Price Regulation and the Cost of Capital

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April 2010

Abstract: This paper investigates how price regulation under moral hazard can affect a regulated firm’s financing decision and its cost of capital. We consider stylized versions of the two most typical regulatory frameworks that have been applied over the last decades by regulators: price cap and cost of service. We show that there is a trade-off between lower operational costs and a higher cost of capital under price cap regulation and higher operational costs and lower cost of capital under cost of service regulation. As a result, when the extent of moral hazard is not significant, price cap regulation generates lower welfare than the cost of service regulation.

JEL Classification: L51

Key-words: regulation and investment; cost of capital; capital structure.

* This paper is part of Fernando Camacho’s PhD project at The University of Queensland, which investigates the relationship between regulation and investment. Camacho acknowledges the financial assistance from the University of Queensland and the Brazilian Development Bank and Menezes the support from the Australian Research Council (ARC Grants DP 0557885 and 0663768). We are thankful to John Panzar and seminar participants at the ANU (RSSS and the School of Economics), The University of Melbourne, Universidad de Chile, PUC-Chile, Auckland University, Hong Kong University and at the CAMA/Brookings Conference on the Economics of Infrastructure for useful comments. The usual disclaimer applies.
1. Introduction

Regulators across the globe have typically used one of two methods for setting prices in regulated industries: Cost of Service (COS) regulation and Price Cap (PC) regulation. In its purest form, COS regulation is an *ex post* mechanism whereby the costs incurred by the firm in providing the service (including the opportunity cost of capital) are computed and the price is set by the regulator to cover these costs. This type of regulation could in principle restrict the firm’s ability to extract economic rents – as prices are set so that the firm can only recover costs, including a competitive rate of return on capital – and avoid adverse selection, as prices are based on actual costs. Of course, COS regulation, however, results in moral hazard in that firms face a diminished incentive to minimise costs.\(^1\)

PC regulation offers a solution to the moral hazard problem. PC regulation is an *ex ante* mechanism that involves the regulation of prices rather than profits. When faced with a fixed price, the firm’s incentive is to produce at the lowest possible cost. In practice, however, regulators cannot fix prices for the entire life of regulated assets, which are typically long-lived. Thus, price cap regulation involves setting maximum prices for a regulatory period (typically 5 years). As a result, although price regulation successfully addresses moral hazard, its implementation reintroduces adverse selection. This follows as the prospective of a price revision at the end of the regulatory period might distort the firm’s behaviour.

The incentives effects of these two different types of price regulation are well documented (e.g., Laffont and Tirole (1993), Joskow (2006, 2007)). However, their impact on the firms’ cost of capital is not as well understood.

The regulatory economics literature (e.g., Laffont and Tirole (1993)) typically (implicitly) assumes that the regulated firm is wholly financed by equity. Moreover, the firm’s cost of capital is exogenous and independent of the nature of price regulation. Therefore, when COS and PC regulation are compared, the costs and benefits associated with each type of regulation are not adequately considered. While a few papers have investigated the relationship between price regulation and the firm’s financing decision and cost of capital, the nature of price regulation has not yet being fully explored.

\(^1\) In addition, COS regulation has been associated with gold platting of assets; when the allowed rate of return, which is the regulator’s estimate of the cost of capital, is greater than the actual cost of capital, then the firm has an incentive to increase profits by increasing its asset base. This is the so called Averch-Johnson (1962) effect.
Taggart (1981) was the first to address the relationship between price regulation and the firm's financing decision and cost of capital. Taggart examines a framework in which the regulator sets the price following the firm's investment and financing decisions but prior to the resolution of demand uncertainty. If the firm issues debt, then the firm faces a bankruptcy risk. The regulator wants to avoid or minimize bankruptcy risks. The author then identifies a price-influence effect of debt; an effect that stems from the firm's perceived ability to influence the choice of the regulated price by altering its capital structure. Taggart, however, does not examine the implications for the equilibrium choice of leverage and its impact on the cost of capital.

Spiegel (1994, 1996) and Spiegel and Spulber (1994, 1997) build on Taggart (1981) to examine the implications of the relationship between bankruptcy risks and the regulated price on equilibrium strategies. In their framework, the cost of equity is equal to the risk-free rate and the cost of debt is determined in competitive debt markets. In the absence of bankruptcy, the cost of debt is also equal to the risk-free rate. There are no direct benefits of debt financing and by borrowing the firm faces a bankruptcy risk. This implies that an unregulated firm would issue no debt. As the regulator seeks to minimise the risk of default, the firm borrows in equilibrium only to force the regulator to set the price above marginal cost. Thus, the cost of capital in equilibrium is higher than the risk-free rate.

The insightful contribution of Spiegel and Spulber was to identify a channel through which price regulation can affect the firm's capital structure and its cost of capital. In their analysis, however, it is socially optimal for the firm to have no debt, as the cost of debt is greater than the cost of equity. In our model, although the cost of debt is also higher than the cost of equity, the firm must issue a minimum level of debt, since it is capital-constrained. Furthermore, we fully explore two stylized models of COS and PC regulation so that prices are either set before or after the firm's investment and financing decisions and uncertainty resolution. This allows us to identify the effects of the nature of price regulation on the regulated firm's cost of capital and contrast it with the effects of moral hazard.

De Fraja and Stones (2004) and Stones (2007) also analyse the relationship between the regulator's pricing decision and the firm's capital structure and cost of capital. However, in their model regulated prices are set before the firm's investment and financing decisions, but prices are contingent on future realised costs. Given that prices are contingent on costs and as a result the firm faces no risk of bankruptcy, the cost of debt is equal to the risk-free rate.

De Fraja and Stones (2004) assume that the cost of equity always exceeds the cost of debt and the cost of equity increases with the level of debt. Consequently, optimal regulated prices are such that the firm issues a positive level of debt, as this yields a lower expected price (due to the lower cost of capital) and a higher consumer surplus. In Stones (2007), the cost of equity is
determined by the covariance of the return to shareholders and the market return, and its value depends on the nature of regulation. For instance, when prices are set \textit{ex ante} to cover the firm’s \textit{ex post} costs (a form of COS regulation), the firm’s capital structure is indeterminate and the cost of capital is equal to the risk-free rate. This follows as the return on equity is constant and does not depend on the state of nature. Thus, the covariance of the return to shareholders and the market return is zero and the cost of equity equals the cost of debt and the risk-free return. In contrast, the cost of debt is endogenously determined in our model and, therefore, may be affected by the nature of regulation.

In contrast to the previous papers, our model fully explores the implications of the timing associated with the price-setting process. Thus, we assume that under COS regulation price is set \textit{ex post} to firm’s investment and financing decisions and uncertainty resolution so that the regulated revenue covers exactly the firm’s operational and capital costs. Under PC regulation, we assume that the regulator sets an \textit{ex ante} price cap before the firm’s investment and financing decisions and uncertainty resolution. This modelling choice allows us to fully explore the contrast between the cost-plus and fixed price nature of regulatory contracts.

