Partial Vertical Integration, Ownership Structure and Foreclosure*

David Gilo, Nadav Levy, and Yossi Spiegel†

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work in progress

Abstract

We study the incentive of firms to acquire partial, controlling, stakes in vertically related firms and then foreclose a downstream rival. We show that partial acquisition of an upstream supplier (partial backward integration) is more likely to occur and lead to foreclosure than full vertical merger, especially when initially, the upstream supplier is held by dispersed shareholders. By contrast, partial acquisition of a downstream customer (partial forward integration) is less profitable than full vertical merger, especially when initially, the downstream customer has a controlling shareholder whose controlling stake is small, and is never profitable when the downstream customer is initially held by dispersed shareholders. We also show that partial backward integration, followed by foreclosure of a downstream rival is less likely when the downstream customer holds a toehold in the upstream supplier, more likely when the acquisition is made by the controlling shareholder of the downstream customer rather than the firm itself, and is as likely when two downstream customers compete for the acquisition of control in the upstream supplier.

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†Gilo: The Buchman Faculty of Law, Tel-Aviv University, email: gilod@post.tau.ac.il. Levy: School of Economics, Interdisciplinary Center (IDC), Herzliya, email: nadavl@idc.ac.il. Spiegel: Recanati Graduate School of Business Administration, Tel Aviv University, email: spiegel@post.tau.ac.il, http://www.tau.ac.il/~spiegel.
1 Introduction

One of the main antitrust concerns that vertical mergers raise is that the merger will result in the foreclosure of either upstream or downstream rivals. While most of the discussion on vertical foreclosure has focused on full vertical mergers, in reality, many firms acquire partial stakes in suppliers (partial backward integration) or in buyers (partial forward integration). A case in point is the U.S. cable industry where several operators acquired partial ownership stakes in cable or television networks (see Waterman and Weiss, 1997, p. 24-32). This situation has raised the concern that non-integrated networks will be denied access to cable systems or will obtain access at unfavorable terms.¹

In this paper we study partial vertical integration and examine the circumstances under which it may occur and lead to “input foreclosure” i.e., the foreclosure of a downstream rival. The main question that we ask is whether firms have an incentive to acquire a partial controlling stake in an upstream supplier or a downstream buyer in the first place, and how this incentive depends on the initial ownership structure of the target firm (the upstream supplier in case of partial backward integration or the downstream buyer in case of partial forward integration).

To address this question we consider a model in which two downstream firms buy inputs from several upstream suppliers. Following integration between one of the downstream firms and one of the upstream suppliers, the upstream supplier may wish to foreclose the remaining downstream rival in order to weaken it and thereby boost its own downstream profit. This input foreclosure however lowers the profit of the integrated upstream supplier, because it now forgoes sales to the downstream rival. Under partial backward integration, part of the upstream loss from input foreclosure is borne by the minority shareholders of the integrated upstream supplier. Since the downstream buyer captures the entire associated gain, input foreclosure is more profitable under partial backward integration than under full vertical integration. We show that partial backward integration, which

¹Recent prominent examples include News Corp.’s (a major owner of TV broadcast stations and programming networks) acquisition of a 34% stake in Hughes Electronics Corporation in 2003, which gave it a de facto control over DirecTV Holdings, LLC (a direct broadcast satellite service provider which is wholly-owned by Hughes), and the 2011 joint venture agreement between Comcast, GE, and NBCU, which gave Comcast (the largest cable operator and Internet service provider in the U.S.) a controlling 51% stake in a joint venture that owns broadcast TV networks and stations, and various cable programming. In the UK, BSkyB (a leading TV broadcaster) acquired in 2006 a 17.9% stake in ITV (UK’s largest TV content producer). The UK competition commission found that the acquisition gave BSkyB effective control over ITN and argued that BSkyB would use it to “reduce ITV’s investment in content” and “influence investment by ITV in high-definition television (HDTV) or in other services requiring additional spectrum.”
leads to input foreclosure, is particularly profitable when the upstream supplier is initially held by dispersed shareholders. In that case, the downstream buyer acquires the minimal stake that ensures control over the upstream supplier at a price that reflects the supplier’s pre-acquisition value. The downstream buyer then internalizes only the reduction in the value of the stake it acquires. The rest of the upstream loss is borne by the remaining shareholders of the upstream supplier, who effectively subsidize the input foreclosure.\(^2\) However, when the upstream supplier has initially a controlling shareholder, the downstream buyer needs to compensate this shareholder for reduction in the value of his entire stake in order to induce him to sell a controlling stake to the downstream buyer. Since this stake may well exceed the minimal stake that ensures control, backward interagration is more costly when the upstream supplier has a controlling shareholder and therefore it is less likely to occur and lead to input foreclosure.

By contrast, input foreclosure is less profitable under partial forward integration than under full vertical integration because part of the downstream gain from foreclosure is now captured by the minority shareholders of the integrated downstream buyer, while the upstream supplier bears the entire cost. We show that this transfer of wealth to the monority shareholders of the upstream supplier renders partial forward integration unprofitable when the downstream buyer is initially held by dispersed shareholders, although it may be profitable when the downstream buyer has a controlling shareholder whose controlling stake is sufficiently large (i.e., there are relatively few minority shareholder who receive a subsidy).

We also consider a few extensions of our basic setup. First, we consider the possibility that the downstream buyer holds a toehold (i.e., an initial non controlling stake) in the upstream supplier before it has the opportunity to acquire full control. We show that the toehold weakens the incentive of the downstream buyer to acquire control over the upstream supplier and use it to foreclose a downstream rival whenever the upstream supplier is initially held by a controlling shareholder, but not if the upstream suppliers is initially held by dispersed shareholders. Second, we show that the controlling shareholder of a downstream firm will prefer to acquire control over an upstream supplier through some firm under his control in which he holds a small stake, rather

\(^2\) Of course, if the rights of the passive shareholders of the upstream supplier are protected effectively, the downstream buyer would be unable to use its control to foreclose the downstream rival. In reality, however, it may be very hard to prevent such foreclosure on the grounds that it expropriates the wealth of passive shareholders. Indeed had the protection of passive shareholders been perfect, antitrust authorities would have no reason to be concened about the possibility of input foreclosure following partial backward integration. The cases mentioned above indicate however that antitrust authorities are concened about this possibility.
than acquire control directly. In particular, an acquisition by a controlling shareholder expands the range of parameters for which input foreclosure occurs. Finally, we show that competition between the two downstream firms for the acquisition of control over an upstream supplier boosts the price paid for the acquired shares, although it does not alter the range of parameters for which foreclosure takes place.

There are three strands in the literature on input foreclosure. Bolton and Whinston (1993) consider a model in which two downstream firms invest in order to boost the quality of their products before they trade with an upstream supplier. Vertical integration strengthens the incentive of the integrated downstream firm to invest and weakens the incentive of nonintegrated firm to invest. As a result, the nonintegrated firm has a lower chance to buy the input when its supply is limited. Bolton and Whinston interpret this situation as input foreclosure. Unlike in Bolton and Whinston (1993), where foreclosure is a by-product of the effect of integration on downstream investments, foreclosure in the other two strands of the literature is due to a deliberate refusal of an upstream supplier to supply the input. In Hart and Tirole (1990), an upstream supplier prefers to deal exclusively with a single downstream firm in order to allow this firm to monopolize the downstream market. The upstream supplier in turn extracts the monopoly downstream profit via a nonlinear contract. The third strand in the literature, due to Ordover, Saloner, and Salop (1990) and Salinger (1988), considers models in which the vertically integrated firm deliberately forecloses downstream rivals in order to raise their costs of buying the input from alternative suppliers. This benefits the integrated downstream firm, who now faces weaker rivals in the downstream market.

Our model is closely related to the third, raising rival cost, strand. Similarly to these models, the upstream supplier in our model also forecloses the downstream rival in order to give the integrated downstream firm a strategic advantage in the downstream market. In our model though, foreclosure does not raise the costs of downstream rivals but rather it diminishes the value that the downstream rival can offer consumers. More importantly, our paper focuses on partial, rather than full vertical integration.

There are only few papers which consider the competitive effects of partial vertical integration. Greenlee and Raskovitch (2006) and Hunold, Röller, and Stahl (2012) consider passive

\footnote{See Rey and Tirole (2007) and Riordan (2008) for literature surveys.}

\footnote{The assumption that the upstream firm can commit to foreclose the downstream rival has criticized as problematic, see Hart and Tirole (1990) and Reiffen (1992) and see Ordover, Salop, and Saloner (1992) for a response. Several papers, including Ma (1997), Chen (2001), Choi and Yi (2001) and Church and Gandal (2000), have proposed models that are immune to this criticism.}
acquisitions, which affect the incentive of the acquirer, but do not have a direct effect on the target’s strategy like in our paper. We are aware of only two papers which consider the acquisition of partial controlling stakes. Baumol and Ordover (1994) show that when a downstream firm controls a bottleneck owner with a partial ownership stake, it has an incentive to divert business to itself, even if downstream rivals are more efficient, because it fully captures the benefits from this diversion, but bears only part of the associated upstream loss.\textsuperscript{5} Spiegel (2013) examines a model in the spirit of Bolton and Whinston (1993), in which foreclosure arises due to the effect of vertical integration on the incentives of the downstream firms to invest in the quality of their products. He shows that relative to full vertical integration, partial vertical integration may either alleviate or exacerbate the concern for vertical foreclosure and examine the implications for consumer welfare. Neither one of these papers, however, examines how the incentive to integrate depends on the ownership structure of the target, which is the main focus of the current paper.

The rest of the paper is organized as follows. The basic model is presented in Section 2. In Section 3 we examine the incentive to engage in “input foreclosure” following partial backward or forward integration. Our main results appear in Section 4, where we examine how the incentive to partially integrate and engage in input foreclosure depend on the initial ownership structure of the target firm. In Section 5 we study three extensions of our basic setup, and in Section 6 we conclude. In the Appendix we show

\section{The model}

Consider two downstream firms, $D_1$ and $D_2$, that use up to $N \geq 1$ differentiated inputs to provide a final good/service to consumers. Each input $i = 1, 2, \ldots, N$ is produced by a single upstream supplier $U_i$. To simplify matters, we assume that the cost that an upstream supplier incurs when it serves a downstream firm is $c$.\textsuperscript{6} Let $\Pi(k,l)$ denote the (reduced form) profit of a downstream firm when it uses $k$ inputs and its rival uses $l$ inputs, before any payments to upstream suppliers. Throughout the analysis we will impose the following assumption:

\textsuperscript{5}Reiffen (1998) examines the stock market reaction to Union Pacific (UP) Railroad’s attempt in 1995 to gain effective control over Chicago Northwestern (CNW) Railroad with a partial ownership stake. However, he finds that CNW’s stock price reacted positively, rather than negatively, to events that made the merger more likely to be consummated. This finding is inconsistent with the idea that UP would have diverted profits from CNW to itself by foreclosing competing railroads.

\textsuperscript{6}We can easily modify this linearity and assume that the cost of serving only one downstream firm is $c_1$ and the cost of serving two is $c_2 > c_1$. 

5
A1 $\Pi(k, l)$ is increasing with $k$ at a decreasing rate and decreasing with $l$

For example, $D_1$ and $D_2$ can be two cable or satellite TV providers, which buy content TV channels in the upstream content market, or online retailers, who sell different brands on their websites. Assumption A1 is then natural in these cases since, other things being equal, a cable TV provider faces a higher demand when it offers more channels while his rival offers less and likewise, an online retailer faces a higher demand when it offers more brands and its rival offers fewer brands.

The sequence of events is as follows. At the outset, all firms are independently owned. Then, either one downstream firm, $D_1$, acquires a controlling stake in upstream supplier $U_1$ (backward integration), or $U_1$ acquires a controlling stake in $D_1$ (forward integration); we denote the minimal ownership stake that gives the acquirer control over the target by $\alpha$.\footnote{Typically the assumption in the literature is that $\alpha = 50\%$. In reality, though, $\alpha$ can be well below 50\%. For example, News Corp. acquired a de facto control in Hughes Electronics Corporation in 2004 by acquiring a 34\% stake; see FCC, MB Docket No. 03-124, Memorandum Opinion and Order, January 14, 2004. In the UK, the Competition Commission concluded that BSkyB’s acquisition of a 17.9\% stake in ITV in 2006, gave BSkyB “the ability materially to influence the policy of ITV which gives rise to common control” (see Paragraph 2 in Competition Commission, 2007, “Acquisition by British Sky Broadcasting Group plc of 17.9 per cent of the Shares in ITV plc”). We will therefore not impose restrictions on $\alpha$.} We will say that integration is partial if $\alpha < 1$. Hence, under partial backward integration, $D_1$ acquires a stake $\alpha \leq \alpha < 1$ in $U_1$, while under partial forward integration $U_1$ acquires a stake $\alpha \leq \alpha < 1$ in $D_1$.

