For the purpose of the current paper, the main difference to the TFP shock will be the distributional implications.\footnote{The effects of a risk shock are very similar to the monetary policy shock, so we omit them here.}

4.2.1 Response of the Aggregate Economy to a Monetary Policy Shock

Figure 5 shows the response of the economy to a 25-basis-point (annualized) monetary tightening. That is, a one-standard-deviation monetary shock. By design, the monetary policy shock is persistent. It therefore raises the expected long-term real rate of interest. Higher expected returns lead households to save more and cut back their spending for consumption by 0.13 percent (second panel in Figure 5). Since nominal prices are rigid, the ensuing fall in aggregate
demand is met by an increase in intermediate goods firms’ markups, validating the fall in activity. Firms invest less in light of the rising opportunity cost and falling demand. At the same time, capacity utilization falls. GDP overall falls by 0.3 percent. A monetary policy tightening strongly affects the labor market (second row): On the one hand, the tightening reduces the demand for labor services and their price. On the other hand, such a monetary policy raises the real rate of interest and therefore makes firms discount the future by more. This further exacerbates the fall in hiring. Along with vacancy posting, the job-finding rate falls markedly (by 1.2 percentage points). As employment falls, the unemployment rate rises by 0.18 percentage point (from a steady-state value of 6 percent to 6.18 percent). The reduced demand for production factors causes a reduction in capacity utilization and output per employed household. Consequently, the wage falls. In the model with borrowing-constrained households, this increase in unemployment and idiosyncratic risk tends to further exacerbate the fall in consumption (second panel, first row of Figure 5). However, it also ensures that investment and the share price do not fall as much as in the representative agent counterpart (third panel, first row of Figure 5 and first panel Figure 6). As the discount rate rises and investment becomes less profitable, the mutual fund pays out free cash flow through dividends.\textsuperscript{11} Only in the medium term do dividends fall below their level

\footnotesize{\textsuperscript{11} Firms cannot retain earnings. This affects income accounting, but should not affect our results regarding consumption inequality and welfare. The timing of dividends matters primarily for households that are at the borrowing constraint. These households, however, do not hold shares in the first place. All other households...}
absent the monetary shock, reflecting that below-average investment in both labor and capital drains the productive resources available to firms. Note that while dividends rise in response to a monetary shock, this does not imply that the same is due for profits.

Importantly, the model has two different sources of \textit{ex-post} profits. The left panel of Figure 7 shows that profits in the intermediate goods (sticky-price) sector rise with tighter monetary policy, because a monetary contraction reduces marginal costs. Due to sticky prices, then, markups in the intermediate goods sector increase. This alone would suggest that wealthier households stand to benefit from contractionary monetary policy. However, such an argument neglects the fact that capital and labor services firms can make profits and/or losses \textit{ex post} as well. And since both employment and capital are investment goods in our model, the losses that these types of firms incur after a monetary contraction can be steep. Both the rental rate of capital $r(X)$ and the rental rate for labor services $h(X)$ fall (not shown). On balance, the profits of all three types of firms (labor, capital, intermediate goods) combined \textit{fall} after a contractionary monetary policy shock, and a little more so than GDP; compare the right panel of Figure 7. In other words, for assessing the distributional effects of monetary it is central to allow for investment opportunities other than sticky-price firms.

Lower rental rates for both labor and capital services mean lower marginal costs. Inflation therefore falls by 0.2 percentage point (annualized); see left panel in Figure 8. By the logic of the Taylor rule, equation (7), a positive monetary shock leads to a persistent increase in the \textit{ex ante} long-term real rate of interest. In the simulations shown here, the increase in the long-term real rate of interest reduces inflation and unemployment in a front-loaded manner. What matters for the contractionary effect of monetary policy is that the central bank commits to keeping the real can undo dividend payments that they consider ill-timed by adjusting the number of shares they hold.
rate of interest higher than usual.

4.2.2 The Effect of a Monetary Shock on Inequality

Figure 9 shows the responses of the Gini indexes for earnings, income, wealth, and consumption to a monetary shock. What is notable here is that shocks that tighten monetary policy raise inequality in the economy. This is consistent with the empirical findings of, for example, Coibion et al. (2012). The effects are of a magnitude similar to a TFP shock (not shown), even though GDP falls four times more after a contractionary TFP shock, but generally less persistent.

5 Heterogeneous Welfare Effects

Next, we look at the welfare consequences of monetary policy. Section 5.1 explores the welfare effects of one-time shocks. While negligible in a representative-agent setting, the welfare costs of a contractionary monetary shock are notable when accounting for heterogeneity. They are an order of magnitude larger for the wealth-poor than for the wealth-rich, and roughly four
Figure 9: Response to a Monetary Shock: Gini Indexes

Notes: Impulse responses of Gini indexes of earnings (not conditioning on being employed), income, wealth, and consumption to a one-standard-deviation monetary policy shock, $D$. The figures show percentage point increases (an increase of “1” on the $y$-axis would increase the earnings Gini from, say, 0.64 to 0.65).

times larger for the unemployed than the employed. Section 5.2 explores the importance of the systematic response of monetary policy to unemployment. A vast majority of the population favors a much stronger response to unemployment than that embedded in the baseline.

