Coordination on List Prices
and Collusion in Negotiated Prices*

Joseph E. Harrington, Jr.† and Lixin Ye‡

1 April 2017

Abstract

A collusive practice in some intermediate goods markets is for sellers to co-
ordinate on list prices but not final prices. We put forth a theory to explain how
coordination on list prices can raise transaction prices even when all customers
pay negotiated prices. Market conditions are identified that are conducive to
firms profitably engaging in this form of collusive practice.

*The comments of participants at the 2016 Hal White Antitrust Conference (Washington, D.C.),
2016 UBC Industrial Organization Conference (Kelowna, British Columbia), and a seminar at the
Norwegian School of Economics are gratefully acknowledged, as is the extremely able research assis-
tance of Ben Rosa and Xingtan (Ken) Zhang. The first author recognizes the financial support of
the National Science Foundation (SES-1148129).
†Department of Business Economics & Public Policy, The Wharton School, University of Penn-
sylvania, Philadelphia, PA 19104, harrij@wharton.upenn.edu
‡Department of Economics, Ohio State University, Columbus, OH 43210, ye.45@osu.edu
1 Introduction

Collusion entails firms coordinating on the prices that they charge customers. In the context of retail markets, firms agree to supracompetitive posted prices and then monitor each other to ensure that those are the prices charged in stores (or online). With intermediate goods markets, it is more complicated because firms set list prices and routinely offer discounts to buyers. While they may agree on list prices, it is really the coordination on final transaction prices that is essential, as noted for the thread cartel:¹

[A cartel member] explained that list prices have more of a political importance than a competitive one. Only very small clients pay the prices contained in the lists. As the official price lists issued by each competitor are based on large profit margins, customers regularly negotiate rebates, but no clear or fixed amount of rebates is granted. ... Therefore, the list prices are essentially "fictitious" prices ... while [rebates] were discussed and agreed during the meetings.

Once having coordinated on list prices and discounts, the challenge is then monitoring for compliance. While list prices are public information, discounts are privately negotiated between a buyer and seller which makes it difficult to determine whether cartel members charged the agreed-upon final transaction prices. The solution pursued by many cartels - including those in the markets for citric acid, lysine, and vitamins - was to agree to an allocation of sales quotas along with final transaction prices, and then monitor for compliance by comparing realized sales to those quotas.²

Contrary to the rather standard collusive practices just described, some intermediate goods cartels coordinated exclusively on list prices and left firms unconstrained

in the discounts that they offered. Furthermore, there was no evidence of sales monitoring to ensure that firms did not gain market share through discounts and, in fact, discounts were regularly given to customers. This pattern occurred in several private litigation cases for which plaintiffs, defendants, and the court often came to different conclusions regarding the efficacy of firms coordinating on list prices.\(^3\)

*Reserve Supply v. Owens-Corning Fiberglas* (1992) concerned possible collusion in the market for insulation. Plaintiffs and defendants put forth conflicting claims regarding the collusive role of list prices:\(^4\)

Reserve points to Owens-Corning and CertainTeed’s practices of maintaining price lists for products and ... asserts that these lists have no independent value because no buyer in the industry pays list price for insulation. Instead, it claims that the price lists are an easy means for producers to communicate and monitor the price activity of rivals by providing a common starting point for the application of percentage discounts. ... Owens-Corning and CertainTeed counter by arguing that the use of list prices to monitor pricing would not be possible because the widespread use of discounts in the industry ensures that list prices do not reflect the actual price that a purchaser pays.

The Seventh Circuit Court expressed skepticism with regards to the plaintiffs' claim:\(^5\)

We agree that the industry practice of maintaining price lists and announcing price increases in advance does not necessarily lead to an inference of price fixing. ... [T]his pricing system would be, to put it mildly, an awkward facilitator of price collusion because the industry practice of providing discounts to individual customers ensured that list price did not reflect the actual transaction price.

---

\(^3\)In addition to the cases mentioned below is *Lum v. Bank of America* 361 F.3d, 217, 231 (3d Cir., 2004). For that case, also see the discussion in Holmes (2004).

\(^4\) *Reserve Supply v. Owens-Corning Fiberglas* 971 F. 2d 37 (7th Cir. 1992), para 61.

\(^5\) Ibid, para. 62.
In a case involving the market for urethane, plaintiffs claimed: 6

[T]hroughout the alleged conspiracy period, the alleged conspirators announced identical price increases simultaneously or within a very short time period. ... [P]urchasers could negotiate down from the increased price. But the increase formed the baseline for negotiations. ... [T]he announced increases caused prices to rise or prevented prices from falling as fast as they otherwise would have.

The Tenth Circuit Court quoted the District Court in supporting this assessment: 7

The court reasoned that the industry’s standardized pricing structure - reflected in product price lists and parallel price-increase announcements - “presumably established an artificially inflated baseline” for negotiations. Consequently, any impact resulting from a price-fixing conspiracy would have permeated all polyurethane transactions, causing market-wide impact despite individualized negotiations.

A final example is a recent cement cartel in the United Kingdom. 8 Annually, cement suppliers sent letters to their customers announcing price increases. However, prices were then individually negotiated with customers and the full price increase was rarely implemented. The Competition and Markets Authority concluded that the price announcement letters served to coordinate on list prices and this impacted the subsequent negotiations: 9

We understand, however, that the prices set out in price increase letters are in practice used as a starting point for negotiations with customers.

---

6 Class Plaintiffs’ Response Brief (February 14, 2014), In Re: Urethane Antitrust Litigation, No. 13-3215, 10th Cir.; pp. 8-9.
7 In Re: Urethane Antitrust Litigation, No. 13-3215 (10th Cir. Sep. 29, 2014); p. 7.
9 Ibid, p. 53.
and that firms generally fail to achieve the prices set out in the price letters, in part because of the rebates offered to large customers. This failure to achieve "list" prices suggests that prices are not simply fixed through this mechanism [that is, price announcement letters].

Though the coordination in list prices did not involve express communication, it is the same pattern as with the cases involving insulation and urethane: Sellers coordinated on list prices but not transaction prices. In commenting on the UK cement case, Justin Coombs, who is head of Compass Lexecon’s London office, posed the question: “How do price announcements help firms coordinate on prices if prices are ultimately individually negotiated?” To our knowledge, there is no theory that provides an answer to that question.

Our objective is to explore the possibility that firms could successfully collude by coordinating only on list prices while leaving firms with complete discretion in setting final transaction prices. The two questions addressed here are: 1) How can coordination in list prices result in supracompetitive transaction prices?; and 2) Having identified a mechanism whereby list prices impact negotiated prices, what market conditions are conducive to firms profitably engaging in this form of collusive practice?

Section 2 describes the model, while a review of some related research is provided in Section 3. There are two steps to developing the theoretical argument. First is establishing an endogenous connection between announced list prices and final transaction prices, which is conducted in Section 4. The second step is showing that firms can jointly raise profits by coordinating their list prices. That is done in Sections 5-7. Section 8 illustrates how this theoretical insight can provide guidance in antitrust cases. Unless otherwise noted, proofs are in the appendix.

2 Model

Consider a market with two sellers offering identical products. A seller may be one of two types, $L$ or $H$, and type $L$ occurs with probability $q$. Sellers’ types are independent. A type $t$ seller’s unit cost is assumed to be a random draw from the cdf $F_t : [\underline{c}_t, \overline{c}_t] \rightarrow [0, 1], t \in \{L, H\}$. $F_t$ is continuously differentiable with positive density everywhere on $(\underline{c}_t, \overline{c}_t)$. The inverse hazard rate function, $h_t(c) \equiv F_t(c)/F'_t(c)$, is assumed to be non-decreasing, $h'_t(c) \geq 0$, which holds for most of the common distributions such as uniform, normal, exponential, logistic, chi-squared, and Laplace. The two cost distributions are ranked in terms of their inverse hazard rates: $h_L(c) > h_H(c)$ for all $c \in (\underline{c}_t, \overline{c}_t)$. Note that the latter condition implies $F_H$ first-order stochastically dominates $F_L$ and, consequently, we will refer to a type $L$ seller as a low-cost type and a type $H$ seller as a high-cost type.

Each buyer is interested in buying 0 or 1 unit and $v$ denotes a buyer’s valuation. There is a continuum of buyers and their valuations are represented by the cdf $G : [\underline{v}, \overline{v}] \rightarrow [0, 1]$. $G$ is continuously differentiable with positive density everywhere on $(\underline{v}, \overline{v})$. A buyer may choose to solicit offers from either 1 or 2 sellers. What exactly it means to “solicit” an offer is described below. A fraction $b \in [0, 1]$ of sellers are assumed to solicit an offer from a single seller and a fraction $1 - b$ from two sellers. Whether a buyer approaches 1 or 2 sellers is assumed to be independent of a buyer’s valuation.\footnote{Though it would be preferable to endogenize the number of sellers that are solicited by a buyer, it is assumed to be exogenous for reasons of tractability. This specification could be trivially rationalized by assuming buyers incur a cost to negotiating with each seller, they vary in this cost, and the cost is independent of a buyer’s valuation. Some buyers have very low cost and thus negotiate with both sellers, while other buyers have a high enough cost that it is optimal to only negotiate with one seller.}

The modelling of the interaction between buyers and sellers is intended to capture many intermediate goods markets for which buyers are industrial customers. Sellers first choose list (or posted) prices. After observing those list prices, each buyer approaches either 1 or 2 sellers to negotiate. A buyer who approaches two sellers is presumed to engage in an iterative bargaining process whereby she uses an offer from...
one seller to obtain a better offer from the other seller. Rather than explicitly model that process, we will use the second-price auction with a reserve price as a metaphor for it. More specifically, a buyer “invites” \( w \) sellers to the auction, where \( w \in \{1, 2\} \). The buyer sets a reserve price and the \( w \) sellers submit bids which, in equilibrium, will equal their cost. We have buyers choose a reserve price so they are not passive, which better mimics negotiation.\(^{12}\) A transaction occurs if the lowest bid is below the buyer’s reserve price. In the case of having chosen just one seller, the mechanism is equivalent to the buyer making a take it or leave it offer. List prices are presumed to be chosen less frequently than negotiated prices and this has the implication that a seller knows its cost type when it chooses its list price but does not know its actual cost until the time of negotiation. In practice, this uncertainty about future cost may be due to volatility in input prices or not knowing the opportunity cost of supply because future inventories or capacity constraints are uncertain.