Consistent with the existing literature, we show that the entrepreneur can be more efficient under PC regulation than under COS regulation. However, unlike the existing literature, we find that PC regulation may yield either a higher cost of capital or a higher rent to the firm than COS regulation. This result is consistent with existing empirical evidence that suggests that firm’s cost of capital under PC is higher than under COS.\footnote{See, for example, Alexander, Mayer and Weeds (1996) and Alexander, Estache and Oliveri (2000).} Which regulation is superior will depend on a comparison of the extent of moral hazard and the effect on the cost of capital that arises from setting prices \textit{ex ante} and creating, therefore, the risk of bankruptcy.

This paper is organized as follows. In section 2 we present the modelling framework and establish our benchmark, namely the optimal choices made by an unregulated monopolist. In section 3 we solve the problem of a regulator that can either use PC or COS regulation to set the price of the regulated firm. Section 4 compares the welfare generated under the two approaches. Section 5 concludes.
2. The Benchmark Model

An infrastructure project requires a fixed investment $I$ and it is completed in one period. The project is undertaken by a risk-neutral entrepreneur who holds cash on hand $H < I$. To fund the project, the entrepreneur must borrow an amount $D \geq I - H$ from risk-neutral lenders.

Lenders behave competitively and are subject to a zero profit constraint; the rate of return expected by lenders is the risk free rate $k_f$. If the entrepreneur borrows $D > I - H$, then the entrepreneur invests the amount $H - (I - D)$ in a Treasury bond that provides the risk-free return $k_f$, that is, the entrepreneur’s opportunity cost is equal to $k_f$. Also, the entrepreneur’s liability is limited and thus the income he derives from the project is nonnegative.

The cost to operate the infrastructure project is $C \in \{\alpha c, c\}$, where $0 < \alpha < 1$ and $c > 0$. The cost depends on the level of effort $E \in \{0, E\}$ undertaken by the entrepreneur as follows:

$$C = \begin{cases} \alpha c, & \text{with probability } p(E) \\ c, & \text{with probability } 1 - p(E) \end{cases},$$

where $p(0) = p(0) > 0$.

The effort level chosen by the entrepreneur is not observable. Consumers are risk-neutral. The infrastructure service provider is a monopolist and faces an inverse demand function characterized by a choke price $P$. At any price less than or equal to $P$ demand is equal to $Q$. At any price greater than $P$ demand is equal to zero.

In the first period ($t = 0$), the entrepreneur chooses the level of effort $E$ and the level of debt $D(E)$ to maximize profit given price $P$ and taking into account the cost of debt $k_f(D(E))$ determined in the debt market. It takes one period to build the network. In the second period ($t = 1$), the infrastructure project is completed, the demand and the operational cost to run the infrastructure are realized and the service is provided to consumers. The sales revenue is then used to cover expenditures in the same order of priority as defined in basic financial statements, namely (1) the operational cost $C$; (2) bondholders; and (3) the entrepreneur.

The expected total welfare at $t = 1$ is equal to $CS + \lambda \pi(E)$, where $CS$ is the expected consumer surplus, $\pi(E)$ is the entrepreneur’s expected profit and $0 < \lambda < 1$ is the weight assigned to the entrepreneur’s profit. The entrepreneur’s expected profit must be non-negative, that is, it must satisfy the participation constraint (otherwise, no investment would take place). To simplify the
analysis we assume that the minimum price that satisfies the firm’s participation constraint for any level of effort \( E \) and level of debt \( D(E) \) is \( P \geq c \). This assumption allows us to focus on the effects of price regulation on the cost of capital and it is consistent with the notion that capital costs represent the bulk of total costs of infrastructure businesses.

We now turn to the case of an unregulated monopolist service provider. We solve the monopolist’s problem backwards. We start at period \( t = 1 \) and calculate the entrepreneur’s net payoff for the two states of nature:

\[
F(E) = \max \{ (P - C)Q - (1 + k^p(D(E), \bullet))D(E) - (1 + k^p)(I - D(E)) - E, (1 + k^p)(I - D(E)) - E \} \quad \text{for } C \in \{c, c_c\}
\]

(1)

The expression for \( F(E) \) follows from the limited liability constraint; if the firm’s revenue is insufficient to pay debt plus interest, the entrepreneur’s maximum loss is equal to the equity and effort used in the project. We can determine the lenders’ payoff in a similar vein:

\[
R(E) = \max \{ (1 + k^d(D(E), \bullet))D(E), (P - C)Q \} \quad \text{for } C \in \{c, c_c\}
\]

(2)

The expression for \( R(E) \) again reflects the limited liability constraint; if the firm’s revenue is insufficient to pay debt plus interest, lenders receive the total revenue as payment.

We now determine the cost of debt. If the firm’s revenue is sufficient to pay debt plus interest in all states of nature (i.e., when \( P \geq (1 + k^d(D(E), \bullet))D(E) \)), there is no default risk and, therefore:

\[
k^d(D(E), \bullet) = k_f.
\]

(3)

In contrast, when the firm’s revenue is insufficient to pay debt plus interest when the realized operational cost is \( c \) (i.e., when \( P \leq (1 + k^d(D(E), \bullet))D(E) \)), the cost of default is determined by using expression (2) as follows:

\[
D(E) = \frac{P(1 + k^d(D(E), \bullet))D(E) + (1 - P)(P - c)Q}{(1 + k_f)}
\]

Rearranging this expression yields:
Note that the cost of equity is equal to $k_f$. Having established the cost of debt and equity, we now determine the monopolist’s choice of capital structure, as per the following Lemma:

**Lemma 1:** The monopolist firm always (weakly) chooses the minimum amount of debt $D(E) = I - H$, independently of being unregulated or regulated.

The proof is straightforward. Recall that the cost of equity is always less than or equal to the cost of debt. Note also that this property does not depend on whether the market is unregulated or not since it comes from the fact that the entrepreneur is capital constrained. Thus the monopolist firm is either indifferent between debt and equity (when the cost of debt is equal to the cost of equity) or strictly prefers equity to debt (when the cost of debt is larger than the cost of equity). For simplicity, we assume henceforth that the entrepreneur chooses $D(E) = D = I - H$.

Having determined that the amount of debt does not depend on the level of effort and also on whether the market is regulated or not, we can now proceed to analyse the monopolist’s optimal choice of effort. We anticipate that the level of effort does depend on whether the market is regulated and also on the type of regulation. In this section we focus on the unregulated monopolist’s decision.