5.1 Welfare Effects of One-Time Shocks

Table 4 summarizes the welfare effects of one-time one-standard-deviation shocks. The welfare gains (positive) or costs (negative) are measured as lifetime consumption equivalents. For example, the entry of “0.47” in row “Top 0.1 percent” and column “TFP” means that the 0.1 percent wealthiest households on average would be willing to permanently pay 0.47 percent of

<table>
<thead>
<tr>
<th>By Wealth</th>
<th>One-standard-deviation shock</th>
<th>TFP</th>
<th>Monetary</th>
<th>Financial</th>
</tr>
</thead>
<tbody>
<tr>
<td>RA, repres. agent</td>
<td>0.26</td>
<td>-0.01</td>
<td>-0.02</td>
<td></td>
</tr>
<tr>
<td>HA, household avg.</td>
<td>0.44</td>
<td>-0.07</td>
<td>-0.12</td>
<td></td>
</tr>
<tr>
<td>Top 0.1 percent</td>
<td>0.47</td>
<td>-0.01</td>
<td>-0.02</td>
<td></td>
</tr>
<tr>
<td>Top 5 percent</td>
<td>0.41</td>
<td>-0.01</td>
<td>-0.02</td>
<td></td>
</tr>
<tr>
<td>80th–95th percentile</td>
<td>0.34</td>
<td>-0.01</td>
<td>-0.03</td>
<td></td>
</tr>
<tr>
<td>60th–80th</td>
<td>0.29</td>
<td>-0.02</td>
<td>-0.03</td>
<td></td>
</tr>
<tr>
<td>40th–60th</td>
<td>0.41</td>
<td>-0.06</td>
<td>-0.11</td>
<td></td>
</tr>
<tr>
<td>30th–40th</td>
<td>0.54</td>
<td>-0.10</td>
<td>-0.20</td>
<td></td>
</tr>
<tr>
<td>Bottom 30 percent</td>
<td>0.60</td>
<td>-0.12</td>
<td>-0.24</td>
<td></td>
</tr>
</tbody>
</table>

Notes: Lifetime consumption equivalent welfare gains from a one-time one-standard-deviation positive TFP, monetary policy, and financial shock, respectively. Row “RA” reports consumption equivalents for a representative household. Row “HA” shows the average consumption equivalent in the population (using population weights).
their consumption to experience a positive one-standard-deviation aggregate TFP shock today. The welfare gains are largest for the wealth-poorest 30 percent of the population, 0.60 percent. The average consumption equivalent across all households (row, “HA”) is 0.44 percent. The wealth-poor are at or very close to their borrowing constraint and thus benefit most from the shock. In addition, they tend to be more impatient. The “upper middle class” gains least. On the one hand these households are far from their borrowing constraint. In addition, they rely mostly on labor income rather than financial income. Wage rigidity, however, imparts more of the TFP gains to capital owners than workers and, thus, to the wealthiest segment of the population. Indeed, at 0.29 percent of lifetime consumption, households around the 70th percentile of wealth gain about as much as would a representative household (compare row “RA”).

The monetary shock would have very small welfare costs if financial markets were complete (0.01 percent of lifetime consumption, see row “RA” and the second column of Table 4), because the way we calibrated it, the shock is rather small: it raises the nominal rate temporarily by 6.25 basis points above the level prescribed by the Taylor rule (0.25 percentage point when annualized). The small size notwithstanding, if markets are incomplete, the monetary shock has sizable distributional consequences. The wealth-poor on average are willing to pay as much as 0.12 percentage point of lifetime consumption to avoid this monetary shock, that is, more than ten times as much as the RA model suggests. The less wealth a household holds, the more its consumption will be affected by the increase in unemployment and unemployment risk and the fall in the wage that the monetary tightening brings about. For the upper middle class and the wealthiest households, instead, the monetary shock carries welfare costs about as small as those under complete markets.\(^\text{12}\)

Wealth, of course, is only one of the characteristics of a household. Figure 10 looks at the welfare costs of a monetary shock in more detail. Each of the four panels in Figure 10 focuses on one specific combination of time preferences ($\beta$) and employment status. In each panel, the x-axis reports a household’s wealth level by decile of the wealth distribution. For each wealth-decile, four bars (dark blue, light blue, green, yellow; from left to right) mark the skill of the household. No bar is shown if the mass of households with that combination is zero. The corresponding mass of the respective groups in the population is reported in Figure 13 in the appendix. The y-axis shows the consumption equivalents.

Several observations are in order. First, as discussed above, lower wealth is associated with higher welfare costs of a monetary shock. Second, and importantly, controlling for wealth, the

\(^{12}\)The financial shock, by construction, affects the economy through the same channels as the monetary policy shock. The size of the financial shock is larger, though.
skill level, and time preferences, the unemployed households are much more negatively affected by an unanticipated monetary tightening than employed households. For example, an unemployed low-beta low-wealth household of high skills ($s_3$) would be willing to forgo 0.45 percent of lifetime consumption to avoid what appears to be a rather small one-time monetary shock; see the green bars (the respective third bars from the left) for wealth deciles 1-3 and 4 in the bottom-left panel of Figure 10. On average, unemployed households’ welfare costs are about three times larger than employed households’ because a monetary shock tends to prolong the unemployment spell of an individual household. This means that lifetime earnings fall and the risk of exhausting savings rises.

Third, and conditioning on all other characteristics, more impatient (low beta) households are affected more negatively by a monetary shock. Conditional on wealth, their higher preference for current consumption means that their welfare is hit particularly hard by a shock that reduces income today and which they will not smooth over time (since the market rate of return is lower than their rate of time preference).
Fourth, and surprisingly perhaps, all else equal the more productive a household is (the higher the skill level $s$) the more a household loses from a monetary contraction. A monetary shock means that labor earnings fall and labor-earnings risk rises for all households. What drives the welfare losses for the high-skilled is that a monetary shock causes them to lose income precisely when they are more productive than usual (idiosyncratic productivity $s$ is persistent but temporary). Therefore, unemployed super-skilled households at the 50th and 60th percentile of the wealth distribution, for example, would be willing to pay as much as, respectively, 0.8 and 0.5 percent of lifetime consumption to forgo the shock. For aggregate welfare, however, these households matter little since only about 0.04 percent of households have these characteristics (Figure 13 in the appendix).