As our focus is on markets for which very few, if any, consumers fail to negotiate the price that they pay, it will make for a more parsimonious analysis if we presume that no transactions take place at list price by supposing a list price is a cheap talk message and not actually a price at which buyers can transact. As in the thread cartel mentioned in the introduction, list prices are then “fictitious” but, as we’ll show, can still be informative and impactful.

The extensive form is as follows:

- Stage 1: Sellers draw types from \( \{L, H\} \) (which is private information to each seller) and choose a list price (which is a cheap talk message) from \( \{l, h\} \).
- Stage 2: Buyers learn their valuations, observe sellers’ list prices, and choose \( w \) sellers where \( w \in \{1, 2\} \).
- Stage 3: Each seller realizes its cost. If a seller is type \( t \) then its cost is a draw from \([c_t, \bar{c}_t]\) according to \( F_t \).

\(^{12}\)As sellers optimally submit bids equal to their costs, it does not matter if the reserve price is hidden or not.
• Stage 4: For each buyer, the $w$ selected sellers participate in a second-price auction with a hidden reserve price. Each seller submits a bid equal to its cost. Transactions and transaction prices are determined as follows:

  – If there are two sellers in the auction and
    * both bids are below the reserve price then the buyer buys from the seller with the lowest bid at a price equal to the second lowest bid.
    * one bid is below the reserve price and the other bid is above the reserve price then the buyer buys from the seller with the lowest bid at a price equal to the reserve price.
    * both bids are above the reserve price then there is no transaction.

  – If there is one seller in the auction and
    * the bid is below the reserve price then the buyer buys from the seller at the reserve price.
    * the bid is above the reserve price then there is no transaction.

A strategy for a seller is a pair of functions: a list price function and a bid function. The list price function maps from $\{L, H\}$ to $\{l, h\}$ and thus has a seller select a list price based on its cost type. In the event a seller is selected by a buyer, a bid function assigns a bid depending on the seller’s cost type, the seller’s cost, the other seller’s list price, and whether the buyer selected one or two sellers. The weakly dominant bidding strategy for a seller is to bid its cost. From hereon, we will think of a strategy for a seller as a list price function and a bid function that has its bid equal to its cost. In that case, there are four strategies, associated with the four ways in which to map $\{L, H\}$ to $\{l, h\}$. For a buyer who is restricted to choosing one seller, a strategy selects a seller and a reserve price conditional on the observed list prices and the buyer’s valuation. If the buyer chooses two sellers, a strategy selects a reserve price conditional on the observed list prices and the buyer’s valuation. The solution concept is perfect Bayes-Nash equilibrium.
3 Literature Review

There is a small body of work that examines firms setting list prices and offering discounts. Chen and Rosenthal (1996), García Díaz, Hernán González, and Kujal (2009), Raskovich (2007), and Lester, Visschers, and Wolloh (2015) have sellers post a list price which is subsequently followed by either discounts or negotiation. Those papers do not consider collusion and the driving forces to their analyses are distinct from that which operates in our model. Gill and Thanassoulis (2016) is the only paper to consider collusion but it assumes firms coordinate on both list and discounted prices. There is no previous research for which firms collude on list prices and compete in discounts or on negotiated prices.

This paper also relates to research on directed search in a market setting. Directed search is present in our model in that list prices may direct buyers to negotiate with certain sellers. One strand of literature concerns indicative bidding which is a two-stage auction process commonly used in the sales of business assets with very high values. In the first stage, potentially interested buyers submit non-binding bids. These bids are meant to be indicative of bidders' interest in the item for sale. The highest of these non-binding bids, in conjunction with decisions regarding bidders' qualifications, are then used to establish a short list of final (second-stage) bidders who participate in a first-price sealed-bid auction. Ye (2007) provides the first study of indicative bidding and allows for costly information acquisition. The analysis suggests that the current design of indicative bidding cannot reliably select the most qualified bidders for the final sale, as there does not exist a symmetric, strictly increasing equilibrium bid function in the indicative bidding stage; hence, bidders do not truthfully reveal their types. By restricting indicative bids to a finite domain, Quint and Hendricks (2015) explicitly model indicative bidding as cheap talk with commitment, and show that a symmetric equilibrium exists in weakly-monotone strategies. But again, the highest-value bidders are not always selected, as bidder types “pool” over a finite number of bids. Our paper differs from Ye (2007) and Quint and Hendricks (2015) mainly in that no entry cost is assumed for bidders, and that
the reserve price in the final selling mechanism depends on the list prices (indicative bids). As such, a separating equilibrium in the cheap-talk stage becomes possible in our setting.

Menzio (2007) considers cheap talk in a search model of a competitive labor market. Employers have private information about the quality of their vacancies and can costlessly communicate with unemployed workers before they engage in an alternating offer bargaining game to determine the wage. It is shown that unless the labor market is either too tight or too slack, there exists an equilibrium in which noncontractual announcements (cheap talk) about compensation is correlated with actual wages and, therefore, it serves to direct the search of workers. As we explain later, our theory encompasses the driving forces in Menzio (2007) though in the context of an imperfectly competitive product market setting.

Finally, Kim and Kircher (2015) introduce cheap talk into an otherwise canonical competing auctions setting. In their model, auctioneers with private reservation values compete for bidders by announcing cheap-talk messages. They show that the choice of the trading mechanism is critical. When the first-price auction is used, there always exists an equilibrium in which each auctioneer truthfully reveals her type. When the second-price auction is used, however, no informative equilibrium exists.

4 Strategies Under Competition and Collusion

A critical element to the ensuing theory is the descriptively realistic assumption that buyers are uncertain whether sellers are competing or colluding.\(^\text{13}\) Let \(\kappa\) (for the German "kartell") denote the probability that buyers assign to firms colluding. Consistent with the low level of documented cartels, it is presumed that \(\kappa > 0\) but

\(^{13}\)That other agents - whether buyers, the competition authority, or potential entrants - are uncertain about whether market outcomes are the product of competition or collusion is assumed, for example, in Harrington (1984), Besanko, and Spulber (1989, 1990), LaCasse (1995), Schinkel and Tuinstra (2006), and Souam (2001).
small. It will become clear where we use the presumption that $\kappa$ is small.\textsuperscript{14} Buyers are assumed to live for only one period and do not observe the history.\textsuperscript{15}

Suppose firms are competing which means that their strategies must form a perfect Bayes-Nash equilibrium for the one-shot game. As this is a cheap talk game, there are always pooling equilibria which, in our setting, means uninformative list prices.\textsuperscript{16} We will focus on equilibria in which a seller’s list price is informative of its cost type. Hence, consider sellers using the separating strategy that has a low-cost (high-cost) type post a low (high) list price:

\[
\phi(t) = \begin{cases} 
  l & \text{if } t = L \\
  h & \text{if } t = H
\end{cases}
\]

(1)

Alternatively, firms are colluding which will mean that they use a pooling strategy whereby a seller announces a high list price regardless of its cost type:

\[
\psi(t) = \begin{cases} 
  h & \text{if } t = H \\
  h & \text{if } t = H
\end{cases}
\]

(2)

The value of coordinating their list prices in this manner is explained below.

Given a prior probability $\kappa$ that firms are colluding and using (2) and probability $1 - \kappa$ that firms are competing and using (1), a buyer’s beliefs as to sellers’ types

\textsuperscript{14}Obviously, evidence is based on discovered cartels and, based on that evidence, the fraction of markets with documented cartels is very small. Of course, there could be many undiscovered cartels so that the frequency of cartels is, in fact, not small. What is important for the analysis is that buyers believe cartels are infrequent which seems highly plausible for most markets.

\textsuperscript{15}Though this assumption is inconsistent with them being industrial buyers, it allows us to avoid a difficult dynamic problem. If buyers were long-lived and observed the past then they would update their beliefs over time regarding the hypothesis that there is collusion. While characterizing buyers’ beliefs over time is not a problem in and of itself, colluding sellers would take into account how their current actions (both with regards to list prices and bids) impacts buyers’ beliefs and the future value of collusion. Thus, it now becomes a dynamic game between buyers and sellers. That is clearly a setting worth examining but is one we leave to future research.

\textsuperscript{16}With those equilibria, a seller’s strategy is to choose $h$ with some probability $s \in [0, 1]$ if type $L$ or $H$, and then bid its realized cost if selected by a buyer. A buyer’s beliefs on a seller’s type are the prior beliefs: If $l$ or $h$ is observed then a seller is type $L$ with probability $q$. A buyer chooses an optimal reserve price based on those prior beliefs.
given their list prices can be derived. When buyers observe either or both sellers posting a low list price, they infer that firms are competing. Letting $m_i$ denote the message and $t_i$ denote the type of firm $i$, respectively, posterior beliefs (conditional on list prices) are:

- If $(m_1, m_2) = (l, l)$ then firms are competing and $\Pr(t_i = L | (m_1, m_2) = (l, l)) = 1, i = 1, 2$.
- If $(m_1, m_2) = (l, h)$ then firms are competing and $\Pr(t_1 = L | (m_1, m_2) = (l, h)) = 1, \Pr(t_2 = H | (m_1, m_2) = (l, h)) = 1$.
- If $(m_1, m_2) = (h, l)$ then firms are competing and $\Pr(t_1 = H | (m_1, m_2) = (h, l)) = 1, \Pr(t_2 = L | (m_1, m_2) = (h, l)) = 1$.