The entrepreneur chooses to undertake $E = \epsilon$ only if the following two constraints are satisfied:

$$\pi(\epsilon) = p(\epsilon)(P - \alpha c)Q + (1 - p(\epsilon))(P - c)Q - (1 + k_f^p(I - H, \bullet))(I - H) - (1 + k_f)H - \epsilon \geq 0$$

$$\pi(0) = p(0)(P - \alpha c)Q + (1 - p(0))(P - c)Q - (1 + k_f^p(I - H, \bullet))(I - H) - (1 + k_f)H \quad \text{(IC)}$$

and

$$\pi(\epsilon) \geq 0 \quad \text{(PR)}$$

If the incentive compatibility (IC) constraint is not satisfied, then the entrepreneur undertakes $E = 0$ as long as $\pi(0) \geq 0$. Note that we can rewrite the IC constraint as follows:

$$\epsilon \leq (1 - \alpha)(p(E) - p(0))E + [k_f^p(I - H, \bullet) - k_f^p(I - H, \bullet)](I - H). \quad \text{(5)}$$
The term \( \epsilon \) is the direct cost whereas \( (1 - \alpha)(p(e) - p(0))cQ \) is the direct benefit of undertaking a positive effort in the form of a lower expected marginal cost. The term \( k_o^o(I - H, \bullet) - k_o^o(I - H, \bullet)(I - H) \) is the difference between the total cost of debt (debt plus interest) when \( E = 0 \) and when \( E = \epsilon \). Expression (5) then states that it will be incentive compatible for the unregulated monopolist to undertake level of effort \( \epsilon \) as long as the resource cost of undertaking such effort is less than or equal to the sum of the expected benefit in terms of lower operational costs plus the change in the total cost of debt associated with a positive effort.

Whether or not the IC constraint is satisfied depends on the choke price. For a sufficiently large \( P \) (i.e., when \( P \geq \frac{(1 + k,)(I - H)}{Q} + c \)), the cost of debt is equal to the risk-free rate under both levels of effort and therefore (5) is reduced to:

\[
\epsilon \leq (1 - \alpha)(p(e) - p(0))cQ. \tag{6}
\]

In contrast, if the risk of default is positive regardless of the level of effort (i.e., \( P < \frac{(1 + k,)(I - H)}{Q} + c \)), the cost of debt is higher than the risk-free rate and then (5) can be rewritten as:

\[
\epsilon \leq (1 - \alpha)(p(e) - p(0))cQ + \frac{(p(e) - p(0))}{p(0)p(e)}\tau Q, \tag{7}
\]

Where \( \tau \in \left(0, \frac{(1 + k,)(I - H)}{Q}\right) \).

Note that \( \frac{(p(e) - p(0))}{p(0)p(e)}\tau Q > 0 \), that is, a positive effort decreases the total cost of debt. The reason is that a positive effort increases the probability of a low cost scenario \( (p(e) > p(0)) \), that is, it decreases the probability of default and also the cost of debt (see equation (4)). Note also that the lower the \( P \) (the higher \( \tau \) ) the higher \( \epsilon \) can be for the entrepreneur to undertake \( E = \epsilon \).

That is, the difference between the total cost of debt when \( E = 0 \) and when \( E = \epsilon \) increases as \( P \) decreases. We have therefore established the following result:
Lemma 2: Table 1 summarises the threshold levels that $\varepsilon$ must satisfy in order for the entrepreneur to undertake $E = \varepsilon$.

Table 1: Threshold Level for $E = \varepsilon$

<table>
<thead>
<tr>
<th>$P$</th>
<th>$\varepsilon \leq$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P = \frac{(1 + k)(I - H)}{Q} + c - \tau$, where $\tau \in \left[0, \frac{(1 + k)(I - H)}{Q}\right]$</td>
<td>$(1 - \alpha)(p(\varepsilon) - p(0))Q + \frac{1}{p(0)p'(\varepsilon)}Q$</td>
</tr>
<tr>
<td>$P \geq \frac{(1 + k)(I - H)}{Q} + c$</td>
<td>$(1 - \alpha)(p(\varepsilon) - p(0))Q$</td>
</tr>
</tbody>
</table>

The characterisation of the unregulated monopolist’s choices of $D$ and $E$ allows us to determine total welfare as follows:

Lemma 3: If the entrepreneur undertakes $E = \varepsilon$, then overall welfare in an unregulated industry $W_M$ is equal to $\lambda \pi(\varepsilon)$. Otherwise, overall welfare is equal to $\lambda \pi(0)$ if $\pi(0) \geq 0$ or 0 if the entrepreneur does not invest.

In the next Section we will compare the outcome of both COS and PC regulation with the outcome of the unregulated monopolist. In particular, under PC regulation the regulator will make full use of the information on Lemma 2 (with $P$ replaced by the regulated price $P_R$) to explore the trade-off between rent extraction and satisfying the IC and participation constraints in order to maximise total welfare.

3. Infrastructure Regulation

In this Section, we assume that the price is set by a risk-neutral regulator who has perfect information about $I$, $P$ and $Q$. At $t = 0$, the regulator reveals whether it will apply a COS or PC regulatory framework. In the case of the former, prices will be set ex post and will be conditional on the realisation of costs, while in the case of the latter, a single price $P_R$ is announced at $t = 0$. At $t = 1$, the regulator observes $C$ but does not observe $E$. The regulator’s objective function is to maximise expected overall welfare at $t = 1$. 

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3.1 Cost of Service Regulation

Under COS regulation, at $t = 1$, after the entrepreneur’s choices of $D$ and $E$, and subsequently to the resolution of cost uncertainty, the regulator sets a price $P$ when $C = \alpha c$ or a price $\overline{P}$ when $C = c$. Thus, the regulated price always covers the operational and capital costs. This is known in advance.

As operational and capital costs are always covered, there is no risk of default and the firm’s cost of capital is always equal to $k_f$. Thus, the entrepreneur is indifferent between any level of debt $D \in [I - H, I]$. As said in the previous section, we assume that the entrepreneur chooses $D = I - H$. The following proposition characterises the choice of effort by the entrepreneur and the optimal (ex post) prices under COS regulation.

**Proposition 1:** Under COS regulation the entrepreneur always chooses $E = 0$. The optimal ex post prices are given by:

$$P = \frac{(1 + k_f)I}{Q} + \alpha c$$  when $C = \alpha c$  and

$$\overline{P} = \frac{(1 + k_f)I}{Q} + c$$  when $C = c$.

The proof of this proposition is in the appendix. This proposition simply states that society bears the full extent of moral hazard under COS regulation but the cost of capital is minimised and equal to the risk-free rate.

Proposition 1 allows us to compute the expected overall welfare from COS regulation evaluated at $t = 1$ as follows:

$$W_{\text{COS}} = p(0)(P - \overline{P})Q + (1 - p(0))(P - \overline{P})Q = P_{\alpha}Q - (1 + k_f)I - (1 - \alpha)p(0)cQ.$$  (8)

3.2 Price Cap Regulation

Under PC regulation, a single price $P_R$ is announced at $t = 0$, before the entrepreneur’s choices of $D$ and $E$, and prior to the resolution of cost uncertainty. The regulator chooses the regulated price $P_R$ to maximize total welfare given by (9) below:

$$\max_{P_R} W_{\text{PC}} = (P - P_R)Q + \lambda \pi_R (E)$$  (9)
Subject to:

\[ \pi_x(E) \geq 0. \text{ (PR)} \]

\[ \pi_x(E) \geq \pi_x(E^*) \text{, where } E, E^* \in \{0, \varepsilon\} \text{ and } E \neq E^*. \text{ (IC)} \]

where

\[
\pi_x(E) = \begin{cases} 
\text{Max} \left[ p(E)(P_x - \alpha e)Q + (1 - p(E))(P_x - e)Q - (1 + k_y)H - E, (1 - p(E))(1 + k_y)H - E \right] & \text{if } P_x \geq \frac{(1 + k_y)(I - H) + c}{Q} \\
\text{Max} \left[ p(E)(P_x - \alpha e)Q + (1 - p(E))(P_x - e)Q - (1 + k_y)H - E, (1 + k_y)H - E \right] & \text{if } P_x \leq \frac{(1 + k_y)(I - H) + c}{Q} 
\end{cases}
\]

Table 2 below shows the optimal price caps for all possible parameter values. The proof is in the appendix.