5.2 Welfare Effects of a Transition to More Accommodative Monetary Policy

So far, we took the monetary policy rule as given. In this section, we consider the welfare effects of changes to the systematic response of monetary policy to unemployment. This response is important because counter-cyclical monetary policy can stabilize unemployment. Not only does this help insure households against having to run down their savings during a persistent unemployment spell, it may also reduce the average unemployment rate itself. We compare the policy implications of our heterogeneous-agent (HA) model with the representative-agent (RA) counterpart and a Campbell and Mankiw (1989)-type saver-spender model (SP). Variants of the latter have been used in New Keynesian models without equilibrium unemployment by Galí et al. (2007) and Iacoviello (2005). In our SP model, some households are assumed to be “spenders,” who consume their entire income. The difference to our model is that households choose if they want to save or not. The central finding of this section is that a large majority of households in the HA model are in favor of a response to unemployment that is twice as strong as would be optimal in the RA model. The SP model provides incomplete guidance only.

The question we ask is: “Would a certain household prefer to live in the baseline economy or have monetary policy change to being more or less responsive to unemployment?” Toward this end we look at a one-time unanticipated change in the interest-rate response to unemployment, $\phi_u$, while keeping the response to inflation fixed at $\phi_\pi = 1.5$. We also keep the target levels for inflation and unemployment in the rule constant. The initial state is the stochastic steady state in the baseline. Welfare is assessed taking transition effects into account.

For the RA model, the optimal unemployment response is $\phi_u = 0.46$. At this value, unemployment fluctuates about 41 percent less than in the baseline, while inflation volatility rises by 71 percent. The reason why price stability is not optimal is that the condition is violated (Hosios,
in two ways: first, wages on average are too high from a social perspective (they remain in the bargaining set, however, and so are privately efficient) and, second, there is wage rigidity. Blanchard and Galí (2010) and Ravenna and Walsh (2012) show this in smaller representative-agent model economies.

Turning to the HA model, the welfare effects of switching to more “dovish” or more “hawkish” monetary policy are presented in Table 5. The left-most column reports on more hawkish policy, the four columns on the right on more dovish policy. The second-to-last row of the table reports the share of households that would be in favor of abandoning the baseline policy rule for a rule with an alternative value of $\phi_u$. In line with the results in the RA model, all households dislike switching to a more hawkish monetary policy ($\phi_u = 0.05$). Indeed, under this policy unemployment volatility would rise by 15 percent. Households with positive target levels of wealth (which includes patient households and higher-skilled impatient households) respond to this by accumulating further savings, which hurts their consumption. It is the wealthiest households that dislike moving to a more hawkish policy most. The wealth-poor instead are partially compensated for the increase in unemployment risk through the higher wages that come with more capital (and, thus, higher labor productivity).

The last four columns of the table show the welfare gains (compared to the baseline) from moving to more accommodative policy. In line with the exercises for the RA model, almost all

<table>
<thead>
<tr>
<th>$\phi_u$</th>
<th>0.05</th>
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<th>0.25</th>
<th>0.5</th>
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<td>RA, repres. agent</td>
<td>-0.09</td>
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<td>0.09</td>
<td>0.12</td>
<td>0.09</td>
<td>0.05</td>
</tr>
<tr>
<td>HA, household avg.</td>
<td>-0.09</td>
<td>—</td>
<td>0.08</td>
<td>0.13</td>
<td>0.13</td>
<td>0.12</td>
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<tr>
<td>Top 0.1 percent</td>
<td>-0.14</td>
<td>—</td>
<td>0.07</td>
<td>0.02</td>
<td>-0.10</td>
<td>-0.23</td>
</tr>
<tr>
<td>Top 5 percent</td>
<td>-0.13</td>
<td>—</td>
<td>0.08</td>
<td>0.06</td>
<td>-0.03</td>
<td>-0.14</td>
</tr>
<tr>
<td>80th–95th percentile</td>
<td>-0.12</td>
<td>—</td>
<td>0.08</td>
<td>0.09</td>
<td>0.04</td>
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<td>60th–80th</td>
<td>-0.11</td>
<td>—</td>
<td>0.09</td>
<td>0.13</td>
<td>0.11</td>
<td>0.08</td>
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<tr>
<td>40th–60th</td>
<td>-0.09</td>
<td>—</td>
<td>0.09</td>
<td>0.16</td>
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<td>30th–40th</td>
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<td>0.08</td>
<td>0.15</td>
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<td>—</td>
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<td>0.14</td>
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<td>0.18</td>
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<td>Percent in favor ...</td>
<td></td>
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<td></td>
</tr>
<tr>
<td>... relative to baseline</td>
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<td>—</td>
<td>99.7</td>
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<td>... relative to next-lower</td>
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<td>100</td>
<td>99.7</td>
<td>92.5</td>
<td>60</td>
<td>48</td>
</tr>
</tbody>
</table>

Notes: Welfare effects of a permanent policy change from $\phi_u = 0.107$ to a different value. See the main text for details.
households (99.7 percent) are in favor of moving to $\phi_u = 0.25$ (second-to-last row in column $\phi_u = 0.25$). The few households that are not are those that are both impatient and lower-skilled but rather wealthy. Asset prices fall on average, hurting those households in particular. Indeed 92.5 percent of households would favor moving toward a still stronger unemployment response of $\phi_u = 0.5$ (see last row, column $\phi_u = 0.5$). This value is close to $\phi_u = 0.46$, a value that was optimal in the RA model. Indeed, for those households that could be deemed reasonably close in characteristics to the representative household (the upper middle class), the welfare gains are comparable to those in the RA model, about 0.13 percent of lifetime consumption. Wealth-richer households would be better off under less accommodative policy.

Employed higher-income households tend to save. As a result, income and wealth are correlated. Nevertheless, there is substantial heterogeneity also within income groups. For example, 81 percent of super-skilled households prefer to move to $\phi_u = 0.5$ instead of $\phi_u = 0.25$ (results not shown in the table). For the low-skilled ($s_1$), the share is higher but still only 93 percent. The households that would gain most from the more accommodative policy are those with low to median levels of wealth. They benefit two-fold from the move to more accommodative policy: first, through a reduced risk of persistent unemployment and, thus, a better ability of smoothing consumption; second, through higher average employment. That is, in spite of the ensuing fall in the need for precautionary savings against unemployment risk, average employment rises. This is in line with the non-linearities in the search and matching model documented by Hairault et al. (2010), by which less volatile unemployment translates into lower unemployment on average. On the other hand, the more monetary policy stabilizes unemployment, the more volatile inflation becomes, increasing the distortion and resource cost caused by the Rotemberg adjustment cost. Assets, thus, will generate lower dividends and, at the same time, have a more procyclical payout structure. Both lead to a fall in asset price.