However, when buyers observe both sellers posting high list prices, they do not know whether sellers are competing (and are high-cost types) or are colluding. Bayesian updating implies:

$$\Pr(t_i = L | (m_1, m_2) = (h, h)) = \frac{\kappa q}{\kappa + (1 - \kappa)(1 - q)}, \quad \Pr(t_i = H | (m_1, m_2) = (h, h)) = \frac{1 - q}{\kappa + (1 - \kappa)(1 - q)}, \quad i = 1, 2.$$  (3)

With these beliefs on sellers’ types, the next step is to derive a buyer’s optimal reserve price conditional on sellers’ list prices and how many sellers are approached. Let $R^w_{m_1, m_2}(v)$ denote the optimal reserve price when a buyer’s valuation is $v$, lists prices are $(m_1, m_2)$, and the buyer approaches $w$ sellers. If $(m_1, m_2) \in \{(l, l), (l, h), (h, l)\}$ then sellers are inferred to be competing in which case a seller’s list price fully reveals its type. When a buyer approaches only one seller, she will randomly choose a seller when $(m_1, m_2) = (l, l)$ and choose the seller with the low list price when $(m_1, m_2) \in \{(l, h), (h, l)\}$. Hence, in all cases, a buyer’s beliefs on the seller’s cost (and bid) is $F_L$. It follows that the optimal reserve price is:

$$R^1_{m_1, m_2}(v) \equiv \arg \max (v - R) F_L (R), \quad \forall (m_1, m_2) \in \{(l, l), (l, h), (h, l)\}. \quad (4)$$
If a buyer instead solicits bids from two sellers, she infers the sellers’ types are \((\phi^{-1}(m_1), \phi^{-1}(m_2))\) where recall \(\phi\) is a seller’s strategy under competition. Therefore,

\[
R^2_{m_1m_2}(v) = \arg \max_R \int_R^{R} \int_R^{R} (v-c_2) dF_{\phi^{-1}(m_2)} (c_2) dF_{\phi^{-1}(m_1)} (c_1) \\
+ \int_R^{R} \int_R^{R} (v-c_1) dF_{\phi^{-1}(m_1)} (c_1) dF_{\phi^{-1}(m_2)} (c_2) \\
+ (v-R) \left[ (1-F_{\phi^{-1}(m_2)} (R)) F_{\phi^{-1}(m_1)} (R) + (1-F_{\phi^{-1}(m_1)} (R)) F_{\phi^{-1}(m_2)} (R) \right] .
\] (5)

Now suppose \((m_1, m_2) = (h, h)\) so buyers remain uncertain regarding whether firms are competing or colluding. Given posterior beliefs (3) as to a seller’s type, a buyer believes a seller chooses its cost according to the mixture cdf \(F_\kappa\):

\[
F_\kappa \equiv \left( \frac{\kappa q}{\kappa + (1-\kappa)(1-q)} \right) \circ F_L + \left( \frac{1-q}{\kappa + (1-\kappa)(1-q)} \right) \circ F_H .
\]

It follows that:

\[
R^1_{hh}(v) \equiv \arg \max_R (v-R) F_\kappa (R) ,
\] (6)

and

\[
R^2_{hh}(v) \equiv \arg \max_R \int_{c_L}^{R} \int_{c_1}^{R} (v-c_2) dF_\kappa (c_2) dF_\kappa (c_1) \\
+ \int_{c_L}^{R} \int_{c_2}^{R} (v-c_1) dF_\kappa (c_1) dF_\kappa (c_2) \\
+ (v-R) 2 (1-F_\kappa (R)) F_\kappa (R) .
\] (7)

where this expression uses the assumption \(c_L \leq c_H\).

When a buyer approaches one seller, Lemma 1 shows that the optimal reserve price is higher when both sellers post high list prices (and thus may be colluding) than when one or both sellers post a low list price (in which case sellers are competing and a seller with a low list price is inferred to have a low-cost distribution).

**Lemma 1** \(R^1_{hh}(v) > R^1_{ll}(v) (= R^1_{lh}(v)), \forall v.\)
For when a buyer approaches both sellers, Lemma 2 shows that the optimal reserve price is increasing in how many sellers posted high list prices. This result does require that the probability of colluding κ is not too high. Otherwise, depending on the prior beliefs on sellers’ cost types (i.e., the value of q), it is possible that \( R_{hh}^2 (v) < R_{lh}^2 (v) \).\(^{17}\) However, \( R_{hh}^2 (v) > R_{lh}^2 (v) > R_{ll}^2 (v) \) regardless of κ.

**Lemma 2** If κ is sufficiently small then \( R_{hh}^2 (v) > R_{lh}^2 (v) > R_{ll}^2 (v) \), ∀v.

### 5 Competition

In determining when a separating equilibrium (under competition) exists, the analysis will examine when \( b = 1 \) (all buyers negotiate with one seller), \( b = 0 \) (all buyers negotiate with both sellers), and finally the general case of \( b \in [0, 1] \).

#### 5.1 All Buyers Negotiate with One Seller

Suppose \( b = 1 \) so that all buyers approach only one seller. Let us derive the conditions for sellers’ competitive strategy (1) to be a perfect Bayes-Nash equilibrium. We have already dealt with a buyer’s beliefs and strategy and just need to derive conditions for a seller’s strategy to be optimal.

A low-cost type seller prefers to choose \( l \) (as prescribed by the competitive strategy) and signal it is a low-cost type if and only if

\[
\left( \frac{q}{2} + 1 - q \right) \int_{v_L}^{v} \int_{c_L}^{R_{lh}^1 (v)} \left( R_{lh}^1 (v) - c \right) dF_L (c) dG (v) \geq \left( 1 - \frac{q}{2} \right) \int_{v_L}^{v} \int_{c_L}^{R_{hh}^1 (v)} \left( R_{hh}^1 (v) - c \right) dF_L (c) dG (v). \tag{8}
\]

On the LHS of the inequality is the payoff from choosing \( l \) (and note that it uses the property \( R_{ll}^1 (v) = R_{lh}^1 (v) \)). A seller posting \( l \) is chosen for sure by the buyer when

\(^{17}\)For example, when \( κ = 1 \), \( R_{hh}^2 (v) \) is based on each seller having a low-cost distribution with probability \( q \). In comparison, \( R_{hh}^2 (v) \) is based on one seller having a low-cost distribution for sure and the other seller having a high-cost distribution for sure. The relationship between reserve prices is ambiguous.
the other seller posted $h$, which occurs when the other seller is type $H$ (and that
occurs with probability $1 - q$); and is chosen with probability 1/2 when the other
seller posted $l$, which occurs when the other seller is type $L$ (and that occurs with
probability $q$). Thus, a seller who chooses a low list price is approached by a buyer
with probability $q/2 + 1 - q$. In that case, the buyer offers a price of $R_{lh}^1(v)$ and the
seller accepts the offer if its realized cost is less than $R_{lh}^1(v)$. If the seller selects a
high list price then it is approached by the buyer with probability 1/2 in the event
that the other seller also posted a high list price, and is not approached when the
other seller posted a low list price. Hence, a seller with list price $h$ assigns probability
$(1 - q)/2$ to being approached by a buyer and, in that situation, is offered $R_{hh}^1(v)$.

If instead a seller is a high-cost type then it prefers to choose $h$ if and only if
\[
\left( \frac{1 - q}{2} \right) \int_{v}^{\infty} \int_{c_{H}}^{R_{hh}^1(v)} \left( R_{hh}^1(v) - c \right) dF_H(c) dG(v) \geq \left( \frac{q}{2} + 1 - q \right) \int_{v}^{\infty} \int_{c_{H}}^{R_{hh}^1(v)} \left( R_{hh}^1(v) - c \right) dF_H(c) dG(v). \]

The expressions are the same as in (8) except that the inequality is reversed and the
cost distribution is $F_H$ instead of $F_L$. Sufficient conditions are later provided such
that (8)-(9) hold so that a separating equilibrium exists when firms compete.

When a buyer selects one seller with which to negotiate, a seller’s list price plays
two roles. First, it affects the likelihood that a seller is selected by a buyer. By setting
a low list price $l$, a seller is selected with probability $1 - (q/2)$, while the probability
is only $(1 - q)/2$ if it chooses a high list price $h$. This effect is referred to as the
inclusion effect in that a lower list price makes it more likely a buyer includes a seller
in the negotiation process. A low list price signals a seller has a low-cost distribution
in which case it is more likely to accept the buyer’s offer (i.e., it is more likely to draw
a cost below the buyer’s offer so that the seller accepts the offer). The inclusion effect
makes posting a low list price attractive because it induces more buyers to approach a
seller and thereby results in more sales. However, there is a countervailing effect from
a seller posting a low list price which is that a buyer negotiates more aggressively
knowing it is more likely the seller’s cost is low given its list price revealed it is a
low-cost type. Referred to as the *bargaining effect*, it manifests itself by a buyer making a lower offer (in the form of a lower reserve price) in response to a low list price compared to a high list price: \( R_{ll}^1(v) < R_{hh}^1(v) \). Though not labelling them as such, the inclusion and bargaining effects are present in Menzio (2007) in the context of a labor market with search.