Given the new ownership structure, each of $N$ upstream suppliers decides whether to supply the input to both downstream firms or to only one. These decisions are publicly observable and irreversible.\footnote{This assumption can be justified as in Church and Gandal (2000) and Choi and Yi (2000), where each upstream firm needs to adapt the input to the special needs of each downstream firm. The assumption allows us to sidestep the “commitment problem,” which arise for example in Ordover, Salop, and Saloner (1990).} The upstream suppliers then make take-it-or-leave-it offers to the downstream firms. Inputs are then procured, and the final product is produced and payoff are realized.

3 Incentives for input foreclosure

In this section, we examine the incentive of $U_1$ to vertically foreclose $D_2$ under four scenarios: (i) $U_1$ is independent of $D_1$, (ii) $U_1$ and $D_1$ are fully integrated (full integration), (iii) $D_1$ has a partial ownership stake in $U_1$ (partial backward integration), and (iv) $U_1$ has a partial ownership stake in $D_1$ (partial forward integration).
Before we turn to the different scenarios, we first consider the second stage of the game, in which the \( N \) upstream suppliers make simultaneous take-it-or-leave-it offers to the two downstream firms. To fix ideas, suppose that downstream firm \( D_i \) already buys \( k - 1 \) inputs and downstream firm \( D_j \) buys \( l \) inputs. The marginal willingness of \( D_i \) to pay for the \( k' \)th input is

\[
\Delta_1 (k, l) = \Pi (k, l) - \Pi (k - 1, l).
\]

This expression represents the incremental profit of \( D_i \) from adding the \( k' \)th input, given that the rival, \( D_j \), uses \( l \) inputs. Assumption A1 implies that \( \Delta_1 (k, l) \) is positive but decreasing with \( k \).

For later use, let us denote the externality that an increase in the number of inputs used by \( D_j \) imposes on \( D_i \)'s profit by

\[
\Delta_2 (k, l) = \Pi (k, l) - \Pi (k, l - 1).
\]

By Assumption A1, \( \Delta_2 (k, l) < 0 \) for all \( k \) and \( l \).

To ensure that selling \( N \) inputs is profitable, we will make the following assumption:

**A2** \( \Delta_1 (N, N) > c \)

While Assumption A2 ensures that selling inputs is profitable if both downstream firms buy all \( N \) inputs, it is possible that an upstream supplier may prefer to sell its input to only one of the two downstream firms. To see why, note that when \( D_j \) increases the number of inputs it uses from \( l - 1 \) to \( l \), the marginal willingness of \( D_i \) to pay for input \( l \) changes by

\[
\Delta_{12} (k, l) = \Delta_1 (k, l) - \Delta_1 (k, l - 1).
\]

Although in general \( \Delta_{12} (N, N) \) could be either positive or negative, the example that we present in the Appendix shows that it is reasonable to assume that \( \Delta_{12} (N, N) < 0 \). That is, the marginal willingness of \( D_i \) to pay for inputs decreases when \( D_j \) is using an extra input. For the sake of concreteness, we will assume that this is indeed the case:

**A3** \( \Delta_{12} (k, l) \leq 0 \) for all \( k, l \)

Given that selling an extra input to \( D_j \) depresses the price that \( D_i \) is willing to pay, at least in principle, it could be that an upstream firm may be unwilling to supply both downstream firms. The following assumption rules out this possibility and ensures that under non-integration, both downstream firms buy all \( N \) inputs:
**A4** $\Delta_1 (k, l) - c > -\Delta_{12} (k, l)$ for all $k, l$

Assumption A4 implies that the maximal profit that an upstream supplier can make by selling an extra input to $D_i$, $\Delta_1 (k, l) - c$, exceeds $-\Delta_{12} (k, l)$ which is the associated loss of profit from selling to $D_j$. With Assumption A4 in place, we prove the following result, which establishes the equilibrium behavior of non-integrated upstream suppliers:

**Lemma 1:** In equilibrium, non-integrated upstream suppliers sell to both $D_1$ and $D_2$, irrespective of whether $D_1$ and $U_1$ are partially or fully integrated and irrespective of whether $U_1$ forecloses $D_2$ or not. If $D_1$ and $U_1$ are integrated and $U_1$ forecloses $D_2$, upstream suppliers 2, ..., $N$ charge $D_1$ a price $\Delta_1 (N, N)$ for the input and charge $D_2$ a price of $\Delta_1 (N - 1, N)$. If $D_2$ is not foreclosed, all upstream suppliers charge $D_2$ a price of $\Delta_1 (N, N)$ and all non-integrated upstream suppliers charge $D_1$ a price of $\Delta_1 (N, N)$.

**Proof:** By Assumption A2, in equilibrium each supplier sells to at least one downstream firm. Now suppose by way of negation that there exists an equilibrium in which $k_1$ suppliers sell exclusively to $D_1$, $k_2$ suppliers sell exclusively to $D_2$, and $N - k_1 - k_2 \geq 0$ suppliers sell to both downstream firms. In this equilibrium, $D_1$ buys $N - k_2$ inputs and $D_2$ buys $N - k_1$ inputs. Hence, the marginal willingness of $D_1$ to pay for inputs is $\Delta_1 (N - k_2, N - k_1)$, while the marginal willingness of $D_2$ to pay for inputs is $\Delta_1 (N - k_1, N - k_2)$. Since the upstream suppliers make take-it-or-leave-it offers to the two downstream firms, in equilibrium, each downstream firm pays a price equal to its marginal willingness to pay. Consequently, the profit of each supplier that sells exclusively to $D_1$ is

$$\Delta_1 (N - k_2, N - k_1) - c.$$ 

If the supplier also sells to $D_2$, its profit becomes:

$$\Delta_1 (N - k_2, N - k_1 + 1) + \Delta_1 (N - k_1 + 1, N - k_2) - 2c.$$ 

Selling to both $D_1$ and $D_2$ is more profitable since

$$[\Delta_1 (N - k_2, N - k_1 + 1) + \Delta_1 (N - k_1 + 1, N - k_2) - 2c] - [\Delta_1 (N - k_2, N - k_1) - c]$$
$$= \Delta_1 (N - k_1 + 1, N - k_2) - c - \left[\Delta_1 (N - k_2, N - k_1) - \Delta_1 (N - k_2, N - k_1 + 1)\right] > 0,$$
$$= -\Delta_{12}(N-k_2,N-k_1+1) > 0,$$
where the inequality follows from Assumption A4. A similar argument applies when suppliers sell exclusively to $D_2$. Hence, in equilibrium, suppliers $2, \ldots, N$ sell to both $D_1$ and $D_2$.

The last part of the lemma follows because $D_1$ and $D_2$ pay input prices that reflect their marginal willingness to pay. ■

Given Lemma 1, we only need to consider the equilibrium behavior of $U_1$. Clearly, if $U_1$ is not integrated with $D_1$ (fully or partially), then its equilibrium behavior is no different than that of other upstream suppliers. Hence, we can report the following Corollary to Lemma 1.

**Corollary 1:** Under non-integration, both $D_1$ and $D_2$ buy all $N$ inputs at a price of $\triangle_1 (N, N)$. The resulting profit of each downstream firm is

$$V_0^D = \Pi (N, N) - N \triangle_1 (N, N),$$

while the profit of each upstream supplier is

$$V_0^U = 2 [\triangle_1 (N, N) - c].$$

Note that the equilibrium profit of each downstream firm is positive since $\Pi (N, N) = \sum_{k=1}^{N} \triangle_1 (k, N)$, so $\Pi (N, N) - N \triangle_1 (N, N) = \sum_{k=1}^{N} [\triangle_1 (k, N) - \triangle_1 (N, N)] > 0$, where the inequality follows because $\triangle_{11} (\cdot, \cdot) < 0$ implies that $\triangle_1 (k, N) > \triangle_1 (N, N)$ for all $k < N$. The equilibrium profit of upstream suppliers is profitable by Assumption A2.

We next characterize the equilibrium under no integration, full integration, partial backward integration, and partial forward integration. As we shall see, in these cases, foreclosure might arise in equilibrium.

### 3.1 Full vertical integration

Under full vertical integration, $D_1$ and $U_1$ fully merge to create a new firm, which we call $DU_1$. Given Lemma 1, we only need to check whether the integrated firm, $DU_1$, is interested in selling the input to $D_2$ and if so, at which price.

To this end, let $w$ denote the price that $D_1$ pays $U_1$ for the input. Since $D_1$ and $U_1$ are fully merged, $w$ is merely a transfer payment within the same organization, and hence it is irrelevant. However, under partial integration, $w$ matters. In particular, when $D_1$ partially controls $U_1$, it would like to set $w$ as low as possible, in which case, $D_1$ essentially expropriates the wealth of $U_1$'s
non-controlling shareholders. In the opposite case where $U_1$ partially controls $D_1$, $U_1$ would like to set $w$ as high as possible, in order to expropriate the wealth of $D_1$’s non-controlling shareholders. This means that in principle, there are two channels through which partial ownership matters: (i) it can lead to the foreclosure of $D_2$, and (ii) it can lead to a distortion of $w$ and hence to a transfer of wealth from $U_1$ to $D_1$ or vice versa. The second channel however can arise even if $D_1$ were a monopoly in the downstream market and hence is not directly related to the interaction between vertical integration and competition which is our main focus. We will therefore “shut down” this channel by assuming that $U_1$ cannot discriminate in favor of or against $D_1$ and hence $w$ must be equal to the price that all other upstream suppliers charge for their inputs.\footnote{For instance, discrimination against $D_1$ could be deemed as an outright violation of the fiduciary duties that $D_1$’s management has towards $D_1$’s non-controlling shareholders. Likewise, discrimination in favor of $D_1$ would be deemed as an outright violation of the fiduciary duties that $U_1$’s management has towards $U_1$’s non-controlling shareholders.} Using Lemma 1, this price is $w = \Delta_1(N, N - 1)$ if $D_2$ is foreclosed, and $w = \Delta_1(N, N)$ if $D_2$ is not foreclosed.

If $DU_1$ sells to $D_2$, then both downstream firms buy all $N$ inputs, so the marginal willingness of each of them to pay for inputs (and hence the price of each input) is $\Delta_1(N, N)$. Hence, the resulting profit of $DU_1$ is $V^D_0 + V^U_0$. If $DU_1$ forecloses $D_2$, then the willingness of $D_1$ to pay for inputs increases to $\Delta_1(N, N - 1)$, so the downstream profit of $DU_1$ becomes

$$V^D_1 = \Pi(N, N - 1) - N\Delta_1(N, N - 1),$$

while the upstream profit of $DU_1$ becomes

$$V^U_1 = \Delta_1(N, N - 1) - c.$$  \hspace{1cm} (4)

Overall, the profit of $DU_1$ when $D_2$ is foreclosed is $V^D_1 + V^U_1$.

Foreclosing $D_2$ is therefore optimal if and only if $V^D_1 + V^U_1 \geq V^D_0 + V^U_0$. Using equations (1), (2), (3), and (4), this is true whenever

$$L < G,$$  \hspace{1cm} (5)

where

$$L \equiv V^U_0 - V^U_1 = \Delta_1(N, N) - c + \Delta_{12}(N, N),$$

and

$$G \equiv V^D_1 - V^D_0 = -\Delta_2(N, N) + N\Delta_{12}(N, N).$$

To interpret (5), note that $L$ represents the loss of upstream profit due to foreclosure: $\Delta_1(N, N) - c$ represents the forgone upstream profit due to the foreclosure of $D_2$, and $\Delta_{12}(N, N) \equiv \Delta_1(N, N) - \Delta_2(N, N)$.
\( \Delta_1 (N, N - 1) < 0 \) is the increase in the price that the upstream unit of \( DU_1 \) charges the downstream unit of \( DU_1 \) when \( D_2 \) is foreclosed (in which case the willingness of \( DU_1 \)'s downstream unit to pay for inputs increases from \( \Delta_1 (N, N) \) to \( \Delta_1 (N, N - 1) \)).\(^{10}\) This price increase cuts the upstream loss from foreclosure, though by Assumption A4, overall, \( L > 0 \).

The effect of foreclosure on the downstream profit of \( DU_1 \) is represented by \( G \). The first term, \( -\Delta_2 (N, N) \equiv - (\Pi (N, N) - \Pi (N, N - 1)) \), is the extra downstream profit that \( DU_1 \) makes due to the foreclosure of \( D_2 \). The second term reflects the idea that once \( D_2 \) is foreclosed, the willingness of \( D_1 \) to pay for inputs increases from \( \Delta_1 (N, N) \) to \( \Delta_1 (N, N - 1) \); since \( D_1 \) buys \( N \) inputs and since we assume that upstream suppliers have all the bargaining power, the total increase in \( DU_1 \)'s payment for inputs is \( N \Delta_{12} (N, N) \). As far as know, this adverse effect of foreclosure on input prices has not been identified earlier in the literature. Of course, this effect is extreme in our model due to our assumption that upstream suppliers have all the bargaining power when they negotiate with downstream firms. This effect will be less extreme if downstream firms were to have some bargaining power too and it would disappear altogether if the downstream firm were to make the upstream suppliers take-it-or-leave-it offers.