Table 5 documents that a majority of households favor moving to more accommodative policy than prescribed by the RA model. In particular, 60 percent of households prefer $\phi_u = 0.75$ over $\phi_u = 0.5$ (64 percent of the low-skilled, falling to 49 percent of the super-skilled). Only around $\phi_u = 1$ does there cease to be a majority for more accommodative policy. Only 48 percent of households favor moving all the way to $\phi_u = 1$ instead of stopping at $\phi_u = 0.75$. Two things are worth noting. First, at $\phi_u = 1$ unemployment is only about a third as volatile as in the baseline, whereas inflation is twice as volatile. Second, the move toward such accommodative policy favors the wealth-poor, whose welfare gain would amount to nearly 0.2 percent of lifetime consumption at the expense of the wealthy. At 0.41 percent of lifetime consumption, welfare differences for the respective groups shown in Table 5 are large.
It is useful to compare the findings above with the unemployment-response coefficients that a household would support in the simpler saver-spender model (no table provided). Thirty percent of households are assumed to be impatient (spenders, with $\beta_L$), based on the share of households with zero net worth in the data.13 Spender households live on their own. The remaining households (savers with a discount factor of 0.99) are assumed to live in representative families that provide for full consumption insurance in the event of unemployment.

As regards the optimal value of the response coefficient, the savers’ welfare is maximized at a value of $\phi_u = 0.42$. Employed spenders prefer a value of $\phi_u = 1.21$, unemployed savers would prefer a value above $\phi_u = 2$. Only some of this is attributable to impatience. Even if spenders had the same preferences as savers, the former would desire more unemployment stabilization, the preferred value being slightly below (above) $\phi_u = 0.9$ for employed (unemployed) spenders. In sum, the simple SP model would capture the rough trade-offs. It fails to provide clear guidance, however, as to the policies favored by the middle-class and so as to the majorities in favor of the differing policies. The fact that most households in the HA economy have a positive target level of wealth (including impatient households with skills $s_3$ and $s_4$; cf. Figure 14) means neither that the median household would favor policies optimal for a “representative” saver household, nor does it mean that the median household favors policies aimed at spenders only.

Returning to the HA model, Figure 11 shows the welfare effects in greater detail. The figure shows welfare for the switch to $\phi_u = 1$. The structure of the figure is the same as that of Figure 10. A larger reaction coefficient $\phi_u$ means that monetary policy reduces the earnings risk in the economy. This benefits, in particular, households that have few assets (or, at least, few assets relative to their target level of wealth) and, again, the unemployed. Indeed, the biggest winners from a more unemployment-focused monetary policy are low-wealth, impatient, super-skilled households. For them, the welfare gain is as much as 0.7 percent of lifetime consumption. High-wealth low-to-high-skilled households lose.

We have also entertained a version with a simple specification of longer-term unemployment. In this, households received benefits of 10 percent of income in each period of unemployment after the first. This increased the welfare gains or costs slightly, but did not change the overall picture as to the winners and losers from more accommodative monetary policy.

---

13 It is not entirely clear what the share of spenders should be in the calibration of the SP model. We also entertained a calibration with 50 percent of the households being spenders, in line with the mass of impatient households in the calibration of the HA model. The results were very similar to those reported above.
6 Conclusions

Monetary policy affects both aggregate economic activity and income risk across households. Toward assessing the distributional effects of conventional monetary policy, we have built a New Keynesian heterogeneous-agent DSGE model that features asset-market incompleteness, heterogeneity in preferences and skills, a frictional labor market, and sticky prices. The model was calibrated to the U.S. in tranquil times. Three main findings emerge.

First, incomplete markets make aggregate consumption considerably more responsive to shocks. Since we allow for physical capital and investment in employment relationships an increase in households’ desire to save need not suppress aggregate demand. Indeed, although there are capital adjustment costs, investment becomes smoother. On net, output volatility rises by 4 percent relative to a representative-agent version of the model.

Second, the average welfare consequences of monetary shocks are an order of magnitude larger with heterogeneous households. Contractionary monetary shocks lead to an increase in earnings, income, wealth, and consumption inequality. Wealth alone is an incomplete guide. Unemployed
households tend to lose about four times as much as employed households. And, conditioning on wealth, it is the households with the highest earnings capability when employed that lose most. In our model, households with these characteristics would be willing to forfeit as much as a full percentage point of consumption to avoid the consequences of a monetary shock that temporarily raises the nominal interest rate by as little as 6.25 basis points (25 basis points annualized).

Third, the median-wealth household prefers monetary policy to deviate notably from price stability. Households agree that there should be some response of monetary policy to unemployment. They disagree about its size, however. The wealthiest prefer a smaller response than in the representative-agent version of our model. The median-wealth household, instead, favors a response to unemployment that is about twice as strong as the optimal response in the representative-agent model. At this policy inflation is twice as volatile as in the baseline. For the wealthiest households implementing this policy instead of their preferred policy would mean a welfare loss equivalent to 0.3 percent of lifetime consumption. Increased costs of inflation to firms lead to falling dividends and asset prices, explaining the welfare losses for the wealthy.

To the best of our knowledge our framework is the first that provides a global solution to a New Keynesian model with search and matching frictions, physical capital, and meaningful household heterogeneity. The paper has, though (albeit intentionally), abstracted from labor-market heterogeneity in unemployment risk and wage risk. We suspect that relaxing the related assumptions might further increase the link between labor-income risk and the business cycle and thus the role for monetary stabilization policy. We leave this for future research.

References


Blanchard, Olivier and Jordi Galí, “Labor Markets and Monetary Policy: A New Keynesian


Curdia, Vasco and Michael Woodford, “Credit Spreads and Monetary Policy,” *Journal of Money, Credit and Banking*, 2010, 42 (S1), 3–35.