In sum, posting a low list price makes it more likely that a buyer negotiates with a seller but then the buyer will demand a lower price in those negotiations. As later shown, a separating equilibrium can exist – and therefore list prices are informative – because only the firm with the low-cost distribution finds it profitable to accept a weaker bargaining position in exchange for attracting more buyers.\(^{18}\)

### 5.2 All Buyers Negotiate with Both Sellers

When all buyers approach both sellers \((b = 0)\), separating equilibria do not exist. The expected profit per buyer to a seller of type \(t_1\) whose list price is \(m_1\) (and thus inferred to be \(\phi^{-1}(m_1)\)) when the other seller’s type and list price are \(t_2\) and \(m_2\), respectively, is

\[
B(m_1, t_1; m_2, t_2) = \int_{c_1}^{\sigma} \int_{c_2}^{\sigma} \int_{\xi_1}^{\xi_2} \left( \min \left\{ R_{m_1m_2}^2(v), c_2 \right\} \right) \left( \min \left\{ R_{m_1m_2}^2(v), c_2 \right\} - c_1 \right) \times dF_{t_1}(c_1) dF_{t_2}(c_2) dG(v).
\]

Recall that a buyer’s optimal reserve price is \(R_{m_1m_2}^2(v)\) given list prices \(m_1\) and \(m_2\). If seller 1’s bid (= cost) is less than \(\min \left\{ R_{m_1m_2}^2(v), c_2 \right\}\) then a buyer with valuation

\(^{18}\)It is also worth noting that signalling here is related to that found in Farrell and Gibbons (1989). They consider a bargaining setting for which one might expect cheap talk to be uninformative because informative equilibria do not exist in games with pure conflict (Crawford and Sobel, 1982). However, two sides of a bargaining situation actually have a mutual interest in communicating that each believes bargaining can result in a Pareto-improving allocation and thus it is worthwhile to bargain. In our setting, a buyer and a seller have a mutual interest of conveying information to enhance the chances that a Pareto-improving sale takes place.
v buys from seller 1 and pays a price equal to \( \min \{ R_{m_1 m_2}^2 (v), c_2 \} \). Hence, the probability that seller 1 makes a sale is weakly increasing in the reserve price \( R_{m_1 m_2}^2 (v) \), as is the profit conditional on making a sale which equals \( \min \{ R_{m_1 m_2}^2 (v), c_2 \} - c_1 \).

For realizations of \( c_2 \) and \( v \) such that \( R_{m_1 m_2}^2 (v) < c_2 \), both are strictly increasing in the reserve price. \( B(m_1, t_1; m_2, t_2) \) is then increasing in the reserve price.

If seller 2 uses (1) then seller 1’s expected payoff from list price \( m_1 \) is

\[
qB(m_1, t_1; l, L) + (1 - q) B(m_1, t_1; h, H).
\]

Given \( B(m_1, t_1; m_2, t_2) \) is increasing in the reserve price, Lemma 2 implies

\[
qB(h, t_1; l, L) + (1 - q) B(h, t_1; h, H) > qB(l, t_1; l, L) + (1 - q) B(l, t_1; h, H), \quad t_1 \in \{ L, H \}.
\]

A seller then prefers to post a high list price regardless of its type. Hence, a separating equilibrium does not exist.

With buyers approaching both sellers, a seller’s list price does not affect the probability of being selected – so there is no inclusion effect – but it does affect how aggressively a buyer negotiates. A seller will always want to signal it is more likely to have a high-cost distribution because it induces a buyer to set a higher reserve price. Compared to posting a low list price, a high list price results in a reserve price of \( R_{hh} (v) \) instead of \( R_{lh} (v) \) (when the other seller is a high cost-type) and \( R_{hl} (v) \) instead of \( R_{ll} (v) \) (when the other seller is a low cost-type). Given that seller 2’s bid is not affected by seller 1’s list price, seller 1’s expected profit is then always higher by posting a high list price. When all buyers negotiate with both sellers, list prices are then uninformative.19

\[19\text{By a similar argument, one can show that semi-pooling equilibria do not exist.}\]

### 5.3 General Case

Thus far, it has been shown that a separating equilibrium may exist when \( b = 1 \), and only pooling equilibria exist when \( b = 0 \). List prices can be informative when they influence a buyer’s decision as to which seller to approach to negotiate a deal. When buyers negotiate with a single seller and a low list price signals a low-cost
distribution, a buyer will choose the seller with the lowest list price because it knows it can expect to negotiate a lower price. Furthermore, only a low-cost firm is willing to post a low list price, and thereby accept a weaker bargaining position, in order to have a higher chance of a buyer approaching it.

Now let us show that if a separating equilibrium exists when \( b = 1 \) then there exists \( b^* \in (0, 1) \) such that a separating equilibrium exists if and only if \( b \in [b^*, 1] \).

Define \( A(m_1, t_1; m_2) \) to be the expected profit to a seller of type \( t_1 \) whose list price is \( m_1 \) when the other seller’s list price is \( m_2 \) and a buyer approaches only that seller:

\[
A(m_1, t_1; m_2) = \int_{v}^{\pi} \int_{\xi}^{\eta} R_{m_1 m_2}^1(v) (R_{m_1 m_2}^1(v) - c) dF_{t_1}(c) dG(v). \tag{11}
\]

When the seller posts a low list price, its expected payoff is independent of the other seller’s list price as buyers believe firms are competing: \( A(l, t_1; l) = A(l, t_1; h) \).

However, when the seller’s list price is high then the payoff does depend on the other seller’s list price, for if it is low then buyers believe sellers are competing and when it is high then buyers are uncertain about whether they face competition or collusion: \( A(h, t_1; l) \neq A(h, t_1; h) \).

When it sets its list price, a seller knows that a fraction \( b \) of buyers will approach one seller (and will choose the one with the lowest list price) and a fraction \( 1 - b \) will approach both sellers. In that case, a type \( L \) seller optimally chooses list price \( l \) if and only if

\[
W(l, L, b) \equiv b \left[ \left( \frac{q}{2} \right) A(l, L; l) + (1 - q)A(l, L; h) \right] + (1 - b) \left[ qB(l, L; l, L) + (1 - q)B(l, L; h, H) \right] \geq b \left( \frac{1 - q}{2} \right) A(h, L; h) + (1 - b) \left[ qB(h, L; l, L) + (1 - q)B(h, L; h, H) \right] \equiv W(h, L, b). \tag{12}
\]

A type \( H \) seller optimally chooses list price \( h \) if and only if

\[
W(h, H, b) \equiv b \left( \frac{1 - q}{2} \right) A(h, H; h) + (1 - b) \left[ qB(h, H; l, L) + (1 - q)B(h, H; h, H) \right] \tag{13}
\]
\[ \geq b \left[ \left( \frac{q}{2} \right) A(l, H; l) + (1 - q)A(l, H; h) \right] + (1 - b) [qB(l, H; l, L) + (1 - q)B(l, H; h, H)] \]
\[ \equiv W(l, H, b) \]

From Section 5.2, if \( b = 0 \) then (12) does not hold (as a type \( L \) seller prefers to choose list price \( h \)) though (13) does hold. Suppose that (12)-(13) are satisfied when \( b = 1 \). Combining these conditions for \( b = 0 \) and \( b = 1 \) delivers:

\[ W(l, L, 1) - W(h, L, 1) > 0 > W(l, L, 0) - W(h, L, 0) \] (14)
\[ W(h, H, 1) - W(l, H, 1) > 0 > W(l, H, 0) - W(h, H, 0) \]

By the linearity of the conditions in (14) with respect to \( b \), it follows that there exists \( b^* \in (0, 1) \) such that (12)-(13) hold if and only if \( b \in [b^*, 1] \). We have then proven:\(^{21}\)

**Theorem 3** If \( \kappa \) is sufficiently small and a separating equilibrium exists for \( b = 1 \) then \( \exists b^* \in (0, 1) \) such that a separating equilibrium exists if and only if \( b \in [b^*, 1] \).

Setting \( b = 1 \) and using (11) in (12)-(13), those conditions can be re-arranged to conclude that a separating equilibrium exists if and only if \( q \in [\underline{q}, \overline{q}] \):

\[ \int_{\underline{w}}^{\overline{w}} \int_{\underline{L}}^{\overline{L}} R_{hh}(v) (R_{hh}(v) - c) dF_L(c) dG(v) - 2 \int_{\underline{w}}^{\overline{w}} \int_{\underline{L}}^{\overline{L}} R_{hl}(v) (R_{hl}(v) - c) dF_L(c) dG(v) \]
\[ \int_{\underline{w}}^{\overline{w}} \int_{\underline{L}}^{\overline{L}} R_{hl}(v) (R_{hl}(v) - c) dF_L(c) dG(v) - \int_{\underline{w}}^{\overline{w}} \int_{\underline{L}}^{\overline{L}} R_{hl}(v) (R_{hl}(v) - c) dF_L(c) dG(v) \]
\[ \equiv \int_{\underline{w}}^{\overline{w}} \int_{\underline{L}}^{\overline{L}} R_{hh}(v) (R_{hh}(v) - c) dF_H(c) dG(v) - 2 \int_{\underline{w}}^{\overline{w}} \int_{\underline{L}}^{\overline{L}} R_{hl}(v) (R_{hl}(v) - c) dF_H(c) dG(v) \]
\[ \int_{\underline{w}}^{\overline{w}} \int_{\underline{L}}^{\overline{L}} R_{hl}(v) (R_{hl}(v) - c) dF_H(c) dG(v) - \int_{\underline{w}}^{\overline{w}} \int_{\underline{L}}^{\overline{L}} R_{hl}(v) (R_{hl}(v) - c) dF_H(c) dG(v) \]
\[ \equiv q \leq q \leq \overline{q} \] (15)

If the probability that the other seller is a low-cost type is too low \( (q < \underline{q}) \) then a low-cost seller prefers to post a high list price in order to induce buyers to set higher reserve price. If the probability that the other seller is a low-cost type is too high \( (q > \overline{q}) \) then a high-cost seller prefers to post a low list price in order to attract more buyers.

Let us derive some sufficient conditions on primitives for (15) to hold. First note that \( q < \underline{q} \) when the optimal reserve price is an interior solution so \( R_{ll}(v) > \underline{c}_L \) for

\(^{21}\)Note that \( \kappa \) is required to be sufficiently small in order for the relationship between reserve prices in Lemma 2 to hold.
some $v$. A sufficient condition for $R_H(v) > c_L$ for some $v$ is $c_L < \bar{v}$. Next note that if $R_H(v) \leq c_H \forall v$ and $c_L < R_{hh}(v)$ for some $v$ then

$$\int_{\bar{v}}^{v} \int_{\xi_H}^{R_{hh}(v)} (R_{hh}(v) - c) dF_H(c) dG(v) > 0 = \int_{\bar{v}}^{v} \int_{\xi_H}^{R_H(v)} (R_H(v) - c) dF_H(c) dG(v)$$

which implies $\bar{q} = 1$. If $c_H < \bar{v}$ then $c_H < R_{hh}(v)$ for some $v$. If $\bar{c}_L < c_H$ (or instead $F_L$ puts sufficient mass below $c_H$) then $R_H(v) \leq c_H \forall v$. (The latter condition is admittedly strong but it is not necessary for existence.) In sum, these conditions give us: $q < 1 = \bar{q}$. Hence, if the probability a seller is a low cost type is sufficiently high then it is an equilibrium for a competitive firm’s list price to signal its cost type.