We can now summarize the discussion as follows:

**Lemma 2:** Suppose that \( D_1 \) and \( U_1 \) fully integrated. Then there exists a unique equilibrium in which the integrated firm \( DU_1 \) forecloses \( D_2 \) if and only if \( G > L \).

Since \( L > 0 \) by Assumption A4 (foreclosure entails a loss of upstream profits), Lemma 2 implies that a necessary (though not sufficient) condition for foreclosure under full integration is that \( G > 0 \). That is, foreclosure must boost the downstream profit of \( DU_1 \). When \( G < 0 \), foreclosure never arises in equilibrium. To make the analysis interesting, we will therefore impose the following assumption:

**A5** \( G \equiv -\Delta_2 (N, N) + N \Delta_{12} (N, N) > 0 \)

### 3.2 Partial backward integration

Now suppose that \( D_1 \) acquires a fraction \( \alpha \in [a, 1) \) of \( U_1 \)'s shares, where \( \alpha \) is the minimal equity stake that gives \( D_1 \) full control over \( U_1 \). The remaining \( 1 - \alpha \) stake in \( U_1 \) is held by dispersed,\(^{10}\) As mentioned earlier, although payments within \( DU_1 \) are mere transfers and hence wash out, it is instructive to write them explicitly since these payments will play an important role when we consider partial vertical integration.
passive, shareholders.

To characterize the equilibrium, we will follow the same steps as in the full integration case. The only difference is that now, $U_1$’s decisions are taken by $D_1$ with the objective of maximizing $D_1$’s profit, plus $\alpha$ times $U_1$’s profit, instead of maximizing the joint profit of $D_1$ and $U_1$ (the difference then is that now $\alpha < 1$, while under full integration, effectively $\alpha = 1$).

Given Lemma 1, we only need to consider $D_1$’s decision on whether to use its control over $U_1$ to foreclose $D_2$. If $D_2$ is not foreclosed then by corollary 1, the profits of $D_1$ and $U_1$ are $V^D_0$ and $V^U_0$, so $D_1$’s overall profit, including its stake in $U_1$’s profit, is $V^D_0 + \alpha V^U_0$. By contrast, if $D_1$ uses its control over $U_1$ to foreclose $D_2$, then, the profits of $D_1$ and $U_1$ becomes $V^D_1$ and $V^U_1$, so $D_1$’s overall profit becomes $V^D_1 + \alpha V^U_1$.

Clearly, $U_1$ will foreclose $D_2$ if and only if $V^D_1 + \alpha V^U_1 \geq V^D_0 + \alpha V^U_0$, which implies in turn that

$$\frac{\alpha (V^U_0 - V^U_1)}{L} < \frac{V^D_1 - V^D_0}{G}. \tag{6}$$

Since $L > 0$ by Assumption A4, (6) is more likely to hold when $\alpha$ is small. This implies in turn that $D_1$ would like to use its control over $U_1$ to foreclose $D_2$ only when $\alpha$ is sufficiently small.

Intuitively, under partial backward integration, $D_1$ bears only a fraction $\alpha$ of the loss $L$ to $U_1$ from foreclosing $D_2$, but it captures the entire downstream gain, $G$. Hence, $D_1$ has a stronger incentive to use its control to foreclose $D_2$, relative to the full information case. Moreover, this incentive becomes stronger as $\alpha$ decreases.

To reduce the number of cases we need consider, we now impose the following assumption:

**A6** $G > \alpha L$

Assumption A6 implies that there exists a range of ownership stakes, $[\alpha, \min\{\frac{G}{L}, 1\}]$, that give $D_1$ control over $U_1$ and induce $D_1$ to use its control over $U_1$ to foreclose $D_2$. Without this assumption, $D_1$ will never find it optimal, when it controls $U_1$, to foreclose $D_2$. We can now summarize the discussion in the following lemma:

**Lemma 3:** Suppose that $D_1$ owns a controlling stake $\alpha \geq \alpha$ in $U_1$. Then there exists a unique equilibrium in which $D_1$ uses its control over $U_1$ to foreclose $D_2$ if and only if $\alpha \in [\alpha, \min\{\frac{G}{L}, 1\}]$. 


3.3 Partial forward integration

Next, we consider the case where $U_1$ acquires a fraction $\alpha \in [\alpha, 1)$ of $D_1$’s shares. As before, $\alpha$ is the minimal equity stake that gives the acquirer, $U_1$ in this case, full control over the target’s operating decisions. The remaining $1 - \alpha$ stake in $D_1$ is held by dispersed, passive, shareholders.

To characterize the equilibrium, note from Corollary 1 that under non-foreclosure, the profits of $D_1$ and $U_1$ are $V^D_0$ and $V^U_0$, so $U_1$’s overall profit, including its stake in $D_1$, is $\alpha V^D_0 + V^U_0$. If $U_1$ forecloses $D_2$, the profits of $D_1$ and $U_1$ become $V^D_1$ and $V^U_1$, so $U_1$’s overall profit becomes $\alpha V^D_1 + V^U_1$. Clearly, $U_1$ will foreclose $D_2$ if and only if $\alpha V^D_1 + V^U_1 \geq \alpha V^D_0 + V^U_0$, which holds whenever

$$\frac{(V^D_0 - V^U_1)}{L} < \alpha \frac{(V^D_1 - V^D_0)}{G}.$$  \hspace{1cm} (7)

Equation (7) shows that $U_1$ would like to foreclose $D_2$ only when $\alpha$ is sufficiently large. Intuitively, under forward integration, $U_1$ bears the entire upstream loss from foreclosing $D_2$, but since it only holds a fraction of $D_1$’s ownership, it captures only a fraction $\alpha$ of the associated downstream gain (the rest of the gain accrues to the passive shareholders of $D_1$). Clearly then, $U_1$ has a stronger incentive to foreclose $D_2$ when $\alpha$ increases.

The next lemma summarizes the discussion:

**Lemma 4:** Suppose that $U_1$ owns a controlling stake $\alpha \geq \alpha_0$ in $D_1$. Then there exists a unique equilibrium in which $U_1$ forecloses $D_2$ if and only if $\alpha G \geq L$. Clearly, if $L > G$, there is no foreclosure in equilibrium.

3.4 Comparison

Having characterized the equilibrium under the full integration, partial backward integration, and partial forward integration, the following proposition follows immediately from Lemmas 2-4:

**Proposition 1:** Partial backward integration expands the range of parameters for which $D_2$ is foreclosed relative to full integration, while partial forward integration shrinks it. Moreover, if foreclosure is profitable under full integration, i.e., $G \geq L$, then partial backward integration always leads to foreclosure, while partial forward integration leads to foreclosure only when $\alpha$ is sufficiently close to 1. And, if foreclosure is not profitable under full integration, i.e., $G < L$, then partial backward integration leads to foreclosure only when $\alpha$ is sufficiently small, while partial forward integration never leads to foreclosure.
Proposition 1 is reminiscent of Baumol and Ordover (1994). It shows that partial backward integration raises the concern for input foreclosure, while partial forward integration alleviates this concern.

We conclude this section by examining how our results change when we relax the assumption that the upstream suppliers can make the two downstream firms take-it-or-leave-it offers. To this end, suppose that if $D_i$ buys $k$ inputs and $D_j$ gets $l$ inputs, $D_i$ pays each upstream supplier a price of $\mu \Delta_1 (k, l)$ for the input, where $\mu \in \left[ \frac{\epsilon}{\Delta_1(N,N)}, 1 \right]$ measures the bargaining power of upstream suppliers. We assume that $\mu \geq \frac{\epsilon}{\Delta_1(N,N)}$ to ensure that the marginal willingness of $D_i$ to pay for inputs exceeds their cost. With these assumptions in place, the post-acquisition values of $D_1$ and $U_1$ are

$$V_{1D} = \Pi (N, N - 1) - N \mu \Delta_1 (N, N - 1), \quad V_{1U} = \mu \Delta_1 (N, N - 1) - c,$$

while their pre-acquisition values are

$$V_{0D} = \Pi (N, N) - N \mu \Delta_1 (N, N), \quad V_{0U} = 2 \left[ \mu \Delta_1 (N, N) - c \right].$$

As a result, the upstream loss from foreclosure becomes

$$L_\mu = V_{0U} - V_{1U} = \mu \Delta_1 (N, N) - c + \mu \Delta_{12} (N, N),$$

and the downstream gain from foreclosure becomes

$$G_\mu = V_{1D} - V_{0D} = -\Delta_2(N, N) + N \mu \Delta_{12} (N, N).$$

By Assumptions A3 and A4, $L_\mu$ is increasing, while $G$ is decreasing with $\mu$. Hence,

**Proposition 2:** An increase in the bargaining power of upstream suppliers vis-a-vis downstream firms shrinks the range of parameters for which $D_2$ is foreclosed.

The intuition for Proposition 2 is simple: an increase in the bargaining power of upstream suppliers vis-a-vis downstream firms boosts the upstream profits and depresses the downstream profits. Since input foreclosure shifts profits from the upstream firm to the downstream firm, an increase in the bargaining power of upstream suppliers makes foreclosure less attractive.

### 4 Input foreclosure under endogenous ownership structure

So far we considered the incentive of a vertically integrated firm to foreclose a downstream rival. We now examine the incentive to vertically integrate in the first place. To this end, we assume
that initially $D_1$ and $U_1$ are not integrated, and then we ask whether $D_1$ would like to acquire a controlling stake, $\alpha \geq \alpha$, in $U$ (backward integration), or $U_1$ would like to acquire a controlling stake, $\alpha \geq \alpha$, in $D_1$ (forward integration), and examine how this incentive depends on the initial ownership structure of the target firm.

Specifically, we consider two cases:

(i) Initially, the target ($U_1$ in the case of partial forward integration and $D_1$ in the case of partial backward integration) has a single controlling shareholder whose stake is $\alpha_C \in [\alpha, 1]$; the remaining $1 - \alpha_C$ stake in $U_1$ (if any) is held by passive shareholders.

(ii) Initially, the target is owned by a mass 1 of atomistic, dispersed, shareholders.

4.1 Backward integration: $U_1$ has initially a single controlling shareholder

In this section we examine the incentive of $D_1$ to acquire a controlling stake $\alpha \in [\alpha, \alpha_C]$ in $U_1$ from $U_1$’s initial controller (whose initial stake is $\alpha_C$). To acquire $\alpha$, $D_1$ makes $U_1$’s controller a take-it-or-leave-it offer $b$, although as we shall see below, the assumption that $D_1$ has all the bargaining power vis-a-vis $U_1$’s controller is not essential. If $D_1$’s offer is accepted, $D_1$ becomes the new controlling shareholder in $U_1$.\footnote{Since the value of $U_1$’s shares is the same to all shareholders other than $D_1$, it is immaterial for the analysis whether the initial controller of $U_1$ retains a minority stake $\alpha_C - \alpha$, or sells this minority stake (fully or partially) to dispersed shareholder.} We begin with the following lemma.

**Lemma 5:** Acquisition of a partial controlling stake in $U_1$, which is not followed by the foreclosure of $D_2$, is profitable for $D_1$ if and only if the post-acquisition price $w$ that $D_1$ pays $U_1$ for the input is below $\Delta_1(N, N)$, which is the price that all other upstream suppliers charge for the input.

**Proof:** We begin by considering the price that $D_1$ needs to pay in order to acquire the controlling stake, $\alpha$, in $U_1$. If $D_2$ is not foreclosed following the acquisition, then the post-acquisition value of $U_1$ becomes

$$\tilde{V}_1^U = \Delta_1(N, N) + w - 2c,$$

where $w - c$ is $U_1$’s profit from selling to $D_1$ and $\Delta_1(N, N) - c$ is the profit from selling to $D_2$. By Corollary 1, the pre-acquisition value of $U_1$, $V_0^U$, is given by (2).
Using these expressions, the minimal acceptable bid, $b^U$, must leave $U_1$’s controller indifferent between accepting and rejecting the offer:

$$b^U + (\alpha_C - \alpha) \hat{V}_1^U = \alpha_C V_0^U.$$  

Hence,

$$b^U = \alpha_C [V_0^U - \hat{V}_1^U] + \alpha \hat{V}_1^U \tag{9}$$

where the second equality follows from (2) and (8). Consequently, if $D_1$ acquires a controlling stake, $\alpha$, in $U_1$, its payoff becomes:

$$[\Pi (N, N) - (N - 1) \Delta_1 (N, N) - w] + \alpha \hat{V}_1^U - \left( \alpha_C [V_0^U - \hat{V}_1^U] + \alpha \hat{V}_1^U \right) \tag{10}$$

where $\Pi (N, N)$ is the downstream profit of $D_1$ and $(N - 1) \Delta_1 (N, N)$ is the total payment of $D_1$ to non-integrated upstream suppliers. If $D_1$ does not acquire a controlling stake in $U_1$, then by Corollary 1, its payoff, $V_0^D$, is given by (1).