**Proposition 2:** Table 2 below summarises the optimal price cap given the IC and PR conditions:

<table>
<thead>
<tr>
<th>Constraints</th>
<th>( p(E) )</th>
<th>Optimal Price Cap</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>IC</strong> ( \varepsilon \leq (1 - \alpha)p(e) - p(0)kQ )</td>
<td>( p(e) &lt; \lambda )</td>
<td>( \frac{(1 + k_y)(I - H)}{Q} + c )</td>
</tr>
<tr>
<td><strong>PR</strong> ( (1 + k_y)H + e \geq (1 - \alpha)p(e)kQ )</td>
<td>( p(e) \geq \lambda )</td>
<td>( \frac{(1 + k_y)(I - p(0))}{Q} + (1 - \alpha)p(e)k )</td>
</tr>
<tr>
<td><strong>IC</strong> ( \varepsilon &gt; (1 - \alpha)p(e) - p(0)kQ )</td>
<td>( p(0) &lt; \lambda )</td>
<td>( \frac{(1 + k_y)(I - p(0))}{Q} + c )</td>
</tr>
<tr>
<td><strong>PR</strong> ( (1 + k_y)H &lt; (1 - \alpha)p(e)kQ )</td>
<td>( p(0) \geq \lambda )</td>
<td>( \frac{(1 + k_y)(I - p(0))}{Q} + (1 - \alpha)p(e)k )</td>
</tr>
<tr>
<td><strong>IC</strong> ( (1 - \alpha)p(e) - p(0)kQ &lt; \varepsilon \leq (1 - \alpha)p(e) - p(0)kQ )</td>
<td>( p(e) &lt; \lambda )</td>
<td>( \frac{(1 + k_y)(I - H)}{Q} + c )</td>
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<td>( \frac{(1 + k_y)(I - p(0))}{Q} + (1 - \alpha)p(e)k )</td>
</tr>
</tbody>
</table>

If \( p(e) < (1 - \alpha)p(e) - 4p(0)kQ + (1 - \alpha)p(e)(1 + k_y)Hkrej \), then

\[
\frac{1 + k_y}{Q} \cdot (1 - p(e))(1 + k_y)H - E. \]

Otherwise, \( \frac{(1 + k_y)(I - H) + c}{Q} \).
Proposition 2 has important implications for the trade-off between rent extraction, optimal effort induction and the regulated firm’s cost of capital. To understand the nature of this trade-off, note that as \( \lambda < 1 \) the regulator will extract the entire entrepreneur’s rent when the rate by which the firm’s cost of capital increases is sufficiently low. Conversely, when the negative impact on the entrepreneur’s rent is higher than the positive impact on consumer surplus, the regulator then sets the minimum price such that the firm’s cost of capital is equal to the risk-free rate. This price leaves a positive rent for the entrepreneur.

Of course, the actual nature of the trade-off will depend on parameter values. Consider \( P_R \) such that \( \left(1 + k_j \right)(I - H) + c \leq P_R \leq P \). It is easy to see that \( \frac{\partial W_{PC}(I - H, \alpha)}{\partial P_R} = -(1 - \lambda)Q < 0 \); a decrease in \( P_R \) increases welfare as the positive impact on consumer surplus \( (Q) \) is higher than the negative impact on the entrepreneur’s profit \( (-\lambda Q) \); a decrease in \( P_R \) does not impact the cost of debt, only the expected revenue, which is always sufficient to pay total cost of debt in all states of nature.

When parameter values are such that debt is paid under all states of nature, the regulated firm’s cost of capital is equal to the risk-free rate, and it does not depend on the level of effort. In this case, a price decrease does not cause any change in the firm’s cost of capital, and thus the regulator will set the lowest possible price, which is \( P_s = \frac{(1 + k_j)I + E}{Q} + (1 - (1 - \alpha)p(E))k \), for \( E \in \{0, E\} \), to extract all expected rent (so that the entrepreneur’s participation constraint is always binding); there is no trade-off between rent extraction, optimal effort choice and the cost of capital. The total welfare is then given by:

\[
W_{PC} = PQ - \left(1 + k_j \right)I - cQ + (1 - \alpha)p(E)kQ - E. \quad (10)
\]

The optimal choice of effort in equation (10) will depend on parameter values as described in rows one \( (E = e) \) and three \( (E = 0) \) of Table 2.

Alternatively, for other parameter values, by reducing the regulated price to extract rents, the regulator affects the firm’s cost of capital and may affect the optimal choice of effort. In fact, consider \( P_R \) such that \( c \leq P_R < \frac{(1 + k_j)(I - H)}{Q} + c \). In this case, \( \frac{\partial W_{PC}(I - H, \alpha)}{\partial P_R} = \left(1 - \frac{\lambda}{p(E)}\right)Q \); whether a decrease in \( P_R \) increases welfare depends on \( p(E) \). The rationale is as follows: a decrease in \( P_R \) has a positive impact on consumer surplus \( (Q) \) and a negative impact on the
entrepreneur’s profit \((-\frac{\lambda}{p(E)} Q\)). Furthermore, the entrepreneur’s profit is reduced because the expected revenue decreases \((-\lambda Q\)) and the cost of debt increases \((-\lambda \frac{(1-p(E))}{p(E)} Q\)); the sum of these two effects is equal to \(-\frac{\lambda}{p(E)} Q\). Thus, if \(p(E) < \lambda\), then a decrease in \(P_r\) reduces welfare. In this case, the rate by which the cost of debt increases \((-\frac{(1-p(E))}{p(E)}\)) is sufficiently high and compensates the fact that the consumers surplus’ weight is higher than the entrepreneur profit’s weight in the welfare function \((\lambda < 1)\). The regulator then sets the minimum price such that the firm’s cost of capital is equal to \(k_j\), that is, \(P_r = \frac{(1+k_j)(I-H)}{Q} + \epsilon\), for \(E \in \{0, \epsilon\}\). The welfare is given by:

\[
W_{rc} = PQ - (1+k_j)(I-H) - cQ + \lambda [(1-\alpha)p(E)kQ - (1+k_j)H - E] \quad (11)
\]

where \((1+k_j)H + \epsilon < (1-\alpha)p(E)k\). Note that in this case the entrepreneur’s expected profit is positive.