In concluding this section, let us present a seller’s expected profit under competition prior to learning its type:

$$E[\bar{\pi}_{\text{comp}}] \equiv b \left[ q^2 (1/2)A(l, L; l) + q(1 - q)A(l, L; h) + (1 - q)^2 (1/2)A(h, H; h) + (1 - b)q^2 B(l, L; L) + q(1 - q)B(l, L; H) + q(1 - q)B(h, H; L) + (1 - q)^2 B(h, H; H) \right].$$

The first bracketed expression pertains to the fraction $b$ of buyers that negotiate with only one seller. With probability $q$, the seller is low cost and chooses list price $l$ which signals to buyers it has a low cost distribution. From these buyers, it will attract half of them when the other seller also posts a low list price (which occurs with probability $q$) and all of them when the other seller’s list price is high (which occurs with probability $1 - q$). In that case, the expected profit earned on each buyer is $A(l, L; l) (= A(l, L; h))$. Now suppose this seller is a high-cost type, which occurs with probability $1 - q$, and thereby chooses list price $h$. For the buyers who approach only one seller, the seller will not attract any of them when the other seller posts a low price and will get half of them when the other seller posts a high list price. A high list price then attracts, in expectation, $(1 - q)/2$ of those buyers, and the seller

---

$^{22}$ That condition implies $\int_{\bar{v}}^{v} \int_{\xi_L}^{R_H(v)} (R_H(v) - c) dF_L(c) dG(v) > 0$ and that the denominator of $q$ is positive. Hence, the numerator is either positive (and less than the denominator) or negative.
earns expected profit of $A(h, H; h)$ per buyer. The second bracketed expression in (16) is the expected profit coming from the fraction $1 - b$ of buyers who negotiate with both sellers where $B(m_1, t_1; m_2, t_2)$ is weighted by the probability that sellers are types $(t_1, t_2)$.\textsuperscript{23}

6 Collusion

After evaluating when coordination on list prices is profitable in Section 6.1, we turn to examining the conditions under which collusion is supported by strategies in an infinitely repeated game in Section 6.2.

6.1 Profitability of Coordinating on List Prices

Expected profit to a seller from using the collusive strategy (2) is

$$E \left[ \pi^{\text{coll}} \right] = b \left[ q(1/2)A(h, L; h) + (1 - q)(1/2)A(h, L; h) \right] + (1 - b) \left[ q^2 B(h, L; h, L) + q(1 - q)B(h, L; h, H) + q(1 - q)B(h, H; h, L) + (1 - q)^2 B(h, H; h, H) \right].$$

For the fraction $b$ of buyers who deal with one seller, each seller will end up negotiating with half of them and earn expected profit per buyer of $A(h, L; h)$ when it is a low-cost type and $A(h, H; h)$ when it is a high-cost type. For the fraction $1 - b$ of buyers who bargain with both sellers, a seller earns $B(h, t_1; h, t_2)$ per buyer when its type is $t_1$ and the other seller’s type is $t_2$.

\textsuperscript{23}It is worth noting that it is critical for the existence of a separating equilibrium that a seller has incomplete information on its cost when it chooses its list price. To see why, consider a buyer who has selected one seller and assume $\kappa = 0$ (though the argument does not rest on that assumption). Suppose that a seller’s type was its cost rather than the distribution from which it chooses cost, and that it chose a list price after learning its cost. Consider two cost types, $c' < c''$, and a seller separating strategy such that it chooses list price $l$ when its cost is $c'$ and list price $h$ when its cost is $c''$. A buyer will then optimally set a reserve price of $c'$ if the list price was $l$ and $c''$ if the list price was $h$. As then a seller’s profit is zero, a type $c'$ would do better to announce a list price $h$ so a buyer infers its cost is $c''$ which results in an expected profit of $((1 - q)/2)(c'' - c') > 0$. Thus, there is no separating equilibrium as a low-cost seller would want to mimic a high-cost seller.
Subtracting (16) from (17) and re-arranging, the incremental profit from collusion is:

\[ E \left[ \pi^{\text{coll}} \right] - E \left[ \pi^{\text{comp}} \right] \]

\[ = \begin{bmatrix}
\left( \frac{q}{2} \right) A(h, L; h) + \left( \frac{1-q}{2} \right) A(h, H; h) \\
- \left( \frac{q^2}{2} \right) A(l, L; l) - q(1-q)A(l, L; h) - \left( \frac{(1-q)^2}{2} \right) (1/2) A(h, H; h)
\end{bmatrix}
\]

\[ + (1-b) \{ q^2 [B(h, L; h, L) - B(l, L; L)] + q(1-q) [B(h, L; h, H) - B(l, L; h, H)] \\
+ q(1-q) [B(h, H; h, L) - B(h, H; L, H)] + (1-q)^2 [B(h, H; h, H) - B(h, H; h, H)] \} \}

Consider the first bracketed term of \( E \left[ \pi^{\text{coll}} \right] - E \left[ \pi^{\text{comp}} \right] \) which is the profit differential (per buyer) associated with the fraction \( b \) of buyers who approach one seller. Re-arranging that term yields

\[ \left( \frac{q^2}{2} \right) [A(h, L; h) - A(l, L; l)] + \left( \frac{q(1-q)}{2} \right) [A(h, L; h) - A(l, L; h)] \]

\[ + \left( \frac{q(1-q)}{2} \right) [A(h, H; h) - A(l, L; h)] \]

When both sellers are high-cost types then, whether colluding or not, they post high list prices. Given expected profit is the same under collusion and competition, there is no term in (19) corresponding to the event when both are high-cost types. The first term in (19) pertains to when both sellers are low-cost types which occurs with probability \( q^2 \). In that case, a seller attracts half of the buyers under both collusion and competition, and makes additional expected profit per buyer under collusion equal to

\[ A(h, L; h) - A(l, L; h) \]

\[ = \int_v \int_{R_{hl}^1(v)} (R_{hh}^1(v) - R_{ll}^1(v)) dF_L(c) dG(v) + \int_v \int_{R_{hl}^1(v)} (R_{hh}^1(v) - c) dF_L(c) dG(v) . \]

The first term in (20) is when the seller’s cost is less than \( R_{hl}^1(v) \). As collusion has both sellers setting a high list price rather than a low list price, a seller ends up selling at \( R_{hh}^1(v) \) instead of \( R_{ll}^1(v) \). Because buyers set a higher reserve price compared to when firms compete in their list prices, the seller earns higher profit of \( R_{hh}^1(v) - R_{ll}^1(v) \) conditional on selling, which we refer to as the price-enhancing effect. The second
term in (20) is when the seller’s cost lies in \( [R^1_{lh}(v), R^1_{hh}(v)] \). Setting a low list price under competition would result in not making a sale because the seller’s bid (which equals its cost) would exceed the buyer’s reserve price of \( R^1_{lh}(v) \). In contrast, under collusion, sellers post high list prices which induces a buyer to set the higher reserve price of \( R^1_{hh}(v) \). Thus, collusion produces profit of \( R^1_{hh}(v) - c \), while competition would have yielded zero profit. Interestingly, collusion allows a Pareto-improving transaction to take place that would not have occurred under competition because collusion causes buyers to bargain less aggressively. This effect we refer to as the \textit{transaction-enhancing effect.}

Next consider when the seller is a low-cost type and the other seller is a high-cost type. Under competition, the seller attracts all buyers and earns \( A(l, L; h) \) per buyer, while under collusion it earns a higher profit per buyer of \( A(h, L; h) \) but only attracts half of the buyers. The second term in (19) captures the half of the market that the seller attracts under both collusion and competition. On those buyers, the profit per buyer is higher by \( A(h, L; h) - A(l, L; h) \), and the associated profit gain is \( b(1/2) [A(h, L; h) - A(l, L; h)] \). However, this gain is offset by an expected loss of \( b(1/2)A(l, L; h) \) corresponding to the half of buyers who no longer solicit a bid from the seller under collusion. That profit loss appears in the third term in (19). But the seller gets those lost buyers back when the tables are turned and it is now a high-cost type and the other seller is a low-cost type. In that event, it would not have attracted any buyers under competition but gets half of the buyers under collusion and earns expected profit of \( b(1/2)A(h, H; h) \). That profit gain is also in the third term in (19). Hence, the net profit impact is \( b(1/2) [A(h, H; h) - A(l, L; h)] \), which gives us the third term in (19). Referred to as the \textit{business-shifting effect}, it is the change in profit associated with half of the buyers no longer soliciting a bid from a firm when it is a low-cost type (under competition) and now soliciting a bid when it is a high-cost type (under collusion). This profit change could be positive or negative. While, ceteris paribus, it is better for a seller to attract a buyer when it is a low-cost type, the buyer’s reserve price is lower. If the third term is non-negative then (19) is
positive which means collusion increases expected profit earned on buyers who solicit one offer. If the third term is negative then the sign of (19) is ambiguous.