The difference between $D_1$’s payoff with and without the acquisition is given by the difference between (10) and (1):

$$\Delta_1 (N, N) - w - \alpha_C [\Delta_1 (N, N) - w] = (1 - \alpha_C) [\Delta_1 (N, N) - w].$$

The last expression is positive if and only if $w < \Delta_1 (N, N)$.

The idea behind Lemma 5 is simple: so long as $w < \Delta_1 (N, N)$, $D_1$ uses its control over $U_1$ to force $U_1$ to sell it the input at an artificially low price. This is profitable for $D_1$ because it expropriates some of the wealth of $U_1$’s passive shareholders. As mentioned in Section 3.1, this motive for partial vertical integration exists even if there is no competition with rival firms either in the downstream or the upstream markets; we will therefore abstract from this consideration by assuming that under integration (full or partial), $D_1$ pays $U_1$ the same input price that it pays other upstream suppliers. That is, $w = \Delta_1 (N, N - 1)$ if $D_2$ is foreclosed, and $w = \Delta_1 (N, N)$ if $D_2$ is not foreclosed.
Now, consider the case where $D_2$ is foreclosed after $D_1$ acquires control over $U_1$. Then, the post-acquisition value of $U_1$ is $V_1^U$, so analogously to (9), the minimal acceptable offer is given by,

$$b^U = \alpha_C V_0^U - (\alpha_C - \alpha) V_1^U$$

$$= \alpha_C [V_0^U - V_1^U] + \alpha V_1^U. \quad (11)$$

That is, $b^U$ is equal to the post-acquisition value of the acquired shares, $\alpha V_1^U$, plus a premium, $\alpha_C L$, which compensates the initial controller of $U_1$ for the loss in the value of his initial stake due to the foreclosure of $D_2$.

But will $D_1$ agree to pay $b^U$ for a controlling stake in $U_1$? To answer this question, note that since $w = \Delta_1 (N, N - 1)$ when $D_2$ is foreclosed, $D_1$’s payoff when it acquires a controlling stake in $U_1$ and then forecloses $D_2$ is:

$$V_1^D + \alpha V_1^U - (\alpha_C L + \alpha V_1^U) = V_1^D - \alpha_C L.$$ 

Notice that this expression depends on $\alpha_C$, which is the size of the controlling stake that the initial controller of $U_1$ holds, but not on the actual size of the acquired stake, $\alpha$. The reason is that $D_1$ pays $U_1$’s controller a fair price for the shares it acquires, plus a premium that fully compensates $U_1$’s initial controller for the drop in the value of the stake that he retains in $U_1$. Since $D_1$ bears the loss of value on the stake it acquires, it fully internalizes the loss to the entire stake $\alpha_C$ due to foreclosure (a fraction $1 - \alpha_C$ of the loss is borne by the passive shareholders of $U_1$).

$D_1$ will therefore acquire a controlling stake in $U_1$ and use it to foreclose $D_2$ if and only if its post-acquisition payoff exceeds its pre-acquisition payoff, which by Corollary 1 equals $V_0^D$:

$$V_1^D - \alpha_C L - V_0^D = G - \alpha_C L \geq 0. \quad (12)$$

Since this condition is independent of the acquired stake, $\alpha$, $D_1$ is indifferent to size of its controlling stake (provided of course that it is above $\alpha$).\(^{12}\) Since $L > 0$ by Assumption A4, it is clear that (12) is more likely to hold when $\alpha_C$ is small.

**Proposition 3:** Suppose that initially, $U_1$ has a single controlling shareholder. Then, in equilibrium, $D_1$ will acquire a controlling stake in $U_1$ and use it to foreclose $D_2$ if and only if $G \geq \alpha_C L$. In

---

\(^{12}\)To the extent that foreclosure is easier when the initial controller of $U_1$ is out of the picture, $D_1$ might as well acquire the entire stake $\alpha_C$ of $U_1$’s initial controller.
particular, the acquisition takes place for all $\alpha C \in [\alpha, 1]$ if $G \geq L$, it never takes place if $G < \alpha L$, and whenever $\alpha L \leq G < L$, it takes place only when $\alpha C$ is sufficiently close to $\alpha$.

Proposition 3 implies that if $U_1$ is initially controlled by a single shareholder and if full vertical integration leads foreclosure, i.e., $G \geq L$, then $D_1$ will surely acquire control in $U_1$ and use it to foreclose $D_2$. On the other hand, when full vertical integration does not lead to foreclosure, i.e., $G < L$, then $D_1$ may still acquire control over $U_1$ and then foreclose $D_2$, but only when the initial stake of $U_1$’s controller is sufficiently small. In other words, the proposition suggests that antitrust authorities should be more concerned with backward integration when the controlling stake is acquired from an initial controller whose controlling stake is relatively small.

Finally, it should be emphasized that the above results would continue to hold even if $D_1$ does not have all the bargaining power vis-a-vis $U_1$’s initial controller. To see why, note that the joint payoff of $D_1$ and $U_1$’s initial controller if the acquisition goes through is $V_1^D + \alpha_C V_1^U$, whereas their joint payoff without an acquisition is $V_0^D + \alpha_C V_0^U$. Hence, their joint surplus from partial backward integration is

$$S_{BI} = \left\{ \begin{array}{ll} V_1^D - V_0^D & \text{if } G \geq L \\ \alpha_C [V_0^U - V_1^U] & \text{if } L > G \end{array} \right.$$  

Hence, transferring control over $U_1$ to $D_1$ is jointly profitable if and only if (12) holds (otherwise, either $D_1$ is not interested in acquiring control over $U_1$, or $U_1$’s initial controller is not interested in selling it).\textsuperscript{13}

4.2 Backward integration: $U_1$’s ownership is initially dispersed

We now turn to the case where $U_1$ is initially held by dispersed shareholders. Specifically, we will follow Grossman and Hart (1980) and assume that $U_1$ is initially held by a continuum of atomistic shareholders, whose total mass is 1. In order to acquire a controlling stake $\alpha$ in $U_1$, $D_1$ makes a tender offer to $U_1$’s initial shareholders at a price that reflects a value $V$ for the entire firm. Below we solve for the equilibrium value of $V$ and also determine whether $D_1$ would wish to make the offer restricted and specify a limit on the stake it is willing to acquire (if the tendered stake exceeds the limit, the submitted shares of each tendering shareholder are prorated). We will say that the

\textsuperscript{13}The relative bargaining power of $D_1$ vis-a-vis $U_1$’s initial controller would matter however if $D_1$ has some fixed cost associated with initiating a takeover. In that case, the lower $D_1$’s bargaining power, the less likely the takeover is.
tender offer succeeds if $D_1$ manages to acquire at least a stake of $\alpha$ (and gains control over $U_1$), and we will say that the tender offer fails if $D_1$ does not acquire a stake of at least $\alpha$.

To characterize the equilibrium, recall that the pre-acquisition value of $U_1$ is $V_0^{U_1}$, and its post-acquisition value is $V_1^{U_1}$, and note that if $V_1^{U_1} < V < V_0^{U_1}$, then it is optimal for each shareholder to tender his shares if the tender offer succeeds (and then get $V > V_1^{U_1}$ for the tendered shares), but not if it fails (in which case the shareholder gets $V < V_0^{U_1}$ for the sold shares). Hence, there exist multiple equilibria in this case. For example, it is an equilibrium for all shareholder to tender their shares (the offer succeeds even if a single shareholder deviates and does not tender his shares), and it is also an equilibrium for all shareholders not to tender their shares (the offer fails even if a single shareholder deviates and does tender his shares). However, since $V_0^{U_1} \geq V_1^{U_1}$, equilibria in which the tender offer fails Pareto dominate equilibria in which the tender offer succeeds. We will therefore assume that whenever $V_1^{U_1} < V < V_0^{U_1}$, a non-tendering equilibrium is played. With this assumption in place, we now prove the following lemma.

**Lemma 6:** Suppose that if $V_1^{U_1} < V < V_0^{U_1}$, then $U_1$’s initial shareholders do not tender their shares. Then in equilibrium, $V = V_0^{U_1}$.

**Proof:** First, notice that if $V \leq V_1^{U_1}$ (the price per share is below the post-acquisition value), then it is a dominant strategy for each shareholder not to tender. And, given the assumption in the lemma, shareholders also do not tender if $V_1^{U_1} < V < V_0^{U_1}$. Hence, the tender offer fails for sure if $V < V_0^{U_1}$. By contrast, if $V \geq V_0^{U_1}$, then it is a weakly dominant strategy for each shareholder to fully tender his shares: if the tender offer succeeds, the shareholder gets $V_0^{U_1}$ on the sold shares, but gets only $V_1^{U_1} < V_0^{U_1}$ on retained shares; if the tender offer fails, the value of the shares is $V_0^{U_1}$ regardless of whether they are tendered. Since the tender offer surely succeeds, it is optimal for $D_1$ to set $V = V_0^{U_1}$, which is the lowest offer that ensures that the tender offer succeeds. \[\blacksquare\]

Lemma 6 implies that $D_1$ has no incentive to acquire control over $U_1$ if $D_2$ is not foreclosed following the acquisition. This is because absent foreclosure, $D_1$’s profit remains $V_0^{D_1}$ and since $D_1$ breaks even on the acquisition (the acquisition price absent foreclosure is $\alpha V_0^{U_1}$, which is also the post-acquisition value of $D_1$’s stake in $U_1$). The lemma also implies that if $D_2$ is foreclosed, $D_1$ wishes to buy the minimal stake, $\alpha$, which ensures control, since the price of the acquired shares exceeds their post-acquisition value.

\[14\] If the offer is conditional on success, the shareholder is indifferent about submitting shares when the offer fails.
The remaining question now is whether the acquisition is profitable for $D_1$ if it does lead to the foreclosure of $D_2$.\footnote{One may wonder whether a large shareholder may wish to acquire a sufficiently large stake from the dispersed shareholders of $U_1$ and use it to oppose $D_1$’s decision to foreclosure $D_2$. Such an action would raise the value of $U_1$ from $V_1^U$ to $V_0^U$. But since the dispersed shareholders of $U_1$ are atomistic, the large shareholder would have to pay them the post-acquisition value of their shares (as in Grossman and Hart, 1980) in order to induce them to submit their shares. As a result, the acquisition is not profitable for the large shareholder.} We address this question in the next proposition:

**Proposition 4:** Suppose that initially, $U_1$’s ownership is dispersed. Then, in equilibrium, $D_1$ will acquire a controlling stake in $U_1$ and will use it to foreclose $D_2$ if and only if $G \geq \alpha L$. If this condition holds, $D_1$ will acquire the minimal stake that ensures control over $U_1$, i.e., $\alpha$, by making a restricted tender offer.

**Proof:** Since $V = V_0^U$, acquiring a controlling stake $\alpha$ in $U_1$ costs $D_1$ a total of $\alpha V_0^U$. Recalling that when $D_2$ is foreclosed $w = \Delta_1 (N, N - 1)$, the post-acquisition payoff of $D_1$ is

$$V_1^D + \alpha V_1^U - \alpha V_0^U = V_1^D - \alpha L.$$ 

Recalling that the pre-acquisition payoff of $D_1$, $V_0^D$, is given by (1), and recalling that $G \equiv V_1^D - V_0^D$, the difference between $D_1$’s post- and pre-acquisition payoff is

$$[V_1^D - \alpha L] - V_0^D = G - \alpha L.$$ 

Since $L > 0$ by Assumption A4, it is clear that $D_1$ would never acquire more than the minimal stake that ensures control, namely, $\alpha$. Acquiring a stake $\alpha$ in $U_1$ is profitable, however, if and only if $G \geq \alpha L$. To ensure that the acquired stake does not exceed $\alpha$, $D_1$ will make his offer restricted and specify that if the tendered stake exceeds $\alpha$, the submitted shares of each tendering shareholder will be prorated such that if $D_1$ acquires exactly a stake of $\alpha$. \hfill ■

Since $\alpha_C \geq \alpha$, Propositions 3 and 4 imply that there is a wider range of parameters for which $D_1$ acquires a controlling stake in $U_1$ and uses it to foreclose $D_2$ when $U_1$ is initially owned by dispersed shareholders, than when $U_1$ has initially a single controlling shareholder. Intuitively, the acquisition of $U_1$ by $D_1$ lowers $U_1$’s value since $D_1$ uses $U_1$ to foreclose $D_2$ and hence it effectively diverts profits from $U_1$ to $D_1$. Although in both cases $D_1$ needs to pay the sellers a price that reflects the pre-acquisition value of $U_1$, in the case of an initial controller, $D_1$ must also compensate the controller for his remaining stake in $U_1$, $\alpha_C - \alpha$. By contrast, in the case of dispersed shareholders,
$D_1$ can acquire a minimal stake, $\alpha$, that ensures control and then effectively expropriate the wealth of the remaining passive shareholders of $U_1$ by foreclosing $D_2$. So long as $\alpha < 1$, part of the upstream loss from foreclosure is borne by the passive shareholders of $U_1$, while the entire downstream gain accrues to $D_1$.