However, if \(p(E) \geq \lambda\), then a decrease in \(P_r\) increases welfare. In this case, the rate by which the cost of debt increases is sufficiently low and is welfare enhancing to reduce prices because the consumer surplus’ weight is higher than the entrepreneur profit’s weight in the welfare function.\(^3\) The regulator then sets the lowest price possible extracting all rent (so that the entrepreneur’s participation constraint is binding), which is \(P_r = \frac{(1+k_j)(I-(1-p(E))H)}{Q} + \frac{p(E)E}{(1-(1-\alpha)p(E)k)}\), for \(E \in \{0, \epsilon\}\). In this case, welfare is given by:

\[
W_{rc} = PQ - (1+k_j)(I-(1-p(E))H) - (1-(1-\alpha)p(E)k)Q - p(E)E \quad . \quad (12)
\]

The optimal choice of effort in equations (11) and (12) will depend on parameter values as described in rows two \((E = \epsilon)\), four \((E = 0)\) and five \((E = \epsilon\) and \(E = 0)\) of Table 2.

\(^3\) Note that if \(p(E) = \lambda\), then welfare is constant within this price range. We assume that if \(p(E) = \lambda\) the regulator sets price as low as possible.
4. The benefits and costs of COS and PC regulatory regimes

We now compare the welfare generated by the two different types of regulatory regimes. First, we note that under COS regulation the entrepreneur always undertakes \( E = 0 \), the firm’s cost of capital is always equal to \( k_j \) and the entrepreneur’s profit is always equal to zero. Under PC regulation, however, the entrepreneur undertakes either \( E = 0 \) or \( E = \varepsilon \), the firm’s cost of capital is higher than or equal to \( k_j \) and the entrepreneur’s expected profit is higher than or equal to zero. As we will see below, whether PC regulation is welfare superior depends on parameter values. Proposition 3 below compares the welfare generated by COS and PC regulatory regimes across all parameter values. The proof is in the appendix.

**Proposition 3:** Table 3 below compares the PC and COS regulatory regimes for all parameter values expressed in terms of the IC and PR constraints under PC regulation.

<table>
<thead>
<tr>
<th>Constraints</th>
<th>( p(E) )</th>
<th>Optimal Regulation</th>
</tr>
</thead>
<tbody>
<tr>
<td>IC ( \varepsilon \leq (1 - \alpha)(p(e) - p(0))kQ ) PR ((1 + k_j)H + \varepsilon \geq (1 - \alpha)p(e)kQ )</td>
<td>-</td>
<td>( W_{PC} \geq W_{COS} )</td>
</tr>
<tr>
<td>IC ( \varepsilon \leq (1 - \alpha)(p(e) - p(0))kQ ) PR ((1 + k_j)H + \varepsilon &lt; (1 - \alpha)p(e)kQ )</td>
<td>( p(e) &lt; \lambda )</td>
<td>( W_{PC} \geq W_{COS} ) if ( \lambda(1 - \alpha)p(e)kQ - (1 + k_j)H \geq \left( (1 - \alpha)DQ - (1 + k_j)H \right) ) ( W_{PC} \geq W_{COS} ) if ( p(e) \leq (1 - \alpha)(p(e) - p(0))kQ + (1 - p(e))(1 + k_j)H )</td>
</tr>
<tr>
<td>IC ( \varepsilon &gt; (1 - \alpha)(p(e) - p(0))kQ ) PR ((1 + k_j)H &lt; (1 - \alpha)p(e)kQ )</td>
<td>-</td>
<td>( W_{PC} = W_{COS} )</td>
</tr>
<tr>
<td>IC ( \varepsilon &gt; (1 - \alpha)(p(e) - p(0))kQ &lt; \varepsilon \leq (1 - \alpha)(p(e) - p(0))kQ ) PR ((1 + k_j)H &lt; (1 - \alpha)p(e)kQ )</td>
<td>-</td>
<td>( W_{PC} &lt; W_{COS} )</td>
</tr>
</tbody>
</table>

Proposition 3 shows that whether PC regulation is welfare superior depends on the trade-off between a higher cost efficiency under PC and a lower cost of capital (or a lower entrepreneur’s rent) under COS. In particular, when under PC regulation the entrepreneur undertakes \( E = \varepsilon \).
the regulator may be able to set a lower price under PC regulation than under COS regulation. The reason is that under PC regulation the regulator is able to extract the rent differential that stems from a positive effort in comparison to a zero level of effort and transfer it to consumer surplus through a lower regulated price.

Conversely, when under PC regulation the firm’s cost of capital is higher than $k_f$ or the entrepreneur’s profit is positive, then the regulator may not be able to set a lower price under PC regulation than under COS regulation. The reason is that the regulator extracts less rent under PC than under COS regulation; by reducing the regulated price to extract rents under PC regulation, the regulator may affect the firm’s cost of capital. When the impact on the firm’s cost of capital is sufficiently low, the regulator will lower the price because the positive impact on consumer’s surplus outweighs the negative impact on firm’s cost of capital (and entrepreneur’s profit). However, if the impact on firm’s cost of capital is sufficiently high, the regulator will set the price so that there is no bankruptcy risk and the cost of capital is equal to the risk-free risk as the negative impact on the firm’s cost of capital (and entrepreneur’s profit) outweighs the positive impact on consumer surplus.

As with Proposition 2, the actual nature of the trade-off depends on parameter values. We have seen that when under PC regulation $P_R$ is such that $\frac{(1+k_f)(I-H)}{Q} + c \leq P_R \leq P$, then the regulated firm’s cost of capital is equal to the risk-free rate (as debt is paid under all states of nature). The regulator then sets the lowest price possible to extract all expected rent. In this case, note that under both types of regulation the firm’s cost of capital is equal to $k_f$ and the entrepreneur’s expected profit is zero. Thus, if the entrepreneur undertakes $E = 0$ under PC regulation, then both types of regulatory regimes provide the same price (consumer surplus) and welfare. However, if the entrepreneur undertakes $E = \varepsilon$ under PC regulation, then this higher level of effort allows the regulator to set a price lower than or equal to the expected price under COS regulation. The optimal choice of effort under PC will depend on parameter values as described in rows one ($E = \varepsilon$) and three ($E = 0$) of Table 3.

Alternatively, for other parameter values, we have seen that by reducing the regulated price to extract rents under PC regulation, the regulator affects the firm’s cost of capital. When $P_R$ is such that $c \leq P_R < \frac{(1+k_f)(I-H)}{Q} + c$, then $\frac{\partial W_c}{\partial p_r} = (\frac{\lambda}{p(E)})Q$ and whether a decrease in $P_R$ increases welfare depends on $p(E)$. Moreover, a decrease in $P_R$ has a positive impact on
consumer surplus ($Q$) and a negative impact on the entrepreneur’s profit ($-\frac{\lambda}{p(E)}Q$) due to a lower revenue ($-\lambda Q$) and a higher cost of capital ($-\lambda \frac{(1-p(E))}{p(E)}Q$).