Returning to the incremental profit from collusion in (18), the second bracketed expression pertains to the $1 - b$ fraction of buyers who solicit bids from both sellers. $B(h, t_1; h, t_2) - B(\phi(t_1), t_1; \phi(t_2), t_2)$ is the difference in expected profit per buyer for a type $t_1$ seller under collusion and under competition. It can be shown that

$$B(h, t_1; h, t_2) - B(\phi(t_1), t_1; \phi(t_2), t_2) = \int_v \int_v R_{hh}^2(v) R_{\phi(t_1)\phi(t_2)}^2(v) \left( c_2 - R_{\phi(t_1)\phi(t_2)}^2(v) \right) dF_t \left( c_1 \right) dF_t \left( c_2 \right) dG(v)$$

$$+ \int_v \int_v R_{hh}^2(v) \int_v R_{\phi(t_1)\phi(t_2)}^2(v) c_2 \left( c_2 - c_1 \right) dF_t \left( c_1 \right) dF_t \left( c_2 \right) dG(v)$$

$$+ \int_v \int_v R_{hh}^2(v) \int_v R_{\phi(t_1)\phi(t_2)}^2(v) \left( R_{hh}^2(v) - R_{\phi(t_1)\phi(t_2)}^2(v) \right) dF_t \left( c_1 \right) dF_t \left( c_2 \right) dG(v)$$

$$+ \int_v \int_v R_{hh}^2(v) \int_v R_{\phi(t_1)\phi(t_2)}^2(v) \left( R_{hh}^2(v) - c_1 \right) dF_t \left( c_1 \right) dF_t \left( c_2 \right) dG(v).$$

When $(t_1, t_2) = (H, H)$, all four terms are zero because, whether colluding or competing, they choose high list prices so the outcome is the same. For any other type pairs, each of these four terms is positive as long as $\kappa$ is sufficiently small so that $R_{hh}^2(v) > R_{hh}^2(v)$. The first and third terms are driven by the price-enhancing effect: Collusion raises the buyer’s reserve price which increases the price seller 1 receives from $R_{\phi(t_1)\phi(t_2)}^2(v)$ to $c_2$ (in the first term) and to $R_{hh}^2(v)$ (in the third term). The second and fourth terms capture the transaction-enhancing effect: By inducing the buyer to have a higher reserve price of $R_{hh}^2(v)$, seller 1 sells for a price of $c_2$ (in the second term) and $R_{hh}^2(v)$ (in the fourth term). There is no business-shifting effects given that these buyers solicit bids from both sellers. Coordination on list prices then always increases profits from buyers who solicit bids from both sellers.

$E \left[ \pi^{coll} \right] - E \left[ \pi^{comp} \right]$ is a weighted average of (21) with weight $1 - b$, which was just shown to be positive (as long as $\kappa$ is sufficiently small), and (19) with weight $b$, for which the sign is ambiguous. It then follows that if $E \left[ \pi^{coll} \right] - E \left[ \pi^{comp} \right] > 0$ for $b = 1$ then $E \left[ \pi^{coll} \right] - E \left[ \pi^{comp} \right] > 0$ for all values of $b$. $E \left[ \pi^{coll} \right] - E \left[ \pi^{comp} \right] > 0$ for
\( b = 1 \) if and only if (19) is positive, which is true if \( q \) is sufficiently close to 1. In sum, if \( \kappa \) is sufficiently small and \( q \) is sufficiently close to 1 then collusion is more profitable than competition for all values of \( b \).

While the focus has been on evaluating the gain in profit from collusion, it is worth noting that the preceding analysis raises the possibility that collusion could also raise expected total surplus. The transaction-enhancing effect is welfare-improving as it expands the set of Pareto-improving transactions and thereby increases both a seller’s profit and a buyer’s net surplus. The welfare effect of the business-shifting effect is ambiguous. That the cost of the seller is higher on average under collusion lowers expected surplus - both by making a transaction less likely and reducing surplus in the event of a transaction - but, holding cost constant, transactions are more likely because the buyer’s reserve price is higher. Finally, the price-enhancing effect is welfare-neutral as it is a transfer from buyers to sellers. In sum, collusion reduces the likelihood that it is the lower cost supplier that makes a sale, which reduces expected surplus, but facilitates trade by making buyers less aggressive, which raises expected surplus. The net effect on welfare depends on which effect dominates. We will return to this issue in Section 7.

6.2 Coordination on List Prices as an Equilibrium

Though list prices do not formally constrain transaction prices, coordination on high list prices influences transaction prices because it induces buyers to negotiate less aggressively as they believe sellers are more likely to have high costs. The impact on buyers’ bargaining behavior is manifested with a higher reserve price which benefits a seller in two ways. First, for those transactions that would have occurred whether firms colluded or competed, a seller receives a higher price because a buyer’s reserve price is higher. Second, the higher reserve price means that a buyer is less likely to cause bargaining to break down which implies a seller earns positive profit under collusion (and the buyer earns positive surplus) because a transaction is consummated that would not have taken place under competition.

While we have shown that coordinating on high list prices can be attractive to
sellers, it has not yet been established that it is an equilibrium for sellers to collude. A variant of the usual Folk Theorem arguments suffices to establish the stability of collusion. Suppose the situation between buyers and sellers repeats itself infinitely often and $\delta \in (0, 1)$ is the common discount factor of sellers. Each period, a seller acquires some partial private information on its cost for the upcoming period. This information acquisition is represented by a seller learning its type. With that knowledge, it then chooses its list price. Each period it receives new information about its cost which is represented by independently drawing a new type.\(^{24}\)

In order to close the model, one additional layer will be added in order to endogenize the probability that buyers attach to firms colluding, $\kappa$. Suppose there is an exogenous Markov process by which a cartel is born (so firms adopt the collusive strategy) and dies (so firms revert to using the competitive strategy). Let $f$ (for "form") denote the probability that a cartel forms out of a competitive industry, and $d$ (for "die") denote the probability that a cartel dies and transforms into a competitive industry.\(^{25}\) Assume time is $-\infty, \ldots, 0, \ldots, +\infty$ and we are at time $t = 0$. As buyers live for only one period and do not observe the history, the probability they assign to firms colluding is the steady-state probability that there is cartel, which is defined by

$$\kappa = \kappa(1 - d) + (1 - \kappa)f \iff \kappa = \frac{f}{f + d}. $$

The strategy profile for sellers is as follows. If sellers are in the competitive state then each chooses a list price according to the separating (stage game) strategy (1).

\(^{24}\)If a period is, say, a quarter then a firm knows its cost distribution for the next three months and, based on those beliefs, chooses a list price. Over the ensuing three months, a seller gets a cost draw whenever a buyer arrives at the seller and it is that cost that is relevant when bargaining with the buyer.

\(^{25}\)While it would be appealing to endogenize cartel birth and death, such a task is beyond the scope of this project. There is very little theoretical research that endogenizes cartel formation and collapse within an infinitely repeated game. With a Bertrand price game, stochastic demand can cause cartel collapse when it results in the lack of existence of collusive equilibria; see Rotemberg and Saloner (1986). That research does not model cartel formation. Harrington and Chang (2009, 2015) assume exogenous cartel birth, as done here, and endogenize cartel death with stochastic demand in the context of the Prisoners' Dilemma.
If sellers are in the cartel state then sellers choose: 1) list price $h$ (regardless of type) as described in (2) if they have always chosen list price $h$ while in the cartel state; and 2) revert to the competitive state and choose a list price according to (1). Once in the competitive state - whether due to exogenous collapse or a deviation (which will not occur in equilibrium) - firms have a probability $f$ in each period of transiting to the cartel state.$^{26}$

Let $V^{\text{coll}}$ denote the value (i.e., expected present value of profits) to a seller when in the cartel state, and $V^{\text{comp}}$ denote the value in the competitive state. They are recursively defined by:

\begin{align*}
V^{\text{coll}} &= E[\pi^{\text{coll}}] + (1 - d)\delta V^{\text{coll}} + d\delta V^{\text{comp}} \quad (22) \\
V^{\text{comp}} &= E[\pi^{\text{comp}}] + (1 - f)\delta V^{\text{comp}} + f\delta V^{\text{coll}}. \quad (23)
\end{align*}

Solving (22)-(23) and simplifying yields

\begin{align*}
V^{\text{coll}} &= \frac{(1 - (1 - f)\delta) E[\pi^{\text{coll}}] + d\delta E[\pi^{\text{comp}}]}{(1 - \delta)(1 - \delta(1 - d - f))} \quad (24) \\
V^{\text{comp}} &= \frac{(1 - \delta(1 - d)) E[\pi^{\text{comp}}] + f\delta E[\pi^{\text{coll}}]}{(1 - \delta)(1 - \delta(1 - d - f))} \quad (25)
\end{align*}

Using (24)-(25) and simplifying, the incremental value to being in the cartel state is:

\begin{equation}
V^{\text{coll}} - V^{\text{comp}} = \frac{E[\pi^{\text{coll}}] - E[\pi^{\text{comp}}]}{1 - \delta(1 - d - f)}. \quad (26)
\end{equation}

Given the strategy for the infinitely repeated game, the equilibrium conditions for firms to collude are:

\begin{equation}
b \left( \frac{1}{2} \right) A(h,t;h) + (1 - b) \left[ qB(h,t;h,L) + (1 - q)B(h,t;h,H) \right] + \delta \left( (1 - d)V^{\text{coll}} + dV^{\text{comp}} \right) \\
\geq bA(l,t;h) + (1 - b) \left[ qB(l,t;h,L) + (1 - q)B(l,t;h,H) \right] + \delta V^{\text{comp}}, \quad t \in \{L,H\}. \quad (27)
\end{equation}

$^{26}$Alternatively, we could assume that reaching the competitive state because of a deviation results in a per period probability $g$ of returning to the cartel state and allow $g$ to differ from $f$. For example, $g = 0$ captures infinite reversion to a stage game Nash equilibrium. As ensuing results are robust to $g \in [0, f]$, it is assumed $g = f$ in order to reduce notation and make for simpler expressions.
The expression on the LHS of the inequality is the payoff to posting a high list price (as prescribed by the collusive strategy), and on the RHS of the inequality is the 
payoff from instead posting a low list price. Note that when a seller deviates by 
setting a low list price, it is ensured of attracting all buyers because the other seller 
is anticipated to post a high list price. Hence, we have \( bA(l, t; h) \) on the RHS and 
\( b \left( \frac{1}{2} \right) A(h, t; h) \) on the LHS.