### 4.3 Forward integration: $D_1$ has initially a single controlling shareholder

We now consider the case where $U_1$ integrates forward by acquiring a controlling stake $\alpha \in [\underline{\alpha}, \alpha_C)$ in $D_1$ from the initial controller of $D_1$. Again, we will assume that the acquirer, here $U_1$, makes a take-it-or-leave-it offer $b$ for the stake $\alpha$, but as in the partial backward integration case, this assumption is not essential. We begin with the following lemma.

**Lemma 7:** Acquisition of a partial controlling stake in $D_1$, which is not followed by a foreclosure of $D_2$, is profitable for $U_1$ if and only if the post-acquisition price, $w$, that $D_1$ pays $U_1$ for the input exceeds $\Delta_1(N,N)$, which is the price that all other upstream suppliers charge for the input.

**Proof:** We begin by considering the price that $U_1$ needs to pay $D_1$’s controlling shareholder in order to acquire a stake $\alpha \in [\underline{\alpha}, \alpha_C)$ in $D_1$. If $U_1$ does not foreclose $D_2$, then the post-acquisition value of $D_1$ is

$$\bar{V}_1^D = \Pi_1(N,N) - (N - 1) \Delta_1(N,N) - w, \quad (13)$$

where $w$ is the price that $D_1$ pays $U_1$ for the input. By Corollary 1, the pre-acquisition value of $D_1$, $V_0^D$, is given by (1).

Analogously to Lemma 5, the minimal acceptable bid is

$$b^D = \alpha_C V_0^D - (\alpha_C - \alpha) \bar{V}_1^D$$

$$= \alpha_C [V_0^D - \bar{V}_1^D] + \alpha \bar{V}_1^D$$

$$= \alpha_C [-\Delta_1(N,N) + w] + \alpha \bar{V}_1^D, \quad (14)$$

where the third equality follows from (1) and (13).

Therefore, if $U_1$ acquires a controlling stake in $D_1$, its payoff becomes:

$$[\Delta_1(N,N) + w - 2c] + a \bar{V}_1^D - \alpha_C [V_0^D - \bar{V}_1^D] + a \bar{V}_1^D$$

$$= \Delta_1(N,N) + w - 2c + \alpha_C [\Delta_1(N,N) - w]. \quad (15)$$

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If $D_1$ does not acquire a controlling stake in $U_1$, then by Corollary 1, its payoff is $V_0^U$, given by (2). The difference between $U_1$’s post- and pre-acquisition payoff is therefore:

$$
\Delta_1 (N, N) + w - 2c + \alpha_C \left[ \Delta_1 (N, N) - w \right] - 2 \left[ \Delta_1 (N, N) - c \right] = -(1 - \alpha_C) \left[ \Delta_1 (N, N) - w \right].
$$

The expression is positive if and only if $w > \Delta_1 (N, N)$. ■

Lemma 7 implies that it pays $U_1$ to acquire a controlling stake in $D_1$ without foreclosing $D_2$ only when it can expropriate the wealth of $D_1$’s passive shareholders by selling the input to $D_1$ at a price that exceeds the price at which it sells the input to $D_2$ and above the price that $D_1$ pays all other upstream suppliers. As before, we shall rule out this possibility; this implies in turn that forward integration is feasible only if it leads to the foreclosure of $D_2$.

We now consider the possibility that $U_1$ acquires a controlling stake $\alpha$ in $D_1$, and then uses it to foreclose $D_2$. Analogously to (14), the minimal $b$ needed to induce $D_1$’s initial controller to accept $U_1$’s offer is,

$$
b^D = \alpha_C V_0^D - (\alpha_C - \alpha) V_1^D = \alpha_C [V_0^D - V_1^D] + \alpha V_1^D \tag{16}
$$

where the third equality follows because $G \equiv V_1^D - V_0^D$. Equation (16) shows that $b^D$ is equal to the post-acquisition value of the acquired shares, $\alpha V_1^D$, minus a discount, $-\alpha_C G$, which is equal to the appreciation in the value of the initial stake of $D_1$’s initial controller due to the foreclosure of $D_2$.

To determine if offering $b^D$ is profitable for $U_1$, notice that $U_1$’s payoff if it gains control over $D_1$ and then forecloses $D_2$ is:

$$
V_1^U + \alpha V_1^D + \alpha_C G - \alpha V_1^D = V_1^U + \alpha_C G.
$$

If $U_1$ does not acquire a controlling stake in $D_1$, its payoff, $V_0^U$, is given by (2). In equilibrium, $U_1$ will acquire a controlling stake in $D_1$ and then foreclose $D_2$ if and only if its post-acquisition payoff exceeds its pre-acquisition payoff:

$$
V_1^U + \alpha_C G - V_0^U = \alpha_C G - L \geq 0. \tag{17}
$$
Since $L > 0$ by Assumption A4, this condition is more likely to hold when $\alpha_C$ is large. Since the condition is independent of the acquired stake, $\alpha$, $U_1$ is indifferent to its final stake in $D_1$, provided that it is high enough to ensure control.

Notice that the joint payoff of $U_1$ and $D_1$’s initial controller if the acquisition goes through is $V_1^U + \alpha_C V_1^D$, whereas their joint payoff without an acquisition is $V_0^U + \alpha_C V_0^D$. Hence, the joint surplus from partial backward integration is,

$$S_{FI} = \alpha_C \left[ \frac{V_1^D - V_0^D}{G} \right] - \left[ \frac{V_0^U - V_1^U}{L} \right].$$

Condition (17) then says that $U_1$ will acquire a controlling stake in $D_1$ if and only if the acquisition is jointly profitable for $U_1$ and $D_1$’s controller. This implies in turn that the relative bargaining powers of the two parties only determine how the joint surplus is divided between them, but not whether the acquisitions will take place.

**Proposition 5:** Suppose that initially, $D_1$ has a single controlling shareholder. Then, in equilibrium, $U_1$ will acquire a controlling stake in $D_1$ and will foreclose $D_2$ if and only if $\alpha_C G \geq L$. In particular, the acquisition takes place for all $\alpha_C \in [\alpha, 1]$ if $\alpha G \geq L$, never takes place if $G < L$, and if $\alpha G \leq L < G$, the acquisition takes place only when $\alpha_C$ is sufficiently close to 1.

Proposition 5 implies that if full vertical integration does not lead to foreclosure, i.e., $G < L$, then it never pays $U_1$ to integrate forward when $D_1$ is initially controlled by a single shareholder. On the other hand, when full vertical integration does lead to foreclosure, i.e., $G \geq L$, then $U_1$ may wish to integrate forward and then foreclose $D_2$, provided that the ownership stake of $D_1$’s initial controller is sufficiently large.

### 4.4 Forward integration: $D_1$’s ownership is initially dispersed

When $D_1$’s shareholders are atomistic, no shareholder is pivotal; since the acquisition boosts the value of $D_1$, it is a dominant strategy for each shareholder to hold on to his share if $V$ is below the post-acquisition value of $D_1$.\(^{16}\) Hence, in equilibrium, $U_1$ must set $V$ equal to the post-acquisition value of $D_1$, implying that it breaks even on the share it acquires.\(^{17}\) Consequently, the acquisition

\(^{16}\)The situation is then different from the one considered in Section 4.2 where the acquisition lowered the target’s value. In that case, whether it is optimal to submit shares or not depends on whether other shareholders submit their shares.

\(^{17}\)This result is just the Grossman and Hart (1980) free rider problem: when a takeover is value-increasing, the acquirer must offer a price that reflects the post-acquisition value of the target, otherwise it is a dominant strategy for
affects $U_1$ only through its direct effect on $U_1$. But since $U_1$’s value either decreases by $L$ if $D_2$ is foreclosed, or does not change if $D_2$ is not foreclosed, $U_1$ has nothing to gain by acquiring control over $D_1$. Hence,

**Proposition 6:** Suppose that initially, $D_1$’s ownership is dispersed. Then forward integration does not take place in equilibrium.

5 Extensions

5.1 Toeholds

We now examine what happens when, at the outset, $D_1$ already holds a non-controlling stake, $\alpha_1 < \alpha$, in $U_1$ (i.e., a toehold). To gain control over $U_1$, $D_1$ must acquire an additional stake $\alpha - \alpha_1$ in $U_1$, such that after the acquisition, its controlling stake in $U_1$ is $\alpha \geq \alpha$.

**Proposition 7:** Suppose that initially, $D_1$ holds a non-controlling stake (toehold), $\alpha_1$, in $U_1$. Then, the toehold has no effect on the equilibrium if $U_1$ is initially held by dispersed shareholders. When $U_1$ has initially a single controlling shareholder, then, in equilibrium, $D_1$ will acquire a controlling stake in $U_1$ and will use it to foreclose $D_2$ if and only if $G \geq (\alpha_C + \alpha_1) L$; in particular, $D_1$’s toehold shrinks the range of parameters for which $D_2$ is foreclosed.

**Proof:** First, suppose that $U_1$ is initially held by a continuum of atomistic shareholders. By Lemma 6, $D_1$ will offer these shareholders a price that reflects a value of $V_0^U$ for the entire firm and will therefore pay a total of $(\alpha - \alpha_1) V_0^U$ for the acquired stake. If $D_2$ is not foreclosed after the acquisition, then the post-acquisition values of $D_1$ and $U_1$ are equal to their pre-acquisition values, $V_0^D$ and $V_0^U$. Hence, $D_1$’s post-acquisition payoff is

$$V_0^D + \alpha V_0^U - (\alpha - \alpha_1) V_0^U = V_0^D + \alpha_1 V_0^U.$$  

Since this is also $D_1$’s payoff without acquisition, $D_1$ has no incentive to acquire control over $U_1$.

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[24] Each shareholder to hold on to his shares. Bagnoli and Lipman (1988) and Holmstrom and Nalebuff (1992) show that the free-rider problem depends crucially on the assumptions that there is a continuum of shareholders rather than finitely many shareholders and on the fact that the acquirer has no private information about the post-acquisition value of the firm.
If the acquisition is followed by the foreclosure of $D_2$, then $U_1$’s profit, $V_1^U$, is given by (4), while $D_1$’s profit, $V_1^D$, is given by (3). Now, $D_1$’s post-acquisition payoff is given by
\[ V_1^D + \alpha V_1^U - (\alpha - \alpha_1) V_0^U = V_1^D + \alpha_1 V_0^U - \alpha \underbrace{V_0^U - V_1^U}_{L}. \]
Since $L < 0$, $D_1$ would only acquire the minimal amount of shares which ensures control, i.e., $\alpha - \alpha_1$, such that its post-acquisition stake in $U_1$ will be $\alpha$. Noting that $D_1$’s pre-acquisition payoff is $V_0^D + \alpha_1 V_0^U$, the acquisition of $\alpha - \alpha_1$ shares in $U_1$ is profitable if and only if
\[ V_1^D + \alpha_1 V_0^U - \alpha \underbrace{V_0^U - V_1^U}_{L} - \underbrace{[V_0^D - \alpha_1 V_0^U]}_{G} = \underbrace{[V_1^D - V_0^D]}_{L} - \alpha \underbrace{[V_0^U - V_1^U]}_{L} \geq 0. \]
This condition, however, is identical to that in Proposition 4, so the existence of a toehold does not affect the equilibrium.

Next, consider the case where $U_1$ is initially controlled by a single shareholder, whose initial stake is $\alpha_C$. If $D_2$ is not foreclosed after the acquisition, then the minimal offer that $D_1$ needs to make to induce $U_1$’s initial controller to accept is given by (9), except that now, the acquired stake is $\alpha - \alpha_1$ (the difference between the final stake and the toehold) rather than $\alpha$. Moreover, since we assume that $w = \Delta_1(N,N)$ when $D_2$ is not foreclosed, the post-acquisition value of $U_1$ equals its pre-acquisition value, i.e., $V_1^U = V_0^U$, so
\[ b^U = (\alpha - \alpha_1) V_0^U, \]
where $V_0^U$ is given by (1). Since $D_1$’s profit absent foreclosure is $V_0^D$, the post-acquisition payoff of $D_1$ is:
\[ V_0^D + \alpha V_0^U - \alpha \underbrace{V_0^U - V_1^U}_{G} = \underbrace{V_0^D - \alpha_1 V_0^U}_{L}. \]
Since this expression is also equal to $D_1$’s payoff without the acquisition, $D_1$ has no incentive to acquire control over $U_1$ if it does not use it to foreclose $D_2$.