Thus, if $p(E) < \lambda$, then the rate by which the cost of debt increases ($-\frac{(1-p(E))}{p(E)}Q$) is sufficiently high and compensates for the fact that the consumers surplus’ weight is higher than the entrepreneur profit’s weight in the welfare function ($\lambda < 1$). The regulator then sets the minimum price such that the firm’s cost of capital is equal to $k_f$. In this case, under both types of regulatory regimes the firm’s cost of capital is equal to $k_f$. However, the entrepreneur’s expected profit is positive under PC regulation and equal to zero under COS regulation. Thus, we have the following: If the entrepreneur undertakes $E = 0$ under PC regulation, then COS regulation is welfare superior. The reason is that under COS the regulator is able to extract the entire economic rent from the entrepreneur and the cost of capital is equal to the risk-free rate whereas under PC the regulator must give a minimum positive rent for the entrepreneur in order to keep the cost of capital equal to the risk-free return. Thus, the regulator extracts less rent under PC than under COS. This rent differential allows the regulator to set a lower expected price under COS than the price under PC regulation. However, if the entrepreneur undertakes $E = \varepsilon$ under PC regulation, then there is a trade-off between a higher level of effort under PC regulation and no economic rent left for the entrepreneur under COS regulation. Whether the regulator will be able to set a lower price under PC than the expected price under COS, will depend on the parameter values.

If $p(E) \geq \lambda$, then the rate by which the cost of debt increases under PC is sufficiently low and it is welfare enhancing to reduce prices because the positive impact on consumer surplus outweighs the negative impact on the firm’s cost of capital (and entrepreneur’s profit). The regulator then sets the lowest price possible to extract all rent (so that the entrepreneur’s participation constraint is binding). In this case, under both types of regulatory regimes the entrepreneur’s expected profit is equal to zero. However, the firm’s cost of capital is higher than $k_f$ under PC regulation and equal to $k_f$ under COS regulation. Thus, we have the following: If the entrepreneur undertakes $E = 0$ under PC regulation, then COS regulation is welfare superior. The reason is the same as in the previous case. That is, because the firm’s cost of capital is higher under PC than under COS (instead of a higher rent under PC than under COS, as in the previous case), the regulator extracts less rent under PC than under COS. This rent differential allows the regulator to set a lower expected price under COS than the price under PC regulation. However, if the entrepreneur undertakes $E = \varepsilon$ under PC regulation, then there is a trade-off between a higher level of effort under PC regulation and a lower cost of capital under COS regulation. Whether the regulator will
be able to set a lower price under PC than the expected price under COS, will depend on the parameter values.

The optimal choice of effort under PC regulation when the cost of debt is higher than the risk-free return (or the entrepreneur’s profit is positive) will depend on parameter values as described in rows two \( (E = \varepsilon) \), four \( (E = 0) \) and five \( (E = \varepsilon \text{ and } E = 0) \) of Table 3.

5. Conclusion

We have investigated the relationship between price regulation and the cost of capital in a two-period model in which the regulator faces moral hazard and the entrepreneur is capital constrained. In our model, the cost of debt is higher than or equal to the cost of equity. Thus, the entrepreneur chooses the minimum level of debt possible.

In contrast to the previous papers, our model fully explores the implications of the timing associated with the price-setting process. Thus, we assume that under COS regulation price is set \textit{ex post} to firm’s investment and financing decisions and uncertainty resolution so that the regulated revenue covers exactly the firm’s operational and capital costs. Under PC regulation, we assume that the regulator sets an \textit{ex ante} price cap before the firm’s investment and financing decisions and uncertainty resolution. This modelling choice allows us to fully explore the contrast between the cost-plus and fixed price nature of regulatory contracts.

Thus, we have seen that when the cost of capital under PC is equal to the risk-free rate, PC regulation generates at least the same welfare as COS regulation. In particular, if the extent of moral hazard is significant, then PC regulation is welfare superior. However, when the cost of capital under PC regulation is higher than the risk-free rate (or the entrepreneur’s profit is positive because the rate by which the cost of capital increases is sufficiently high), then we have the following: if the extent of moral hazard is insignificant, then COS regulation is welfare superior; if the extent of moral hazard is significant, then there is a trade-off between a higher cost efficiency under PC regulation and a lower cost of capital or lower economic rent under COS regulation.

In summary, this paper has provided a channel through which PC regulation affects the cost of capital of the regulated firm. Therefore, it has shown that any comparison between PC and COS regulatory regimes has to take into account the trade-off between higher cost of capital and less moral hazard.
References


Appendix

Proof of Proposition 1:

Under COS regulation the regulator does not observe $E$ and sets $\overline{P}$ when $C = \alpha c$ and $\underline{P}$ when $C = c$ such that the regulated price always covers operational and capital costs. Under such prices, the entrepreneur’s profit is given by:

$$\pi = \begin{cases} 
\pi_H = (P - \alpha c)Q - (1 + k_f)I - E, & \text{if } C = \alpha c \\
\pi_L = (\overline{P} - c)Q - (1 + k_f)I - E, & \text{if } C = c
\end{cases}$$

Fix $P$ and $\overline{P}$ at any level. From the equation above, the entrepreneur never chooses $E = \varepsilon$, as he can always guarantee higher profits by choosing $E = 0$. It follows that the welfare maximising regulated ex post prices (that guarantee zero profits and maximise consumer surplus) are equal to

$$P = \frac{(1 + k_f)I}{Q} + \alpha c \quad \text{and} \quad \overline{P} = \frac{(1 + k_f)I}{Q} + c.$$  

Proof of Proposition 2:

We begin this Proof by showing that there are only five cases to examine. Then, we proceed to state these cases. First, assume that the minimum price that satisfies the participation constraint is higher than or equal to $\left(\frac{(1 + k_f)I - H}{Q}\right) + c$. Recall from Lemma 2 that in this case the entrepreneur chooses only one level of effort $E \in \{0, \varepsilon\}$ within this price range. Indeed, the decision of undertaking a positive effort depends only on the resource cost of undertaking such effort ($\varepsilon$) being less than or equal to the expected benefit in terms of lower operational costs ($((1 - \alpha)(p(\varepsilon) - p(0))kQ$). These cases are stated as Cases 1 and 2 below.