Rearranging (27) and substituting using (26), (27) becomes:

\[
\left( \frac{\delta(1 - d)}{1 - \delta(1 - d - f)} \right) \left( E[\pi_{\text{coll}}] - E[\pi_{\text{comp}}] \right) 
\geq b \left[ A(l, t; h) - \left( \frac{1}{2} \right) A(h, t; h) \right] 
\]

\[
+ (1 - b) \left[ q (B(l, t; h, L) - B(h, t; h, L)) + (1 - q) (B(l, t; h, H) - B(h, t; h, H)) \right].
\]

Consider the second-bracketed term on the RHS of (28) which pertains to those 
buyers who approach both sellers. This expression is the difference in expected profit per buyer between having a buyer think a seller is a low-cost type for sure and is a 
low-cost type with probability \( \frac{\kappa q}{\kappa + (1 - \kappa)(1 - q)} \). It is always negative because a buyer’s 
beliefs do not affect the bid of the other seller (recall that sellers are not coordinating 
their bids) but do affect a buyer’s reserve price. Hence, expected profit per buyer is 
lower by deviating to a low list price. Next turn to the first term on the RHS of (28), 
\( b \left[ A(l, t; h) - \left( \frac{1}{2} \right) A(h, t; h) \right] \). If a separating equilibrium is presumed to exist then, 
for a type \( H \), if \( b = 1 \) then (13) take the form:

\[
\left( \frac{1 - q}{2} \right) A(h, H; h) > \left( \frac{1 - q}{2} \right) A(l, H; h) \Longleftrightarrow 
\]

\[
\left( \frac{1}{2} \right) A(h, H; h) - A(l, H; h) > \left( \frac{q}{2} \right) [A(h, H; h) - A(l, H; h)].
\]

It can be shown that \( A(h, H; h) - A(l, H; h) > 0 \). Hence, the RHS of (29) is positive 
which implies

\[
\left( \frac{1}{2} \right) A(h, H; h) - A(l, H; h) > 0.
\]

Thus, the first bracketed term on the RHS of (28) is negative for a high-cost type. 
As the RHS of (28) is negative for a high-cost type and the LHS is positive as long 
as collusion is more profitable than competition, the equilibrium condition holds for
a high-cost type. However, for a low-cost type seller, \( \frac{1}{2} A(h, L; h) - A(l, L; h) \) need not be negative. For those buyers who negotiate with one seller, it is possible that a low-cost type seller may earn higher expected profit by posting a low list price and attracting all of them rather than setting a high list price as prescribed by the collusive strategy. For (28) to be assured of holding for a low-cost type, the LHS must then be sufficiently great.

The LHS of (28) is the difference in the future value between setting the collusive list price \( h \) and deviating with a list price \( l \). If we let the probability of cartel birth and death become very small and firms to become very patient then

\[
\lim_{d,f \to 0, \delta \to 1} \frac{\delta(1 - d)}{1 - \delta(1 - d - f)} = +\infty.
\]

Thus, as long as collusion is profitable, \( E[\pi^{\text{coll}}] > E[\pi^{\text{comp}}] \), it is an equilibrium for firms to coordinate on high list prices when cartel birth and death are sufficiently rare and firms are sufficiently patient.

Let us now pull together the analysis of Section 6. In order to produce the phenomenon of sellers coordinating their list prices and resulting in supracompetitive transaction prices, collusion must be feasible (i.e., a separating equilibrium exists, which requires (15) to hold), collusion must be profitable (i.e., \( E[\pi^{\text{coll}}] > E[\pi^{\text{comp}}] \), which requires (18) to be positive), and collusion must be stable (i.e., it is an equilibrium outcome in an infinitely repeated game, which requires (28) to hold). It was shown that: 1) collusion is feasible if \( \varepsilon_L \leq \varepsilon_H, \kappa \) is sufficiently small, \( q \) is sufficiently close to 1, and \( b \) is sufficiently close to 1; 2) collusion is profitable if \( \kappa \) is sufficiently small and \( q \) is sufficiently close to 1; and 3) collusion is stable if \( d \) and \( f \) are sufficiently small and \( \delta \) is sufficiently close to 1. Given \( \kappa = \frac{f}{d+f} \), we also need \( f \) to be sufficiently small relative to \( d \) for \( \kappa \) is to be sufficiently small; that is, cartel death is sufficiently less likely than cartel birth so that cartels are infrequent.\(^{27}\) Let us emphasize that

\(^{27}\)While admittedly speculative, the evidence suggests this assumption is plausible. A very small fraction of markets have documented cartels, which is consistent with a low value for \( f \). At the same time, the estimated annual probability of death is around 0.17 (Harrington and Wei, 2015). If a period is a quarter then this translates into \( d = 0.046 \) which is reasonably low but probably high relative to the probability of cartel formation.
these are sufficient (not necessary) conditions for firms to coordinate their list prices. In the next section, alternative conditions are provided under a particular class of distributions on costs and values.

7 Collusion for a Class of Parametric Distributions

Assume $b = 1$ (so all buyers negotiate with one seller) and $\kappa = 0$ (so the prior probability of collusion is zero). By the analysis in Section 6, the ensuing results will approximate the case when $b$ is close to one and $\kappa$ is close to zero.\(^{28}\) Suppose valuations and costs have support $[0, 1]$. Valuations are uniformly distributed: $G(v) = v$. The cdf for a low-cost type is $F_L(c) = e^c$ and for a high-cost type is $F_H(c) = e^\beta$, where $0 < \alpha < \beta$ so the inverse hazard rate ranking is satisfied: $h_L(c) = c/\alpha > c/\beta = h_H(c)$. Recall that a seller is a low-cost type with probability $q$.

**Theorem 4** Under the assumptions of Section 7, collusion is feasible if and only if

\[ q(\alpha, \beta) \equiv \frac{\beta^{\alpha+1}}{(\beta+1)^{\alpha+1}} - 2 \frac{\alpha^{\alpha+1}}{(\alpha+1)^{\alpha+1}} \leq q \leq \frac{\beta^{\beta+1}}{(\beta+1)^{\beta+1}} - 2 \frac{\alpha^{\beta+1}}{(\alpha+1)^{\beta+1}} \equiv \overline{q}(\alpha, \beta) \quad (31) \]

and is profitable if $\alpha < 1$.

Given the distributions, the necessary and sufficient conditions for a separating equilibrium described in (15) take the form in (31). It can be shown that $\alpha < \beta$ implies the RHS of (31) exceeds the LHS. For example, $[q(0.5, 2), \overline{q}(0.5, 2)] = [0.453, 0.857]$. A sufficient condition for collusion to be profitable is that the low-cost distribution is concave, $\alpha < 1$. Collusion is stable as long as $\delta$ is close to one.

For when $(\alpha, \beta) \in [0, 1] \times [0, 2]$, Figure 1 reports the range of values for $q, \overline{q}(\alpha, \beta) - q(\alpha, \beta)$, such that collusion is feasible and profitable (where the latter holds because $\alpha < 1$).\(^{30}\) Depending on the values for $(\alpha, \beta)$, there can be a wide range of values for $q$ such that firms can effectively and profitably coordinate their list prices.

\(^{28}\) If $\kappa > 0$ or $b < 1$ then there is no longer closed-form solutions for optimal reserve prices and, therefore, no closed-form solutions for $q$ and $\overline{q}$.

\(^{29}\) The proofs of all the results in this section are provided in an Online Appendix.

\(^{30}\) $q(\alpha, \beta)$ and $\overline{q}(\alpha, \beta)$ are constrained to lie in $[0, 1]$. Hence, more exactly, Figure 1 reports max $\{\min \{\overline{q}(\alpha, \beta), 1\}, 0\}$ $- \min \{\max \{q(\alpha, \beta), 0\} 1\}$. 

30
Figure 1: Range of Values for $q$ for which Collusion is Feasible and Profitable

Let us now examine whether collusion can be welfare-improving. Let $\Delta(q)$ denote the difference between expected total surplus under collusion and under competition, where its dependence on $q$ is made explicit.\footnote{In the Online Appendix, the expression for $\Delta(q)$ is provided.} For $(\alpha, \beta) \in [0,1] \times [0,2]$, Figure 2 reports the maximum welfare difference,

$$
\overline{\Delta}(\alpha, \beta) \equiv \max \{ \Delta(q) : q \in [q(\alpha, \beta), \overline{q}(\alpha, \beta)] \},
$$

and the minimum welfare difference,

$$
\underline{\Delta}(\alpha, \beta) \equiv \min \{ \Delta(q) : q \in [q(\alpha, \beta), \overline{q}(\alpha, \beta)] \}.
$$

Figure 2 shows that $\overline{\Delta}(\alpha, \beta) > 0$ for most values of $(\alpha, \beta)$ so collusion improves welfare for some values of $q$. In addition, for some values of $(\alpha, \beta)$, $\underline{\Delta}(\alpha, \beta) > 0$ so welfare is higher under collusion for all values of $q$ (for which collusion is feasible and profitable). By reducing the aggressiveness of buyers, collusion enhances the total surplus in the market by resulting in more Pareto-improving transactions and that
can more than compensate for the higher cost under collusion.

Figure 2 - Welfare Difference Between Collusion and Competition

8 Tying Theory to Cases: Antitrust Guidance

The feasibility of collusion relies on existence of a separating equilibrium under competition so that list prices are informative and, therefore, the coordination on high list
prices can distort buyers’ beliefs and thereby lead them to set higher reserve prices. Recall that critical to the existence of a separating equilibrium is the inclusion effect, for it leads a low-cost seller (but not a high-cost seller) to set a low list price in order to attract more buyers, even though those buyers bargain more aggressively. In our simple set-up, the inclusion effect is stronger when more buyers negotiate with only one of the two sellers.