Now suppose that following an acquisition, $D_1$ uses its control over $U_1$ to foreclose $D_2$. Then, the minimal acceptable offer that $D_1$ needs to make is given by (11), except that now, $\alpha - \alpha_1$ replaces $\alpha$. Consequently, $D_1$’s post-acquisition payoff becomes,
\[ V_1^D + \alpha V_1^U - \alpha \underbrace{[\alpha C L + (\alpha - \alpha_1) V_1^U]}_{G} = V_1^D - \alpha C L + \alpha_1 V_1^U, \]
where $V_1^U$ is given by equation (4). Noting that without acquisition, $D_1$’s payoff is $V_0^D + \alpha_1 V_0^U$, and using the definitions of $L$ and $G$, $D_1$ will make the offer if and only if his post-acquisition payoff
exceeds his pre-acquisition payoff, i.e.,

$$V^D_1 - \alpha_C L + \alpha_1 V^U_1 - [V^D_0 + \alpha_1 V^U_0] = G - (\alpha_C + \alpha_1) L \geq 0.$$ 

Intuitively, when $U_1$'s ownership is initially dispersed, $D_1$ must pay the initial shareholders a price equal to the pre-acquisition value of $U_1$. As a result, $D_1$ bears the entire loss in the value of these shares. When $D_1$ already owns a toehold, it needs to buy fewer shares to gain control over $U_1$, so it saves on the cost of acquiring shares, but this saving is exactly offset by the decline in the value of its toehold. Hence, $D_1$ bears a cost equal to $\alpha L$, irrespective of how $\alpha$ is divided between the toehold, $\alpha_1$, and the acquired stake, $\alpha - \alpha_1$. As a result, the toehold does not affect the equilibrium.

By contrast, when $U_1$ initially has a controlling shareholder, a toehold in $U_1$ makes it more costly for $D_1$ to foreclose $D_2$, since it lowers the value of $D_1$'s toehold. In other words, $D_1$ internalizes a larger part of the loss of upstream profit and hence is more reluctant to incur these loses in order to boost its downstream profits. From a competitive standpoint, this implies in turn that a toehold is actually pro-competitive when initially $U_1$ has a controlling shareholder.

### 5.2 Acquisition by a controller

So far we have assumed that vertical integration arises when $D_1$ buys a controlling stake in $U_1$ or $U_1$ buys a controlling stake in $D_1$. However, there are cases in which the controlling shareholders of a firm (rather than the firm itself) buys a controlling stake in a vertically related firm, either directly, or through other firms that are under his control. For example, in 2000, Vivendi, which already held a controlling 49% stake in Canal+ (a major European producer of pay-television channels, with a significant presence in the distribution of films and the licensing of broadcasting rights) acquired Seagram, which owned Universal Studios Inc., as well as Polygram’s music activities. A second example is the 2009 offer of International Petroleum Investment Company (IPIC) to acquire a 70% stake in MAN Ferrostaal, which held a controlling 30% stake in Eurotecnica Melamine (the sole supplier and licensor of high pressure technology used in melamine production). The European Commission expressed the concern that after the acquisition, IPIC, which was the controlling shareholder of Agrolinz Melamine International (AMI) (one of the leading melamine producers

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18See [http://ec.europa.eu/competition/mergers/cases/decisions/m2050_en.pdf](http://ec.europa.eu/competition/mergers/cases/decisions/m2050_en.pdf)
world-wide), would foreclose AMI’s competitors from Eurotecnica’s technology. In this subsection we study how acquisitions by controllers affect the concern for foreclosure.

For the sake of concreteness, suppose that the controlling shareholder of $D_1$ controls $m \geq 1$ firms; one of these firms is $D_1$, while the other firms are from other industries. The stakes that the controller holds in the $m$ firms are $\beta_1, \ldots, \beta_m$, where $\beta_1$ is the controller’s stake in $D_1$ and $\beta_2, \ldots, \beta_m$ are his stakes in firms 2, $\ldots$, $m$.

$D_1$’s controller can now acquire a controlling stake in $U_1$ either directly, through $D_1$, or through firms 2, $\ldots$, $m$. If he acquires the stake through firm $i = 0, 1, \ldots, m$ (“firm 0” means that the controller acquires a controlling stake in $U_1$ directly, so naturally, $\beta_0 = 1$) at a price $b$, and uses his control over $U_1$ to foreclose $D_2$, then his payoffs becomes

$$\beta_1 V_i^D + \beta_i \left[ \alpha V_i^U - b \right] + \sum_{j=2, \ldots, m} \beta_j V_j,$$

where $V_j$ is the value of firm $j = 2, \ldots, m$. If the controller does not acquires a controlling stake in $U_1$, his payoff is

$$\beta_1 V_0^D + \sum_{j=2, \ldots, m} \beta_j V_j.$$

The difference between the controller’s payoff with and without acquisition is then

$$I = \beta_1 \left[ \frac{V_i^D}{G} - \frac{V_0^D}{G} \right] + \beta_i \left[ \alpha V_i^U - b \right]. \quad (18)$$

From (11) we know that if $U_1$ has initially a controlling shareholder, then $b = \alpha C L + \alpha V_1^U$. Hence,

$$I = \beta_1 \left[ \frac{V_i^D}{G} - \frac{V_0^D}{G} \right] + \beta_i \left[ \alpha V_i^U - \alpha C L - \alpha V_1^U \right] = \beta_1 \left[ \frac{V_i^D}{G} - \frac{V_0^D}{G} \right] - \beta_i \alpha C L. \quad (19)$$

Likewise, we know from Lemma 6 that if $U_1$’s ownership is initially dispersed, then $b = \alpha V_0^U$. Hence,

$$I = \beta_1 \left[ \frac{V_i^D}{G} - \frac{V_0^D}{G} \right] + \beta_i \left[ \alpha V_i^U - \alpha V_0^U \right] = \beta_1 \left[ \frac{V_i^D}{G} - \frac{V_0^D}{G} \right] + \beta_i \alpha \left[ \frac{V_i^U}{G} - \frac{V_0^U}{G} \right]. \quad (20)$$

Note immediately that without foreclosure, $G = L = 0$, so acquisition has no effect on the controller’s payoff. Moreover, note that $I$ is independent of $\alpha$ in (19), but is decreasing with $\alpha$ in (20). Hence, $D_1$’s controller is indifferent to the precise stake he acquires in $U_1$ if he acquires it from an initial controller in $U_1$, but will prefer to acquire the minimal stake $\alpha$ that ensures control if $U_1$’s

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19 See http://ec.europa.eu/competition/mergers/cases/decisions/m5406_20090313_20212_en.pdf
ownership is initially dispersed. Moreover, since $I$ is decreasing with $\beta_i$ and increasing with $\beta_1$ in both (19) and (20), $D_1$’s controller will prefer to acquire the controlling stake in $U_1$ through the firm in which he holds the minimal controlling stake and will have a stronger incentive to acquire control over $D_2$ when his controlling stake in $D_1$ is large. These observations imply the following result:

**Proposition 8:** Suppose that $D_1$ has controlling shareholder who also owns controlling shares in firms from other industries and let $k$ be the firm in which his controlling stake, $\beta_k$, is the lowest, i.e., $\beta_k \leq \beta_i$ for all $i = 1, \ldots, m$. Then, in equilibrium, $D_1$ will acquire a controlling stake in $U_1$ through firm $k$ and will use it to foreclose $D_2$ if and only if $\beta_1 G \geq \beta_k \hat{\alpha} L$, where $\hat{\alpha} = \alpha_C$ when $U_1$ has initially a single controlling shareholder, and $\hat{\alpha} = \underline{\alpha}$ when $U_1$ is initially held by dispersed shareholders. In the latter case, $D_1$ will acquire the minimal stake that ensures control over $U_1$, i.e., $\underline{\alpha}$, by making the tender offer restricted.

Since by definition, $\beta_k \leq \beta_1$, it is clear that the ability of $D_1$’s controller to choose whether to acquire a controlling stake in $U_1$ through $D_1$ or through another firm which he controls, expands the range of parameters for which $D_2$ is foreclosed (unless $D_1$ happens to be the firm in which the controller has the lowest controlling stake among all firms under his controls). Moreover, so long as $\beta_k < 1$, the controller will not acquire a controlling stake in $U_1$ directly, but rather through firm $k$. Intuitively, when the controller has a small stake in firm $k$, a large fraction of the upstream loss from foreclosing $D_2$ is borne by the passive shareholders of $k$. And, when $\beta_1$ is large, a large fraction of the associated downstream gain accrues to the controller. Hence, acquiring a controlling stake in $U_1$ and using it to foreclose $D_2$ is more attractive when $\beta_k$ is small and $\beta_1$ is large.

### 5.3 Competition for the acquisition of a controlling stake in $U_1$

In this section we maintain the assumption that $U_1$ is the only potential upstream target, but we now consider the possibility that both $D_1$ and $D_2$ will compete for acquiring $U_1$. We begin by considering the case where $U_1$ has a single controlling shareholder.

Suppose that $D_1$ and $D_2$ simultaneously offer $U_1$’s initial controller payments $b_1$ and $b_2$ for controlling stakes $\alpha_1 \geq \underline{\alpha}$ and $\alpha_2 \geq \underline{\alpha}$ in $U_1$, and suppose that the controller accepts $D_i$’s offer. Using $\hat{V}^U_1$ to denote the post-acquisition payoff of $U_1$, the payoff of $U_1$’s controller becomes

$$b_i + (\alpha_C - \alpha_i) \hat{V}^U_1 = \alpha_C \hat{V}^U_1 + \left(\frac{b_i - \alpha_i \hat{V}^U_1}{\phi_i}\right).$$
where \( \hat{V}_1^U = V_1^U \) if \( D_j \) is foreclosed and \( \hat{V}_1^U = V_0^U \) if \( D_j \) is not foreclosed, and \( \phi_i \) is the premium that \( D_i \) pays for the acquired stake above and beyond its post-acquisition value. Since \( \alpha_C \hat{V}_1^U \) is constant, it is obvious that \( U_1 \)'s controller will accept the offer with the highest premium \( \phi_i \), provided that his resulting payoff, \( \alpha_C \hat{V}_1^U + \phi_i \), is at least as high as \( \alpha_C V_0^U \), which is his payoff if he rejects both offers. Hence, acceptable offers must be such that

\[
\alpha_C \hat{V}_1^U + \phi_i \geq \alpha_C V_0^U, \quad \implies \quad \phi_i \geq \alpha_C \left[ V_0^U - \hat{V}_1^U \right]. \tag{21}
\]

Using \( \hat{V}_1^D \) to denote the post-acquisition profit of \( D_i \), the associated payoffs of \( D_i \) if its offer is accepted is

\[
\hat{V}_1^D + \alpha_i \hat{V}_1^U - b_i = \hat{V}_1^D - \phi_i,
\]

where \( \hat{V}_1^D = V_1^D \) if \( D_j \) is foreclosed and \( \hat{V}_1^D = V_0^D \) if \( D_j \) is not foreclosed. The payoff of \( D_j \), denoted \( \hat{V}_j^D \), is either \( V_0^D \) if it is not foreclosed and

\[
V_F^D = \Pi (N - 1, N) - (N - 1) \Delta_1 (N - 1, N), \tag{22}
\]

if it is foreclosed. With these payoffs in place, we now prove the following result:

**Proposition 9:** Suppose that \( D_1 \) and \( D_2 \) compete for the acquisition of a controlling stake in \( U_1 \) and assume that \( U_1 \) has initially a single controlling shareholder. Then,

(i) if \( G \geq \alpha_C L \), then in equilibrium, \( D_1 \) and \( D_2 \) will set \( \phi_1 = \phi_2 = V_1^D - V_F^D \) and \( U_1 \) will accept one of these offers at random; the acquirer, \( D_i \), will use its control over \( U_1 \) to foreclose the downstream rival \( D_j \).

(ii) if \( G < \alpha_C L \), then in equilibrium, neither downstream firm acquires a sufficiently small controlling stake in \( U_1 \) such that after the acquisition the rival will be foreclosed.

**Proof:** First, note that \( G \geq \alpha_C L \) implies that \( G \geq \alpha_i L \) for all \( \alpha_i \in [\underline{\alpha}, \alpha_C] \), so once \( D_i \) gains control over \( U_1 \), it will use it to foreclose \( D_j \) (\( D_i \)'s gain from foreclosure exceeds its share in \( U_1 \)'s loss). Hence, the post-acquisition payoffs are \( \hat{V}_1^D = V_1^D \) and \( \hat{V}_j^D = V_F^D \). The highest premium that \( D_i \) will offer is therefore

\[
\phi_i = V_1^D - V_F^D,
\]

which is the difference between \( D_i \)'s payoff if it acquires control over \( U_1 \) and if \( D_j \) acquires it. Noting that \( V_F^D < V_0^D \) (foreclosure hurts \( D_j \)), and that by assumption, \( G \geq \alpha_C L \), and recalling
that \( L \equiv V_0^U - V_1^U \) and that \( \tilde{V}_1^U \) is either equal to \( V_1^U \) (if \( D_j \) is foreclosed) or to \( V_0^U > V_1^U \) (if \( D_j \) is not foreclosed), it follows that

\[
\phi_i = V_1^D - V_F^D \geq \underbrace{V_1^D - V_0^D}_{G} \geq \alpha_C L \geq \alpha_C \left[ V_0^U - \tilde{V}_1^U \right].
\]

Hence, by (21), the offer will be accepted by \( U_1 \)'s controller. Consequently, in equilibrium, \( \phi_1 = \phi_2 = V_1^D - V_F^D \), and \( U_1 \)'s controller accepts one of the two offers at random.