Second, assume that the minimum price that satisfies the participation constraint is lower than $\left(\frac{(1 + k_f)I - H}{Q}\right) + c$. We know from Lemma 2 that when price is lower than $\left(\frac{(1 + k_f)I - H}{Q}\right) + c$ the decision of undertaking a positive effort depends on the resource cost of undertaking such effort ($\varepsilon$) being less than or equal to the sum of the expected benefit in terms of lower operational costs ($((1 - \alpha)(p(\varepsilon) - p(0))kQ$) plus the change in the total cost of debt associated with a positive.
effort \( (k_{o}^2(I-H,\epsilon) - k_{o}^1(I-H,\epsilon)) \). Moreover, we know that \( (k_{o}^2(I-H,\epsilon) - k_{o}^1(I-H,\epsilon)) \) is positive and increases as price decreases. Thus, if the threshold to undertake a positive effort is reached at a specific price, the entrepreneur will undertake a positive effort for all prices lower or equal to this price. Thus, we have three cases to consider: In the first two cases the entrepreneur chooses only one level of effort \( E \leq 0 \) for all \( P_{R} \) such that the PR constraint is satisfied. In the third case, there is a price \( P_{R} < \frac{(1+k)^{(I-H)}}{Q} + c \) such that if \( P_{R} > P_{E} \) the entrepreneur undertakes \( E = 0 \) and if \( P_{R} \leq P_{E} \) the entrepreneur undertakes \( E = \epsilon \). These cases are stated below as Case 3, 4 and 5.

**Case 1:** \( \epsilon \leq (1-\alpha)(p(\epsilon) - p(0))kQ \) and \( (1+k,I+\epsilon)H \geq (1-\alpha)p(\epsilon)kQ \).

Lemma 2 states that if \( \epsilon \leq (1-\alpha)(p(\epsilon) - p(0))kQ \), then the entrepreneur undertakes \( E = \epsilon \) for all \( P_{R} \) such that the participation constraint is satisfied. If \( (1+k,I+\epsilon)H \geq (1-\alpha)p(\epsilon)kQ \), then the minimum price that satisfies the participation constraint is

\[
P_{R} = \frac{(1+k)(I-H)}{Q} + c
\]

(we can find that by substituting \( P_{R} \) in \( \pi_{R}(\epsilon) \)). We have seen that within the price range \( \left[ \frac{(1+k)(I-H)}{Q} + c, P_{R} \right] \) we have

\[
\frac{\partial \pi_{R}(I-H,\epsilon)}{\partial P_{R}} = -(1-\lambda)Q < 0.
\]

Thus, the optimal price cap is the minimum price that satisfies the participation constraint, which is equal to \( \frac{(1+k,I+\epsilon)}{Q} + (1-(1-\alpha)p(\epsilon)k) \) (we find this price by setting \( \pi_{R}(\epsilon) = 0 \)).

**Case 2:** \( \epsilon \leq (1-\alpha)(p(\epsilon) - p(0))kQ \), \( (1+k,I+\epsilon)H < (1-\alpha)p(\epsilon)kQ \).

Lemma 2 states that if \( \epsilon \leq (1-\alpha)(p(\epsilon) - p(0))kQ \), then the entrepreneur undertakes \( E = \epsilon \) for all \( P_{R} \) such that the participation constraint is satisfied. If \( (1+k,I+\epsilon)H < (1-\alpha)p(\epsilon)kQ \), then the minimum price that satisfies the participation constraint is

\[
P_{R} = \frac{(1+k)(I-H)}{Q} + c
\]

(we can find that by substituting \( P_{R} \) in \( \pi_{R}(\epsilon) \)). We have seen that within the price range \( \left[ \frac{(1+k)(I-H)}{Q} + c, P_{R} \right] \) we have

\[
\frac{\partial \pi_{R}(I-H,\epsilon)}{\partial P_{R}} = \left\{ \frac{1}{p(\epsilon)} - \frac{\lambda}{Q} \right\} Q.
\]

Thus, if \( p(\epsilon) < \lambda \), then the optimal price cap is

\[
\frac{(1+k,I-H)}{Q} + c.
\]

If \( p(\epsilon) \geq \lambda \), then the optimal price cap is the minimum price that satisfies the
participation constraint \( \frac{(1+k_J)(I - (1-p(e))H) + p(e)ε}{Q} + (1-(1-α)p^*(e))k_e \): (we find this price by setting \( π_R^*(e) = 0 \)).

**Case 3:** \( ε > (1-α)(p(e) - p(0))k_e Q \) and \((1+k_J)H ≥ (1-α)p(0)k_e Q\).

Lemma 2 states that if \( ε > (1-α)(p(e) - p(0))k_e Q \), then the entrepreneur undertakes \( E = 0 \) for all \( P_R ≥ \frac{(1+k_J)(I - H)}{Q} + c \) such that the participation constraint is satisfied. If \((1+k_J)H ≥ (1-α)p(0)k_e Q\), then the minimum price that satisfies the participation constraint is \( P_R ≥ \frac{(1+k_J)(I - H)}{Q} + c \) (we can find that by substituting \( P_R \) in \( π_R^*(0) \)). We have seen that within the price range \( \left[ \frac{(1+k_J)(I - H)}{Q} + c, P \right] \) we have \( \frac{∂W_{Rc}(I - H, P)}{∂P_R} = -(1-λ)Q < 0 \). Thus, the optimal price cap is the minimum price that satisfies the participation constraint, which is equal to \( \frac{(1+k_J)(I - H)}{Q} + (1-(1-α)p(0))k_e \). (we find this price by setting \( π_R^*(0) = 0 \)).

**Case 4:** \( ε > (1-α)(p(e) - p(0))k_e Q + \frac{(p(e) - p(0))}{p(e)}[(1-α)p(0)k_e Q - (1+k_J)H] \) and \((1+k_J)H < (1-α)p(0)k_e Q\).

Lemma 2 states that if \( ε > (1-α)(p(e) - p(0))k_e Q + \frac{(p(e) - p(0))}{p(e)}[(1-α)p(0)k_e Q - (1+k_J)H] \), then the entrepreneur undertakes \( E = 0 \) for all \( P_R ≥ c \) such that the participation constraint is satisfied. If \((1+k_J)H < (1-α)p(0)k_e Q\), then the minimum price that satisfies the participation constraint is \( P_R < \frac{(1+k_J)(I - H)}{Q} + c \) (we can find that by substituting \( P_R \) in \( π_R^*(0) \)). We have seen that within the price range \( \left[ c, \frac{(1+k_J)(I - H)}{Q} + c \right] \) we have that \( \frac{∂W_{Rc}(I - H, P)}{∂P_R} = \left( 1 - \frac{λ}{p(0)} \right)Q \). Thus, if \( p(0) < λ \), then the optimal price cap is \( \frac{(1+k_J)(I - H)}{Q} + c \). If \( p(0) ≥ λ \), then the optimal price cap is the minimum price that satisfies the participation constraint, which is equal to \( \frac{(1+k_J)(I - (1-p(0))H)}{Q} + (1-(1-α)p^*(0))k_e \) (we find this price by setting \( π_R^*(0) = 0 \)).
**Case 5:** \( \epsilon > (1 - \alpha)(p(e) - p(0))kQ \), \( \epsilon \leq (1 - \alpha)(p(e) - p(0))kQ + \frac{(p(e) - p(0))}{p(0)}[(1 - \alpha)p(e)kQ - (1 + k_j)H - \epsilon] \) and \((1 + k_j)H < (1 - \alpha)p(0)kQ\).