A  The Mutual Fund’s Discount Factor

This appendix gives the exact formula for the mutual fund’s discount factor. Notice that in equilibrium knowledge of $X$ and $X'$ implies knowledge of $\tilde{X}'$. The stochastic discount factor is defined as follows:

\[
Q(X, X') = \int_{\{1\} \times S \times B \times A} \sum_{\beta' \in B} \pi_B(\beta, \beta') \sum_{s' \in S} \pi_S(s, s') \beta \left( g_a(X, 1, s, \beta, a) \right) \frac{(1-\lambda f(X')) u_c(g_a(X', 1, s', \beta', g_a(X, 1, s, \beta, a)))}{u_c(g_a(X, 1, s, \beta, a))} \, d\mu(e, s, \beta, a)
\]

\[
+ \int_{\{1\} \times S \times B \times A} \sum_{\beta' \in B} \pi_B(\beta, \beta') \sum_{s' \in S} \pi_S(s, s') \beta \left( g_a(X, 0, s, \beta, a) \right) \frac{(\lambda f(X')) u_c(g_a(X', 1, s', \beta', g_a(X, 0, s, \beta, a)))}{u_c(g_a(X, 1, s, \beta, a))} \, d\mu(e, s, \beta, a)
\]

\[
+ \int_{\{0\} \times S \times B \times A} \sum_{\beta' \in B} \pi_B(\beta, \beta') \sum_{s' \in S} \pi_S(s, s') \beta \left( g_a(X, 0, s, \beta, a) \right) \frac{(1-f(X')) u_a(g_a(X', 0, s', \beta', \beta_a, g_a(X, 0, s, \beta, a)))}{u_a(g_a(X, 0, s, \beta, a))} \, d\mu(e, s, \beta, a),
\]

Above, $u_c(c)$ marks the marginal utility of consumption.

B  Definition: Recursive Equilibrium

Definition 1 (Recursive equilibrium) A recursive equilibrium is a set of functions $G(X)$, $\tilde{G}(X)$, $\tilde{G}(\tilde{X})$, $W(X, e, s, \beta, a)$, $g_a(X, e, s, \beta, a)$, $g_c(X, e, s, \beta, a)$, $f(\tilde{X})$, $p_a(X)$, $d_a(X)$, $w(X)$, $\tau(X)$, $h(X)$, $Q(X, X')$, $J_L(X, s)$, $V(\tilde{X})$, $r(X)$, $J_K(X, k)$, $i(X)$, $v(X)$, $K'(X)$, $P(X)$, $y_j(X, P_j)$, $J_I(X, P_{j-1})$, $k_j(X)$, $\ell_j(X)$, $P_j(X)$, $\Pi_j(X)$, $y(X)$, $R(X)$ such that:

1. Given $\tilde{G}(X), f(\tilde{X}), w(X), p_a(X), d_a(X), G(X)$, and $\tau(X)$, value function $W(X, e, s, \beta, a)$ is a solution to the household’s problem. $g_a(X, e, s, \beta, a)$ and $g_c(X, e, s, \beta, a)$ are the associated optimal decision rules.

2. Given $h(X), w(X), Q(X, X')$, and $G(X)$, $J_L(X, s)$ solves the problem of a labor agency. $V(\tilde{X})$ satisfies the free-entry condition in the labor agency sector. $f(\tilde{X})$ is consistent with $V(\tilde{X})$.

3. Given $r(X), Q(X, X')$, and $G(X)$, $J_K(X, k)$ solves the problem of a capital-producing firm. $i(X), v(X),$ and $K'(X)$ are the associated optimal decision rules.

4. Given $r(X), v(X), h(X), P(X), y_j(X, P_j)$, and $Q(X, X')$, value function $J_I(X, P_{j-1})$ solves the problem of an intermediate good producer. $k_j(X), \ell_j(X), P_j(X)$, and $\Pi_j(X)$ are the associated optimal decision rules.
5. Given $P(X)$ and $P_j$, $y_j(X, P_j)$ and $y(X)$ are the optimal decisions of final good producers.

6. The aggregate discount factor $Q(X', X)$ satisfies equation (10).

7. $d_a(X)$ satisfies the flow budget constraint of mutual funds (9).

8. The wage per efficiency unit of labor is given exogenously by $w(X)$ (5).

9. The labor tax $\tau(X)$ satisfies the government budget constraint (8).

10. The nominal interest rate $R(X)$ satisfies the Taylor rule (7).

11. The aggregate laws of motion $G(X)$, $\tilde{G}(X)$, and $\hat{G}(\tilde{X})$ are consistent with the relevant optimal decision rules.

12. All market clearing conditions are satisfied.

C Solution Algorithm

This appendix outlines the solution method of an equilibrium with aggregate uncertainty. The method is a version of the method developed by Krusell and Smith (1998) and is closely related to the solution method based on reference distributions described in Reiter (2002) and Reiter (2010).14

1. Following Reiter (2010) we approximate the aggregate state of the economy by $X = (K, N, Z, \zeta_F, \zeta_R)$ and assume that there is a distribution selection function $\hat{\mu}$, a mapping from $X$ into the space of all distributions on the household state variables. We approximate such a distribution following Young (2010) as a histogram on the product of skill state, the discount factor, employment state and a grid on the wealth distribution. All agents use this function to construct their forecasts about the evolution of the economy. We discretize $X$ and interpolate between points using a Smolyak approximation (see Krueger and Kubler 2004).

2. Solve the model without aggregates shocks and follow the steps in Reiter (2010) to construct a first guess for the distribution selection function $\hat{\mu}$.