Next, suppose that \( G < \alpha_C L \). Now, an acquisition will lead to the foreclosure of \( D_j \) only if \( D_i \) acquires a sufficiently small controlling stake, \( \alpha \), such that \( G > \alpha L \), but not if it acquires the entire stake \( \alpha_C \). However if \( D_i \) acquires a sufficiently small stake such that \( G > \alpha L \), then by (21), it must set \( \phi_i \geq \alpha_C \left[ V_0^U - V_1^U \right] = \alpha_C L \). But then \( D_i \)'s payoff is

\[
V_1^D - \phi_i \leq V_1^D - \alpha_C L < V_0^D,
\]

where the strict inequality follows since \( G < \alpha_C L \). Therefore, there does not exist an equilibrium in which \( D_i \) alone makes a bid for \( U_1 \) and then uses its control over \( U_1 \) to foreclosure \( D_j \).

Finally, we need to check that there does not exist an equilibrium in which both \( D_1 \) and \( D_2 \) offer to acquire controlling stakes in \( U_1 \), such that \( G \geq \alpha_1 L \) and \( G \geq \alpha_2 L \) (if \( D_i \)'s offer is accepted, then \( D_j \) will be foreclosed). Such an equilibrium might exists since each \( D_i \) fears that if it will not acquire control, its rival will, and will use its control in \( U_1 \) to foreclose it. Now, assume by way of negation that such an equilibrium exists. Since these offers lead to foreclosure, \( U_1 \) would accept \( D_i \)'s offer if \( \phi_i \geq \phi_j \) (\( i \) offers a higher premium than \( j \)) and \( \phi_i \geq \alpha_C L \). The resulting payoff of \( D_i \) is \( V_1^D - \phi_i \). If instead \( D_j \)'s offer is accepted, then \( D_i \) will be foreclosed and its post-acquisition payoff will be \( V_F^D \). Hence, the highest premium that \( D_i \) will offer is \( \phi_i = V_1^D - V_F^D \). In equilibrium (if it exists) both \( D_1 \) and \( D_2 \) must offer \( V_1^D - V_F^D \), otherwise the firm with the higher offer can cut its offer slightly. Hence, the payoff of both \( D_1 \) and \( D_2 \) in equilibrium is \( V_F^D \) and the payoff of \( U_1 \)'s controller is \( \alpha_C V_1^U + V_1^D - V_F^D \).

Now suppose that \( D_i \) deviates and offers \( b_C \) for the entire stake of \( U_1 \)'s controller, \( \alpha_C \). If the offer is accepted, \( D_j \) would not be foreclosed, since by assumption, \( G < \alpha_C L \). The deviation is more profitable for \( U_1 \)'s controller's than \( D_j \)'s offer provided that

\[
\underbrace{b_C}_{\text{Accepting } D_j \text{'s offer}} \geq \underbrace{\alpha_C V_1^U + V_1^D - V_F^D}_{\text{Accepting } D_i \text{'s offer}}.
\]

If \( D_i \) makes the minimal acceptable bid, its payoff under the deviation is

\[
V_0^D + \alpha_C V_0^U - \underbrace{(\alpha_C V_1^U + V_1^D - V_F^D)}_{b_C} = \alpha_C L - G + V_F^D,
\]

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which exceeds $D_i$’s payoff in the candidate equilibrium since by assumption, $G < \alpha C L$.  

Intuitively, competition between $D_1$ and $D_2$ induces the two firms to offer more money to $U_1$’s controller because each firm fears that if it does not acquire a controlling stake in $U_1$, its rival will acquire it and will use it to foreclose it. However, competition between $D_1$ and $D_2$ does not alter the range of parameters for which acquisition which leads foreclosure takes place.

We now turn to the case where initially, $U_1$’s ownership is dispersed. The two downstream firms $D_1$ and $D_2$ make simultaneous tender offers to $U_1$’s shareholders. Each offer specifies the maximal stake $\overline{\alpha}_i$ that $D_i$ offers to acquire and a price that reflects a value $V_i$ for the entire firm. The shareholders observe the two offers and decide whether to tender to $D_1$, to $D_2$, or hold on to their shares. If more than $\overline{\alpha}_i$ shares are tendered to $D_i$, then the tendered shares are prorated. $D_i$ obtains control over $U_1$ if it acquires more shares than $D_2$ and if its final stake is at least $\alpha$.

**Proposition 10:** Suppose that $D_1$ and $D_2$ compete over the acquisition of a controlling stake in $U_1$, who initially has a dispersed ownership and assume that if $V_1^U < V_i < V_0^U$, then $U_1$’s initial shareholders do not tender their shares to $D_i$. Then,

(i) if $G > \alpha L$, then in equilibrium, $D_i$ offers to acquire $\alpha$ shares for a price that reflects value $V_i = V_1^U + \frac{V_1^D - V_F^D}{\alpha}$ for $U_1$ and $D_j$ offers to acquire $\alpha_j > 0$ shares at $V_j = V_1^U + (V_1^D - V_F^D)$. All shareholders tender to $D_i$ who gains control of $U_1$ and then uses it to foreclose $D_j$.

(ii) if $G < \alpha L$, then in equilibrium neither $D_1$ nor $D_2$ make a tender offer to $U_1$’s shareholders.

**Proof:** Suppose that $G > \alpha L$. If $D_i$ gains control over $U_1$, then $U_1$’s shareholders would tender shares only if the offered price, $V_i$, exceeds the post-acquisition value of $U_1$ (otherwise they are better-off holding on to their shares). Hence, $D_j$ would not acquire any shares, while $D_i$ would prefer to acquire the minimal stake that ensures control. The resulting post-acquisition payoffs of $D_i$ and $D_j$ are therefore:

$$\Pi_i = \alpha (V_1^U - V_i) + V_1^D, \quad \Pi_j = V_F^D.$$  

In equilibrium, it must be that $\Pi_i = \Pi_j$, otherwise $D_j$ would outbid $D_i$ slightly, and gain control over $U_1$. Hence,

$$\alpha (V_1^U - V_i) + V_1^D = V_F^D, \quad \Rightarrow \quad V_i = V_1^U + \frac{V_1^D - V_F^D}{\alpha}. \quad (23)$$
Note that since $\frac{V_1^D - V_F^D}{\alpha} > \frac{V_0^D - V_0^D}{\alpha} \equiv \frac{G}{\alpha} \geq L$, then $V_i > V_1^U + L = V_0^U$. Hence all shareholders will tender to $D_i$.

To compute $V_j$, notice that in equilibrium, shareholders must be indifferent between tendering to $D_i$ and tendering to $D_j$, otherwise $D_i$ can profitably lower $V_i$. Since $D_i$’s offer is restricted and therefore prorated, the payoff of each shareholder per share if he tenders to $D_i$ is $\alpha V_i + (1 - \alpha) V_1^U$. If the shareholder tenders to $D_j$, his payoff per share is $V_j$ (recall that in equilibrium no shareholder tenders to $D_j$, so the shareholder’s submission will not be prorated). Hence, using (23),

$$V_j = \frac{\alpha}{V_i} \left( V_1^U + \frac{1}{\alpha} (V_1^D - V_F^D) \right) + (1 - \alpha) V_1^U = V_1^U + V_1^D - V_F^D. \quad (24)$$

We now show that there do not exist equilibria in which neither $D_1$ nor $D_2$ gains control over $U_1$. To this end, assume by way of negation that there exists an equilibrium in which $\alpha_1 < \alpha$ and $\alpha_2 < \alpha$. Since neither downstream firm controls $U_1$, neither firm is foreclosed, so the downstream profits of $D_1$ and $D_2$ are both equal to $V_0^D$. Moreover, both firms pay a price of $V_0^U$ for the shares they acquire in $U_1$ (if any); this price is the minimal price needed to induce $U_1$’s shareholders to tender shares and neither firm would have an incentive to offer a higher price for the shares. Hence, the two firms break even on the shares they acquire in $U_1$ so their payoffs are both equal to $V_0^D$.

Now, holding $\alpha_j$ fixed, suppose that $D_i$ deviates and acquires a stake $\alpha$ in $U_1$, which ensures that it controls $U_1$. Since $G > \alpha L$, $D_i$ will use its control over $U_1$ to foreclose $D_j$. The resulting payoff of $D_i$ is then $V_1^D + \alpha (V_1^U - V_0^U) > V_0^D$, where the inequality follows since $G > \alpha L$. The deviation then is profitable and it upsets the putative equilibrium.

Likewise, there do not exist equilibria in which $\alpha_1 = \alpha_2 > \alpha$. In these equilibria, $D_1$ and $D_2$ have joint control over $U_1$, so neither firm is foreclosed. As before, each firm pays a price of $V_0^U$ for the shares it acquires, so the payoff of each firm equals its downstream profit $V_0^D$. Now, suppose that $D_i$ deviates by raising $\alpha_i$ slightly to $\alpha_i'$, while continuing to pay $V_0^U$ for the acquired shares. Following the deviation, $D_i$ obtains controls over $U_1$, and since $G > \alpha L$ it uses its control to foreclose $D_j$. The resulting payoff of $D_i$ is then $V_1^D + \alpha_i' (V_1^U - V_0^U)$ implying that the deviation is profitable and it upsets the putative equilibrium.

If $\alpha_1 = \alpha_2 = 1/2$, then again neither firm is foreclosed but now $D_i$ cannot raise $\alpha_i$ without offering a higher price for the shares it buys. If $V_i > V_j$, then $D_i$ can offer to buy $\alpha_i > 1/2$, obtain controls over $U_1$ and use it control to foreclose $D_j$. The resulting payoff of $D_i$ is then $V_1^D + \alpha_i (V_1^U - V_0^U)$ implying that the deviation is profitable and it upsets the putative equilibrium.
If \( V_i > V_j \), then \( D_i \) can offer to buy \( \alpha_i > 1/2 \) at a price slightly above \( V_i \). \( D_i \) then obtains controls over \( U_1 \) and use it control to foreclose \( D_j \). The resulting payoff of \( D_i \) is then \( V_i^D + \alpha_i (V_i^U - V_0^U) \) implying that the deviation is profitable and it upsets the putative equilibrium.

Finally, suppose that \( G < \alpha L \). Then, an acquisition of a controlling stake in \( U_1 \) does not lead to foreclose, so neither downstream firm has an incentive to acquire control over \( U_1 \).

Proposition 10 is very similar to Proposition 4: the condition under which foreclosure emerges remains the same whether or not \( D_1 \) competes with \( D_2 \) for the acquisition of control over \( U_1 \).

6 Conclusions

We consider the effects of partial vertical integration on the foreclosure of downstream competitors. Our analysis shows that firms often have incentives to acquire partial rather than full stakes in vertically related firms and that partial acquisitions may often increase the risk of anticompetitive foreclosure of rivals beyond what is posed by a full vertical integration. The profitability of partial acquisitions depends on the ownership structure of the target firm. We show that an acquisition of an upstream supplier may be unprofitable when the supplier has a single owner, but be profitable when the supplier has a controlling shareholder who holds less than 100\% of the shares and provided the controller’s initial share is not too high. Partial acquisition is yet more profitable when the shares of the supplier are held by dispersed shareholders.

The profitability of anticompetitive vertical mergers is higher under partial integration due to the potential to expropriate the target’s minority shareholders. When minority shareholders are present the acquirer has to pay the shareholders only a partial compensation for the eventual loss of upstream sales due to the foreclosure of the his downstream rivals. We explore some additional implications of this basic intuition and find that it implies that the acquisition of a non-controlling toehold in a supplier often lowers the risk of eventual foreclosure of rivals.

From an antitrust perspective, our analysis suggests that antitrust authorities should view partial acquisitions of control in upstream suppliers as posing a potential bigger anticompetitive risk than similar acquisitions in which all shares are acquired. Antitrust authorities should be particularly wary when the acquisition is carried out through a tender offer to dispersed shareholders. On the other hand, an acquisition of a noncontrolling share in an upstream supplier does not facilitate the eventual acquisition of full control and, at least from this perspective, should not be
viewed as harmful.

The analysis is also relevant to the large literature on corporate government and the protection of minority shareholders. Our paper suggests that firms with a large share of minority shareholders are particularly attractive targets to value-decreasing acquisitions by their downstream buyers, who may then abuse their control to foreclose product-market competitors to the disadvantage of the target’s minority shareholders. The paper thus formalizes the wide-spread notion that the foreclosure of rivals can be an important source of private benefits to acquirers in takeovers in which vertically-related firms are involved.