Lemma 2 states that if \( \epsilon > (1 - \alpha)(p(e) - p(0))kQ \) and \( \epsilon \leq (1 - \alpha)(p(e) - p(0))kQ + \frac{(p(e) - p(0))}{p(0)}[(1 - \alpha)p(e)kQ - (1 + k_j)H - \epsilon] \), then the entrepreneur undertakes two levels of effort within the price range \( \left[ \frac{(1 + k_j)(I - (1 - \alpha)p(e)H) + p(e)e}{Q} + \frac{(1 + k_j)(I - H)}{Q} + c \right] \).

More specifically, there is a price \( \overline{P}_k \) such that if \( P_k > \overline{P}_k \) the entrepreneur undertakes \( E = 0 \) and if \( P_k \leq \overline{P}_k \) the entrepreneur undertakes \( E = \epsilon \). If \( (1 - \alpha)p(0)kQ > (1 + k_j)H \), then the minimum price that satisfies the participation constraint is \( P_k = \frac{(1 + k_j)(I - H)}{Q} + c \) (we can find that by substituting \( P_k \) in \( \pi_k(0) \)).

We have seen in Cases 2 and 4 that if \( p(e) < \lambda \) and \( p(0) < \lambda \), then the optimal price cap is \( \frac{(1 + k_j)(I - H)}{Q} + c \). Thus, if \( p(0) < p(e) < \lambda \), then the optimal price cap is \( \frac{(1 + k_j)(I - H)}{Q} + c \). We have also seen that if \( p(e) \geq \lambda \) and \( p(0) \geq \lambda \), then the optimal price cap is the minimum price that satisfies the participation constraint. Thus, if \( \lambda < p(0) < p(e) \), then the optimal price cap is \( \frac{(1 + k_j)(I - (1 - \alpha)p(e)) + p(e)e}{Q} + \frac{(1 - (1 - \alpha)p^2(e))}{Q} \).

It is easy to see that if we have \( p(0) < \lambda \leq p(e) \), then the optimal price cap will be \( \frac{(1 + k_j)(I - H)}{Q} + c \) or \( \frac{(1 + k_j)(I - (1 - \alpha)p(e)) + p(e)e}{Q} + \frac{(1 - (1 - \alpha)p^2(e))}{Q} \), as \( \frac{\partial W_{\pi_k}(D(0), \epsilon)}{\partial P_k} > 0 \) and \( \frac{\partial W_{\pi_k}(D(e), \epsilon)}{\partial P_k} < 0 \). By calculating \( (12) \cdot (11) \), where \( E = \epsilon \) in \( (12) \) and \( E = 0 \) in \( (11) \), we find that \( \frac{(1 + k_j)(I - (1 - \alpha)p(e)) + p(e)e}{Q} + \frac{(1 - (1 - \alpha)p^2(e))}{Q} \) is welfare superior if:

\[
p(e)e < (1 - \alpha)(p^2(e) - \lambda p(0))kQ + (1 + k_j)(\lambda - p(e))H \quad (13)
\]

Otherwise, \( \frac{(1 + k_j)(I - H)}{Q} + c \) is the optimal price cap. \( \square \)

**Proof of Proposition 3:**
We proceed to prove Proposition 3 using the cases obtained in Proposition 2.

Case 1: \( \varepsilon \leq (1 - \alpha)(p(\varepsilon) - p(0))kQ \) and \( (1 + k_f)_H + \varepsilon \geq (1 - \alpha)p(\varepsilon)kQ \).

We know from Proposition 2 that under these market conditions the optimal price cap is 
\[
P_a = \frac{(1 + k_f)H + \varepsilon}{Q} + (1 - (1 - \alpha)p(p))k \] .

By taking the difference between (10) \( (E = \varepsilon) \) and (8) we find that the PC always generates at least the same welfare as the COS since 
\( \varepsilon \leq (1 - \alpha)(p(\varepsilon) - p(0))kQ \).

Case 2: \( \varepsilon \leq (1 - \alpha)(p(\varepsilon) - p(0))kQ \) and \( (1 + k_f)_H + \varepsilon < (1 - \alpha)p(\varepsilon)kQ \).

We know from Proposition 2 that if \( p(\varepsilon) < \lambda \) the optimal price cap is 
\[
P_a = \frac{(1 + k_f)(H - H)}{Q} + p(\varepsilon)k \].

By taking the difference between (11) \( (E = \varepsilon) \) and (8) we find that the PC will generate at least the same welfare as the COS if 
\( P_cQp - (1 - \alpha)p(\varepsilon)kQ \).

Case 3: \( \varepsilon > (1 - \alpha)(p(\varepsilon) - p(0))kQ \) and \( (1 + k_f)_H \geq (1 - \alpha)p(0)kQ \).

We know from Proposition 2 that under these market conditions the optimal price cap is 
\[
P_a = \frac{(1 + k_f)H}{Q} + (1 - (1 - \alpha)p(0))k \].

By taking the difference between (10) \( (E = 0) \) and (8) we find that the PC will generate the same welfare as the COS.

Case 4: \( \varepsilon > (1 - \alpha)(p(\varepsilon) - p(0))kQ \) and \( (1 + k_f)_H < (1 - \alpha)p(0)kQ \).

We know from Proposition 2 that if \( p(0) < \lambda \) the optimal price cap is 
\[
P_a = \frac{(1 + k_f)(H - H)}{Q} + \varepsilon \].

By taking the difference between (11) \( (E = 0) \) and (8) we find that the PC is welfare inferior as we never have 
\( \lambda[(1 - \alpha)p(0)kQ - (1 + k_f)_H] \geq [(1 - \alpha)p(0)kQ - (1 + k_f)_H] \). We know from Proposition 2 that if \( p(0) \geq \lambda \) the optimal price cap is 
\[
P_a = \frac{(1 + k_f)(H - p(0)_H)}{Q} + (1 - (1 - \alpha)p(0))k \].

By taking the difference between (12) \( (E = 0) \) and (8) we find that the PC is welfare inferior as we never have 
\( (1 - \alpha)p(0)kQ \leq (1 + k_f)_H \).
Case 5: $\epsilon > (1 - \alpha) (p(e) - p(0)) k Q \cdot \epsilon \leq (1 - \alpha) (p(e) - p(0)) k Q + \frac{(p(e) - p(0))}{p(0)} [(1 - \alpha) p(e) k Q - (1 + k_f) H - \epsilon] \quad \text{and} \quad (1 + k_f) H < (1 - \alpha) p(0) k Q$.

We know from Proposition 2 that if $p(e) < \lambda$ the optimal price cap is $P^* = \frac{(1 + k_f) (I - H)}{Q} + c$ (See Proof of Case 4). If $p(0) \geq \lambda$, the optimal price cap is $P^* = \frac{(1 + k_f) (I - (1 - \alpha) p^2(e) H) + p(e) c}{Q} + (1 - (1 - \alpha) p^2(e)) c$. If the former is the optimal price cap then see Proof of Case 4. If the latter is the optimal price cap then see Proof of Case 2. □