14 In earlier versions of the paper we used an approach closer to Krusell and Smith (1998), in which we forecasted the expectation terms in the firms’ Euler equations and asset prices. The current method allows for a faster solution of the model, but results were close to each other when we compared both methods.
3. Form an initial guess for the following: the price of the asset, \( P_a(X) \), and the terms 
\[ \mathbb{E}Q(X,X')\phi P(\Pi(X') - \Pi(X)), \mathbb{E}Q(X,X')\frac{1}{\Pi(X')}, \]
\[ \mathbb{E}Q(X,X') \left[ r'(X')v' + \frac{1}{2v'/K'} \left( 1 - \delta(v') + \zeta \left( \frac{v'}{K} \right) \right) - \frac{v}{K} \right], \text{ and } \mathbb{E}[Q(X,X')(1-\lambda)J_L(X',s')] \]
each as functions of \( X \). For a shorthand, we denote these guesses by \( \Sigma(X) \).

4. Given these initial guesses, perform the following steps

(a) Given \( \Sigma(X) \) use a numerical equation solver to obtain the solution to the firms’ and 
government’s equations on the grid.

(b) Interpolate the static choices.

(c) Given the solutions obtained in the previous step, iterate on the value function of the 
households.

   i. Set a guess for the value function \( W(X,e,s,\beta,a) \).

   ii. Use the Bellman equation to update the value function.

   iii. If the updated value function is close to the guess, this step is done. The optimal decision rules 
   \( a' = g_a(X,e,s,\beta,a) \) and \( c = g_c(X,e,s,\beta,a) \) are obtained. 
   Otherwise, go back with an updated value function.

(d) Use \( \hat{\mu} \) and the solutions to the firms’ and government’s problems along with the 
optimal decision rules to compute the discount factor (10) on the grid. Use this to 
update \( \Sigma(X) \) to \( \Sigma'(X) \). \( P_a(X) \) is updated by solving for the market-clearing price at 
each grid point using the reference distribution. If \( \Sigma(X) \) and \( \Sigma'(X) \) are close, go to 
the next step, otherwise use a weighted average of \( \Sigma(X) \) and \( \Sigma'(X) \) and start with 
the firms’ and government’s equations again.

(e) Simulate the model. Notice that, for each period a market-clearing \( p_a \) has to be found, 
in the same manner as in Krusell and Smith (1997).

   i. Set the initial state and the initial type distribution. Use the steady-state values 
as the initial guess.

   ii. At the beginning of period \( t \), draw a new set of shocks. We have the aggregate 
state in period \( t \), \( (K_t,N_t,Z_t,D_t) \).

   iii. Set a guess for the share price \( \hat{p}_a \), using the forecasting function with \( P_a(X) \).

\[ ^{15} \text{For the initial guess, we solved the representative-agent version of our model (setting } \beta = 0.99 \text{ so as to match}
\text{the same real rate). Alternatively, we could have started with the choices in the heterogeneous-agent model}
\text{in the steady state and guessed } Q(X,X') = 0.99. \]
iv. Conditional on \( \hat{p}_a \), and the aggregate state variables in period \( t \), solve the problem of the households.

v. Check market clearing. Compute the excess demand for the shares. If it is zero, a market-clearing price in period \( t \), \( p_{a,t} \), is obtained for period \( t \). \( K_{t+1} \) and \( N_{t+1} \) can be computed. Go to the next step. Otherwise, update \( \hat{p}_a \) and go back to the previous step.

vi. Update the type distribution and aggregate state variables using \( p_{a,t} \) and go to period \( t + 1 \).

vii. Keep simulating until period \( T = T_0 + T_1 = 500 + 3000 \) periods.

(f) The previous step generates a time series of household distributions \( \{\mu\}_{t=0}^{T} \). Drop the first \( T_0 \) periods. Using the time series for \( t = T_0 + 1, \ldots, T \) construct a new reference distribution function \( \hat{\mu}' \) following Reiter (2002).

(g) Compare \( \hat{\mu} \) and \( \hat{\mu}' \). If they are close, an equilibrium is obtained. Stop. Otherwise, update \( \hat{\mu} \) and return to the firms’ and government’s problem.

In practice, as \( \zeta_R \) and \( \zeta_F \) have the same persistence under our calibration, the only variable affected differently by the two shocks is the nominal interest rate. Therefore, it is possible to solve the model while merging them into one state variable. During simulations of the model we distinguish between the two shocks in order to capture the right movements in the nominal interest rate given our calibration.

D Data and Details on the Calibration

This appendix provides further details on the data used and the calibration.

D.1 Data in Table 3 of the Main Text

The data we compare the model to in Table 3 are either quarterly already or are quarterly averages of monthly data. They are seasonally adjusted, if necessary. Unless noted otherwise, their source is the St. Louis Fed’s FRED II database. We start with series that cover the period 1979Q1 to 2013Q3. After filtering them, we drop the first and last 20 quarters to arrive at a sample covering the period 1984Q1 to 2008Q3. Nominal variables are deflated by the GDP deflator, which we also use as our measure of inflation, \( \Pi \). Personal consumption expenditures, \( c \) include total durable and non-durable consumption expenditures as well as services. Investment, \( i \), is gross private domestic investment. Our measure of GDP is the sum of consumption and investment. Capacity utilization, \( v \), is measured by the quarterly average of the Board of Governors’ headline index of
industrial capacity utilization. We use the civilian unemployment rate, $U(X)$, among those 16 years of age and older. Employment, $N(X)$, is one minus this measure. We measure vacancies $V$ using Barnichon’s (2010) composite help-wanted index. The data counterpart to the job-finding rate, $f$, is the quarterly average of the monthly transition probability from unemployment to employment in the Current Population Survey (CPS). The data are adjusted for time aggregation as in Shimer (2012). The wage, $W(X)$ is computed as wage and salary accruals from the national accounts divided by the GDP deflator. The interest rate, $R$, is the quarterly average of the effective federal funds rate.

D.2 Details on the Calibration

Figure 12 shows the wealth Lorenz curve for U.S. households aged 21-65 as a red dashed line. The data are from the 2004 Survey of Consumer Finances. For the construction of the wealth variable, we follow Díaz-Giménez et al. (2011). A solid line shows the model's nonstochastic steady-state counterpart.