For the purpose of drawing some insight into the market conditions conducive to coordination on list prices, let us extrapolate our finding by conjecturing, more generally, that the inclusion effect is stronger when buyers negotiate with a smaller fraction of sellers. As each negotiation takes time and effort, a buyer is likely to negotiate with a smaller fraction of sellers when a market has more sellers. One also expects buyers to negotiate with fewer sellers when the magnitude of the expenditure over which negotiation is taking place is smaller (in which case the extent of possible savings from negotiation are less). Thus, a market for a low-expenditure item, such as some minor raw material in an industrial buyer’s production process, which is sold by many sellers should have a strong inclusion effect. Hence, collusion is likely to be feasible. Assuming the logic of our result extends, coordination on list prices would then be an effective form of collusion in that market. In contrast, a market for a high-expenditure item, such as some complex piece of equipment, sold by a small number of sellers is a market for which the inclusion effect is weak and, therefore, coordination on list prices is not likely to be effective.

To illustrate how this intuition can be applied to assess the credibility of a claim that coordination on lists price is an effective collusive strategy, let us consider two cases: large turbine generators around 1960 and urethane around 2000. A turbine generator is a device that converts mechanical energy into electrical energy. The particular market under consideration is large turbine generators purchased by electric utilities. It is a substantial custom-made piece of equipment which could cost in excess of $10 million in the early 1960s (which is around $80 million in 2016 dol-

\[32\] For details of the turbine generators market and legal case, see Sultan (1974) and Harrington (2011).
turbine generators. In light of the item’s high expense to a buyer and the presence of only two sellers, it is quite likely that a buyer would negotiate with both sellers. In such a market, our theory suggests that list prices would be uninformative because most buyers would negotiate with both sellers. Therefore, coordinating on list prices would be an ineffective method of collusion because a seller’s list price would have little effect on a buyer’s beliefs about a seller’s cost and thus have minimal impact on bargaining and final transaction prices. While GE and Westinghouse did collude in this market, it is notable that they did so by first coordinating on a policy of not offering discounts in which case list prices became actual transaction prices. GE then acted as a price leader on list prices. Absent a move to a “no negotiation” policy, the theory of this paper suggests that coordination on list prices would have been ineffective.

Returning to a case discussed in the Introduction, the urethane market would appear to have features more conducive to coordination on list prices. Polyurethanes are used in various consumer and industrial products including mattress foams, insulation, sealants, and footwear. BASF, Bayer, Dow Chemical, Huntsman, and Lyondell either pled guilty or were convicted of coordinating their list prices over 2000-03. Their market shares in the sub-markets for polyether polyols, toluene diisocyanate (TDI), and methylene diphenyl diisocyanate (MDI) are shown in Figure 3. In contrast to turbine generators with two sellers, buyers of various categories of polyurethane had four or five suppliers from which to choose. While, at present, we do not have any data on the level of expenditure for a buyer, it is probably not a big ticket item like a large turbine generator. It would then seem unlikely that a buyer would negotiate with all or almost all sellers. If that is so then the inclusion effect might be significant enough to support informative list prices which would be the basis for

\[33\] The ensuing facts are from In re: Urethane Antitrust Litigation, 768. F.3d 1245 (10th Cir. 2014) and Class Plaintiffs’ Response Brief, In re: Urethane Antitrust Litigation, (10th Cir.), February 14, 2014.

\[34\] Cartel members controlled the entire market for MDI and TDI and 79% of the market for polyether polyols.
firms effectively colluding by coordinating their list prices.

Figure 3: Market Shares in Urethane

![Market Shares in Urethane](image)

Source: Class Plaintiffs’ Response Brief,

*In re: Urethane Antitrust Litigation*, (10th Cir.), February 14, 2014

9 Concluding Remarks

The primary contribution of this paper is to show that coordination on list prices can be an effective form of collusion even though firms are left unconstrained in the final prices they offer buyers. By coordinating on high list prices, sellers can cause buyers to believe that costs are likely to be high which will induce buyers to bargain less aggressively and that will serve to raise final negotiated prices. Notably, sellers continue to bargain in a competitive manner. We also offer some initial insight for what types of markets are suitable for this type of collusive practice.

In concluding, we offer two directions for future research. The model assumed that buyers live for only one period and lack information prior to their arrival in the market. This assumption was made for purposes of tractability in order to focus on the phenomenon of coordination on list prices. Of course, industrial buyers live for many periods which then means there is a game between buyers and sellers. Buyers use data over time to assess whether firms are colluding, and sellers (if they are colluding) adjust their behavior in order to balance higher profits earned from
collusion against buyers becoming more confident that there is collusion. The latter is detrimental to a cartel because it means buyers will negotiate more aggressively if they think high list prices are less likely to signal high cost, and there is the possibility that buyers may pursue private litigation to claim damages. There is no research that models equilibrium behavior in a dynamic game in which colluding sellers try to avoid detection and buyers try to detect collusion. While technically challenging, it is a research direction that is likely to shed new light on cartel pricing dynamics.\footnote{Besanko and Spulber (1990) consider a static game of incomplete information between a (possible) cartel and customers who are trying to determine whether there is a cartel. Harrington (2004, 2005) and Harrington and Chen (2006) examine the impact of detection for the price path in a dynamic setting but customers are represented by a detection technology and thus are not strategic.}

A natural question is why firms would choose to coordinate only on list prices rather than go that additional step and coordinate on final prices, especially given that express communication on either list or final prices is per se illegal. The answer may lie with a cartel’s concern about detection. For two reasons, detection by customers or the competition authority may be less likely when firms only coordinate on list prices. First, coordinating on final prices along with a market allocation (and the monitoring of sales) requires more extensive and frequent communication among cartel members which enhances the chances of the cartel’s discovery. Second, customers might be less inclined to think that firms are colluding when they offer different final prices, even though their list prices are similar. Competition in final prices could make buyers less inclined to suspect collusion. A topic for future research is to understand when firms prefer to coordinate on final prices and when they instead prefer to coordinate only on list prices.
10 Appendix

Proof of Lemmas 1 and 2: First, it can be verified that given \( h_L(c) > h_H(c) \), we have \( h_L(c) > h_\kappa(c) > h_H(c) \) if \( \kappa \in (0, 1) \).

To show Lemma 1, the first-order conditions of (4) and (6) are given by

\[
v - R_{m1m2}^1(v) = h_L(R_{m1m2}^1(v)), \quad \forall (m_1, m_2) \in \{(l, l), (l, h), (h, l)\}
\]
\[
v - R_{hh}^1(v) = h_\kappa(R_{hh}^1(v))
\]

It is easily verified that

\[
R_{m1m2}^1(v) = \frac{1}{1 + h_L'(R_{m1m2}^1(v))} > 0, \quad \forall (m_1, m_2) \in \{(l, l), (l, h), (h, l)\}
\]

So \( R_{m1m2}^1(v) \) is increasing in \( v \), \( \forall (m_1, m_2) \in \{(l, l), (l, h), (h, l)\} \).

To show that \( R_{hh}^1(v) > R_{ll}^1(v) = R_{ll}^1(v) \) \( \forall v \), suppose the negation so that \( R_{hh}^1(v) \leq R_{ll}^1(v) \) for some \( v \). It follows that

\[
0 \leq -(R_{hh}^1(v) - R_{ll}^1(v)) = h_\kappa(R_{hh}^1(v)) - h_L(R_{ll}^1(v)) \leq h_\kappa(R_{hh}^1(v)) - h_L(R_{ll}^1(v)) < 0
\]

which is a contradiction.

Next to show Lemma 2, when \( (m_1, m_2) \in \{(l, l), (h, h)\} \), the first-order condition from (5) and (7) are given by

\[
v - R_{ll}^2(v) = h_L(R_{ll}^2(v))
\]
\[
v - R_{hh}^2(v) = h_\kappa(R_{hh}^2(v))
\]

So we have \( R_{ll}^2(v) = R_{ll}^1(v) \) and \( R_{hh}^2(v) = R_{hh}^1(v) \). When \( (m_1, m_2) \in \{(l, h), (h, l)\} \), say, when \( (m_1, m_2) = (l, h) \), the first-order condition from (5) becomes

\[
0 = (1 - F_H(R_{hh}^2)) f_L(R_{ll}^2) \left[ (v - R_{ll}^2) - h_L(R_{ll}^2) \right]
+ (1 - F_L(R_{ll}^2)) f_H(R_{ll}^2) \left[ (v - R_{ll}^2) - h_H(R_{ll}^2) \right].
\]

Given the assumption that \( h_L(R_{ll}^2) > h_H(R_{ll}^2) \), we have

\[
(v - R_{ll}^2) - h_L(R_{ll}^2) < 0 < (v - R_{ll}^2) - h_H(R_{ll}^2), \quad (34)
\]

37
As \((v - R_{li}^2) - h_L(R_{li}^2) = 0\) (from (32)) then (34) implies \((v - R_{li}^2) - h_L(R_{li}^2) < (v - R_{li}^2) - h_L(R_{li}^2)\). As \((v - R_{li}^2) - h_\kappa(R_{li}^2) = 0\) (from (33)) then (34) implies \((v - R_{li}^2) - h_\kappa(R_{li}^2) < (v - R_{li}^2) - h_H(R_{li}^2)\). Those two conditions imply \(R_{li}^2 + h_L(R_{li}^2) > R_{li}^2 + h_L(R_{li}^2)\) and \(R_{li}^2 + h_\kappa(R_{li}^2) > R_{li}^2 + h_H(R_{li}^2)\). When \(\kappa\) is sufficiently small, we have \(R_{li}^2 + h_\kappa(R_{li}^2) > R_{li}^2 + h_\kappa(R_{li}^2) > R_{li}^2 + h_H(R_{li}^2)\) by continuity of \(h_\kappa(R_{li}^2)\) in \(\kappa\).

Given that \(h_\kappa'(z) \geq 0\), we have the strict monotonicity of \(z + h_\kappa(z)\), \(t \in \{L, H\}\). Thus \(\exists \kappa > 0\) such that if \(\kappa \in [0, \kappa]\) then \(R_{li}^2(v) > R_{li}^2(v) > R_{li}^2(v), \forall v\).
References