![Figure 12: Wealth Lorenz Curve](image)

E State Dependence of Impulse Responses

This appendix documents the state dependence of the impulse responses and the extent to which heterogeneity amplifies the state dependence.

E.1 TFP Shocks: State Dependence

Figure 15 analyzes the extent to which a TFP shock has different effects depending on the state of the business cycle. We consider three such states: a deep recession (dashed line), a boom
Table 6: “Wall Street’s” and “Main Street’s” Income Sources

<table>
<thead>
<tr>
<th>Wealth percentile</th>
<th>0-5</th>
<th>5-20</th>
<th>20-40</th>
<th>40-60</th>
<th>60-80</th>
<th>80-95</th>
<th>95-100</th>
</tr>
</thead>
<tbody>
<tr>
<td>Data: 2004</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Labor income</td>
<td>92</td>
<td>83</td>
<td>91</td>
<td>89</td>
<td>89</td>
<td>81</td>
<td>55</td>
</tr>
<tr>
<td>Financial income</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>5</td>
<td>6</td>
<td>14</td>
<td>41</td>
</tr>
<tr>
<td>Transfers</td>
<td>7</td>
<td>16</td>
<td>8</td>
<td>6</td>
<td>5</td>
<td>6</td>
<td>3</td>
</tr>
<tr>
<td>Model (steady-state)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Labor income</td>
<td>96</td>
<td>96</td>
<td>97</td>
<td>97</td>
<td>81</td>
<td>57</td>
<td>32</td>
</tr>
<tr>
<td>Financial income</td>
<td>0</td>
<td>0</td>
<td>0.1</td>
<td>2</td>
<td>18</td>
<td>42</td>
<td>68</td>
</tr>
<tr>
<td>Transfers</td>
<td>4</td>
<td>4</td>
<td>3</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0.3</td>
</tr>
</tbody>
</table>

Notes: Share of income coming from labor and financial income, respectively, by percentile of the wealth distribution. The data are from the Survey of Consumer Finances (2004), for households aged 21-65. “Financial income” includes the categories financial income, business income, and capital gains/loss. Labor income does not include social security or pensions.

Table 7: Second Moments – Comparison HA, RA, and Saver-Spender Variants

<table>
<thead>
<tr>
<th></th>
<th>heterog. hh. (HA)</th>
<th>represent. hh. (RA)</th>
<th>Saver-Spender SP30</th>
<th>Saver-Spender SP50</th>
</tr>
</thead>
<tbody>
<tr>
<td>GDP (GDP)</td>
<td>1.69</td>
<td>1.62</td>
<td>1.66</td>
<td>1.68</td>
</tr>
<tr>
<td>Consumption (c)</td>
<td>1.02 0.69</td>
<td>0.89 0.71</td>
<td>0.94 0.70</td>
<td>0.98 0.70</td>
</tr>
<tr>
<td>Investment (i)</td>
<td>5.28 0.73</td>
<td>5.86 0.71</td>
<td>5.66 0.72</td>
<td>5.51 0.72</td>
</tr>
<tr>
<td>Capacity utilization (v)</td>
<td>0.96 0.24</td>
<td>0.83 0.27</td>
<td>0.89 0.25</td>
<td>0.95 0.26</td>
</tr>
<tr>
<td>Employment N(X)</td>
<td>0.65 0.64</td>
<td>0.62 0.66</td>
<td>0.64 0.65</td>
<td>0.66 0.65</td>
</tr>
<tr>
<td>Unemployment U(X)</td>
<td>10.9 0.65</td>
<td>10.2 0.67</td>
<td>10.7 0.66</td>
<td>10.9 0.66</td>
</tr>
<tr>
<td>Vacancies (V)</td>
<td>8.94 0.07</td>
<td>8.35 0.10</td>
<td>8.68 0.08</td>
<td>8.97 0.07</td>
</tr>
<tr>
<td>Job finding rate (f)</td>
<td>5.37 0.38</td>
<td>5.08 0.40</td>
<td>5.26 0.39</td>
<td>5.42 0.37</td>
</tr>
<tr>
<td>GDP(X)/N(X)</td>
<td>1.14 0.62</td>
<td>1.10 0.63</td>
<td>1.11 0.63</td>
<td>1.12 0.62</td>
</tr>
<tr>
<td>Wage W(X)</td>
<td>0.51 0.62</td>
<td>0.50 0.63</td>
<td>0.50 0.63</td>
<td>0.51 0.62</td>
</tr>
<tr>
<td>Inflation Π[1]</td>
<td>0.67 -0.32</td>
<td>0.67 -0.40</td>
<td>0.68 -0.35</td>
<td>0.62 0.69 -0.31</td>
</tr>
<tr>
<td>Nominal rate R[1]</td>
<td>0.97 -0.14</td>
<td>0.96 -0.25</td>
<td>0.98 -0.19</td>
<td>1.00 -0.14</td>
</tr>
</tbody>
</table>

Notes: Same as Table 3, but listing the saver-spender model variants with 30 percent (“SP30”) and 50 percent (“SP50”) of spenders instead of the data.

(dotted line), and the stochastic steady state (circles). The deep recession state is obtained as follows. Starting at the stochastic steady state, we feed a sequence of five periods of negative one-standard-deviation TFP shocks, and one-standard-deviation contractionary monetary and financial shocks into the model. The state of the economy after that sequence of shocks is the “deep recession state.” The boom state is the result of the same sequence of shocks but with the opposite sign. The first row shows the response of the model economy with a representative...
Figure 13: Mass of Households by Idiosyncratic State

Figure 14: Savings Policy Functions

Notes: Savings policy as a function of current wealth. Policy functions evaluated in the stochastic steady state. x-axis: wealth as a share of annual potential labor income, Wealth: $p_a \cdot a/(4 \cdot s \cdot w)$, where $s \in S$. y-axis: saving as a share of annual potential labor income, Saving: $p_a \cdot (a' - a)/(4 \cdot s \cdot w)$. Positive values thus mean accumulation of wealth. Negative values mean dissaving.
agent, the second row the economy with heterogeneous households. We start by discussing the

**Figure 15: Effect of State of the Economy on IRFs to TFP Shock**

**Representative household**

<table>
<thead>
<tr>
<th>GDP</th>
<th>Consumption</th>
<th>Unempl. Rate</th>
<th>Inflation</th>
</tr>
</thead>
<tbody>
<tr>
<td>steady state rep</td>
<td>recession rep</td>
<td>boom rep</td>
<td>steady state rep</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>0.5</td>
<td>0</td>
</tr>
<tr>
<td>0.6</td>
<td>0.7</td>
<td>0.2</td>
<td>−0.15</td>
</tr>
<tr>
<td>0.8</td>
<td>0.6</td>
<td>0.5</td>
<td>−0.25</td>
</tr>
<tr>
<td>1</td>
<td>0.5</td>
<td>0.4</td>
<td>−0.35</td>
</tr>
<tr>
<td>1.2</td>
<td>0.4</td>
<td>0.3</td>
<td>−0.40</td>
</tr>
</tbody>
</table>

**Heterogeneous households**

<table>
<thead>
<tr>
<th>GDP</th>
<th>Consumption</th>
<th>Unempl. Rate</th>
<th>Inflation</th>
</tr>
</thead>
<tbody>
<tr>
<td>steady state het</td>
<td>recession het</td>
<td>boom het</td>
<td>steady state het</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>0.5</td>
<td>0</td>
</tr>
<tr>
<td>0.6</td>
<td>0.7</td>
<td>0.2</td>
<td>−0.15</td>
</tr>
<tr>
<td>0.8</td>
<td>0.6</td>
<td>0.5</td>
<td>−0.25</td>
</tr>
<tr>
<td>1</td>
<td>0.5</td>
<td>0.4</td>
<td>−0.35</td>
</tr>
<tr>
<td>1.2</td>
<td>0.4</td>
<td>0.3</td>
<td>−0.40</td>
</tr>
</tbody>
</table>

*Notes:* Impulse response to a 1 standard deviation TFP shock, Z. First row: model economy with representative households. Second row: model economy with heterogeneous households. In each of the panels, the response indicated by circles is the response starting from the steady state (as in figure 2) and the response indicated by dashes starts from a deep recession state. The dotted line marks responses starting in a boom. Whenever the figure shows percent responses, it normalizes the response by the steady-state value of the respective variable. This means that larger percent responses also mean larger responses in levels.

representative-agent economy. A sequence of expansionary shocks means that the unemployment rate is lower than in the non-stochastic steady state. This makes hiring more costly for labor-services firms. The costs for labor services increase. As a result, an expansion in TFP has only half the impact on the unemployment rate in a boom as in a recession. Similarly, inflation falls only half as much in a boom as in a recession. Relatively higher costs for labor services mean that, in order to take advantage of the TFP shock, the mutual funds increasingly invest more in physical capital in a boom than they would in a recession (not shown). As a result, consumption responds by less (relative to consumption in the non-stochastic steady state) in the boom than in the recession. Again, the response of output is affected to a lesser extent.

The model economy with heterogeneous households inherits the state-contingence of the responses from the representative-agent economy. What is interesting, however, is that the heterogeneous-agent economy alters the response of consumption in deep recessions. First, relative to the steady state, consumption responds by less on impact in the heterogeneous-agent economy because more households find themselves close to the borrowing constraint in the recession. As a result, they take advantage of the rising incomes that emerge from the expansionary
TFP shocks by consuming but also rebuilding their savings. Consumption in a deep recession, therefore, only gradually builds up in response to an expansionary TFP shock.

In sum, in both the RA and HA economies there is notable state dependence of the responses. Responses of consumption, unemployment and inflation to a TFP shock are much larger in a recession than in a boom. The difference between the HA and RA models’ responses is state-dependent as well.

E.2 Monetary Shocks: State Dependence

Figure 16 analyzes the extent to which the impulse responses to a contractionary monetary shock are state-dependent. Similar to the results we obtained for the TFP shock, the impact of a monetary shock is state-dependent, and considerably so. In the heterogeneous-agent economy, the response of GDP to a contractionary monetary shock is 20 percent larger (GDP falls by 0.05 percentage points more) in a boom than in a recession. The representative-agent economy shows less (if any) of such state dependence in the response of GDP. Both the RA and HA models have considerable state-dependence in the inflation response, however. Inflation falls almost three times as much in response to a monetary shock if that shock hits in a boom than if it hits in a recession. This is so even though the response of the unemployment rate is almost 50 percent larger in a deep recession than in a steep boom. Putting this in slightly different terms, the responses shown here suggest that the “sacrifice ratio” (measured as the rise in unemployment for a given fall in inflation) is considerably lower in boom times than in recessions. Or, putting it still differently, monetary policy (through an expansionary monetary shock) can more easily increase output without having to jeopardize price stability in recessions than in booms. Heterogeneity further increases the scope for state-dependent responses to monetary shocks. This is most clearly visible again for the consumption response: in a recession, more households will be close to the borrowing constraint or, more generally, farther below their target level of wealth. As a result, those households are less susceptible to the intertemporal substitution that a monetary tightening causes. Actually, on impact, in the HA model consumption falls by less in a recession than in a boom (or in the steady state). Over time, however, in the HA economy, the consumption response is considerably stronger and much more persistent in a recession than in a boom.
Figure 16: Effect of State of the Economy on IRFs to a Monetary Shock

Representative household

Heterogeneous households

Notes: Impulse response to a one-standard-deviation monetary shock, $D$. First row: model economy with representative households. Second row: model economy with heterogeneous households. In each of the panels, the response indicated by circles is the response starting from the steady state (as in figure 5) and the response indicated by dashes starts from a deep recession state. The dotted line marks responses starting in a boom. Whenever the figure shows percent responses, it normalizes the response by the steady-state value of the respective variable. This means that larger percent responses also mean larger responses in levels.