7 Appendix

Following are three examples. The first example motivates the assumptions that we impose in Sections 2 and 3 on the downstream profit functions. The second and third examples show that our basic setup is consistent with variants of the two main raising your rivals costs models of input foreclosure: In Ordover, Salop and Saloner (1990), and Salinger (1988).

7.1 An example

Suppose that firms $D_1$ and $D_2$ are located at the two ends of a unit line and compete by setting prices. Consumers are uniformly distributed on the unit line. The utility of a consumer located at a distance $x$ from firm $D_1$ if he buys from $D_1$ is given by

$$U_1(x) = v \log (n_1 + 1) - tx - p_1,$$

where $v \log (n_1 + 1)$ is the “quality” of $D_1$, which increases with the number of inputs, $n_1$, it uses, $t > 0$ is the transportation cost per unit of distance, and $p_1$ is the price that $D_1$ charges. If the consumer buys from $D_2$, his utility is

$$U_2(x) = v \log (n_2 + 1) - t (1 - x) - p_2.$$

If the consumer does not buy at all, his utility is 0.

Assuming that the market is fully covered, the location of the indifferent consumer between $D_1$ and $D_2$ is

$$x^*(p_1, p_2, n_1, n_2) = \frac{1}{2} + \frac{p_2 - p_1 + v \log (n_1 + 1) n_2}{2t}.$$ (25)
Assuming in addition that $D_1$ and $D_2$ pay a fixed price for the inputs and normalizing their additional costs to 0, the gross profits of $D_1$ and $D_2$ are given by

$$\pi_1 = p_1 x^* (p_1, p_2, n_1, n_2), \quad \pi_2 = p_2 (1 - x^* (p_1, p_2, n_1, n_2)).$$

Solving for the Nash equilibrium prices, we obtain:

$$p_1^* (n_1, n_2) = t + \frac{v}{3} \log \left( \frac{n_1 + 1}{n_2 + 1} \right),$$
$$p_2^* (n_1, n_2) = t - \frac{v}{3} \log \left( \frac{n_1 + 1}{n_2 + 1} \right).$$

To avoid uninteresting complications, we shall assume that $\left| \log \left( \frac{n_1 + 1}{n_2 + 1} \right) \right| < \frac{3t}{v}$ for all $n_1$ and $n_2$; this assumption ensures that $p_1^* (n_1, n_2)$ and $p_2^* (n_1, n_2)$ are both nonnegative. Substituting $p_1^* (n_1, n_2)$ and $p_2^* (n_1, n_2)$ in (25), and the substituting the result in $U_1 (x)$, reveals that in equilibrium, the utility of the indifferent consumer is

$$U_1 (x^* (n_1, n_2)) = \frac{-3t + v \log (n_1 + 1) + v \log (n_2 + 1)}{2}.$$

The market is covered, as we assume, provided that $U_1 (x^* (n_1, n_2)) \geq 0$. This inequality holds if $\frac{3t}{v} < \log (n_1 + 1) + \log (n_2 + 1)$.

Substituting $p_1^* (n_1, n_2)$, $p_2^* (n_1, n_2)$ in the profit functions and using (25), the profit of a downstream firm when it uses $k$ inputs and its rival uses $l$ inputs (e.g., the profit of $D_1$ when $n_1 = k$ and $n_2 = l$) is

$$\Pi (k, l) = \left[ t + \frac{v}{3} \log \left( \frac{k + 1}{l + 1} \right) \right]^2.$$

Our assumption that $\left| \log \left( \frac{n_1 + 1}{n_2 + 1} \right) \right| < \frac{3t}{v}$ ensures that $\Pi (k, l)$ is increasing with $k$ and decreasing with $l$ as Assumption 1 states.

Now,

$$\Delta_1 (k, l) \equiv \Pi (k, l) - (k - 1, l)$$
$$= \frac{v}{6t} \log \left( \frac{k + 1}{k} \right) \left[ 2t + \frac{v}{3} \log \left( \frac{k (k + 1)}{(l + 1)^2} \right) \right],$$

and

$$\Delta_2 (k, l) \equiv \Pi (k, l) - (k, l - 1)$$
$$= \frac{v}{6t} \log \left( \frac{l}{l + 1} \right) \left[ 2t + \frac{v}{3} \log \left( \frac{(k + 1)^2}{l (l + 1)} \right) \right].$$
Assumption 3 holds since
\[
\Delta_{12} (l, k) \equiv \left( \Pi (l, k) - (l - 1, k) \right) - \left( \Pi (l, k - 1) - (l - 1, k - 1) \right)
\]
\[
= \frac{v^2}{9t} \log \left( \frac{k + 1}{k} \right) \log \left( \frac{l}{l + 1} \right) < 0.
\]
Assumption 2 holds if
\[
= \frac{v}{6t} \log \left( \frac{N + 1}{N} \right) \left[ 2t + \frac{v}{3} \log \left( \frac{N}{N + 1} \right) \right] > c,
\]
while Assumption A4 holds if
\[
= \frac{v}{6t} \log \left( \frac{k + 1}{k} \right) \left[ 2t + \frac{v}{3} \log \left( \frac{k (k + 1) l^2}{(l + 1)^4} \right) \right] > c,
\]
for all \( k \) and all \( l \). Moreover, Assumption A5 holds if
\[
- \frac{v}{6t} \log \left( \frac{N}{N + 1} \right) \left[ 2t + \frac{v}{3} \log \left( \frac{N + 1}{N} \right) \right] + N \frac{v^2}{9t} \log \left( \frac{N + 1}{N} \right) \log \left( \frac{N}{N + 1} \right)
\]
\[
= - \frac{v}{3} \log \left( \frac{N}{N + 1} \right) - \frac{v^2}{18t} \log \left( \frac{N}{N + 1} \right) \log \left( \frac{N + 1}{N} \right) + N \frac{v^2}{9t} \log \left( \frac{N + 1}{N} \right) \log \left( \frac{N}{N + 1} \right)
\]
\[
= - \frac{v}{3} \left[ \frac{v}{6t} \log \left( \frac{N + 1}{N} \right) - 1 \right] \log \left( \frac{N}{N + 1} \right) > 0.
\]
Since \( \log \left( \frac{N}{N + 1} \right) < 0 \), this inequality holds if the square bracketed term is negative, i.e., if
\[
\frac{3t}{v} > \left( N - \frac{1}{2} \right) \log \left( \frac{N + 1}{N} \right).
\]

### 7.2 A variant of Ordover, Salop and Saloner (1990)

In this example we develop a variant of the Ordover, Salop and Saloner (1990) model (henceforth OSS) which is consistent with our basic setup. OSS consider two upstream suppliers, \( U_1 \) and \( U_2 \), which produce a homogenous input and sell it to two symmetric downstream firms, \( D_1 \) and \( D_2 \), which produce substitute products and compete by setting prices. Since \( U_1 \) and \( U_2 \) engage in Bertrand competition in the upstream market, their profit under non-integration is 0. By definition then, an upstream suppliers cannot lose from vertical integration. Clearly the OSS setting is
extreme. To make it more realistic and ensure that \( U_1 \) and \( U_2 \) earn a profit before integration, we will modify the OSS setting slightly by assuming that the upstream costs are random.\(^{20}\)

Specifically, we assume that the per unit cost of each upstream supplier \( i \), \( c_i \), is either high, \( c \), or low, \( \varepsilon \), with equal probabilities, independently across the two suppliers (OSS assume that \( U_1 \) and \( U_2 \) have the same per unit cost, which is deterministic). Given their cost realizations, \( U_1 \) and \( U_2 \) set the prices of their respective inputs. Then downstream firms, \( D_1 \) and \( D_2 \), buy the inputs, convert each unit of input to one unit of the final product, at no additional cost, set their respective prices, and sell to final consumers.

Let \( w_1 \) and \( w_2 \) be the prices that \( D_1 \) and \( D_2 \) pay for the input. Since inputs are converted to outputs on a 1:1 basis, \( w_1 \) and \( w_2 \) are also the marginal costs of \( D_1 \) and \( D_2 \). The profit of each downstream firm \( i \) is then given by

\[
\pi_i = (p_i - w_i) q_i (p_i, p_j),
\]

where \( p_i \) and \( p_j \) are the downstream prices and \( q_i (p_i, p_j) \) is firm \( i \)'s quantity. Since the products of \( D_1 \) and \( D_2 \) are (imperfect) substitutes, \( q_i (p_i, p_j) \) decreases with \( p_i \) and increases with \( p_j \).

The equilibrium price of each downstream firm \( i \) is \( p_i(w_i, w_j) \) and its corresponding profit is \( \pi_i(w_i, w_j) \).

**Lemma 1:** \( \pi_i(w_i, w_j) \) decreases with \( w_i \) and assuming that \( p_i \) increases with \( w_i \), \( \pi_i(w_i, w_j) \) also increases with \( w_j \).

**Proof of Lemma 1:** Let \( \hat{w}_i > w_i \) and let \( q_i(w_i, w_j) \equiv q_i(p_i(w_i, w_j), p_j(w_i, w_j)) \). Then by revealed preferences,

\[
\begin{align*}
\pi_i(w_i, w_j) &= (p_i(w_i, w_j) - w_i) q_i(w_i, w_j) \\
&\geq (p_i(\hat{w}_i, w_j) - w_i) q_i(\hat{w}_i, w_j) \\
&> (p_i(\hat{w}_i, w_j) - \hat{w}_i) q_i(\hat{w}_i, w_j) \\
&= \pi_i(\hat{w}_i, w_j).
\end{align*}
\]

\(^{20}\)Another possibility is to assume that the inputs are imperfect substitutes. However, this modelling approach would require us to specify how the two inputs are combined into a final product, which would add another layer of complication to the model, which we avoid with our approach.
Moreover,
\[
\pi_i(w_i, \bar{w}_j) = (p_i(w_i, \bar{w}_j) - w_i) q_i(w_i, \bar{w}_j) > (p_i(w_i, w_j) - w_i) q_i(w_i, w_j) > (p_i(w_i, w_j) - w_i) q_i(w_i, w_j) = \pi_i(w_i, w_j),
\]
where the first inequality follows because \( p_j(w_i, \bar{w}_j) > p_j(w_i, w_j) \) and because the two final products are substitutes, so \( p_j(w_i, \bar{w}_j) > p_j(w_i, w_j) \) implies that \( q_i(w_i, \bar{w}_j) > q_i(w_i, w_j) \) and the second inequality follows by revealed preference.

### 7.2.1 Nonintegration

Since the input is homogenous, both input prices under nonintegration are equal to \( c \) if \( c_1 = c_2 = c \) and \( \bar{c} \) if \( c_1 = c_2 = \bar{c} \). When \( c_i = c \) and \( c_j = \bar{c} \), \( U_i \) can always undercut \( U_j \) slightly and sell to both \( D_1 \) and \( D_2 \), so in equilibrium, only \( U_i \) sells the input. We will assume that the difference between \( \bar{c} \) and \( c \) is not too large in the sense that \( U_i \) will prefer to set the input price at \( \bar{c} \).

Assuming that in case of a tie, \( D_1 \) and \( D_2 \) buy from the lowest cost supplier (and if costs are the same, they randomize their purchases), it follows that in equilibrium,

\[
w_1 = w_2 = \begin{cases} 
c & \text{if } c_1 = c_2 = c, \\
\bar{c} & \text{otherwise.}
\end{cases}
\]

Let \( \bar{q} \equiv q_1(\bar{c}, \bar{c}) = q_2(\bar{c}, \bar{c}) \) be the equilibrium output levels when \( w_1 = w_2 = \bar{c} \), and define \( q \) similarly. The associated downstream prices are \( \bar{p} \equiv p_1(\bar{c}, \bar{c}) = p_2(\bar{c}, \bar{c}) \) and \( p = p_1(c, c) = p_2(c, c) \).

Since the input is converted to output on a 1:1 basis, \( \bar{q} \) and \( q \) are also the demands for the input. The expected profit of each supplier is then:

\[
V^U_0 = \frac{1}{4} \times 2(\bar{c} - c) \bar{q} = \frac{(\bar{c} - c) \bar{q}}{2}.
\]

This expression reflects the fact that a nonintegrated supplier \( U_i \) earns a positive profit only when \( c_i = c \) and \( c_j = \bar{c} \); the probability of this event is \( \frac{1}{4} \). The supplier then sets a price of \( \bar{c} \) and sells \( \bar{q} \) units to each downstream firm. The associated expected profits of \( D_1 \) and \( D_2 \) is

\[
V^D_0 = \frac{3}{4} \pi_1(\bar{c}, \bar{c}) + \frac{1}{4} \pi_1(c, c),
\]

where

\[
\pi_1(\bar{c}, \bar{c}) = \bar{q} (\bar{p} - \bar{c}), \quad \pi_1(c, c) = q (p - c).
\]

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7.2.2 Integration

When $U_1$ and $D_1$ integrate, $U_1$ supplies $D_1$ at cost, unless $c_1 = \bar{c}$ and $c_2 = \underline{c}$, in which case $U_2$ sells the input to $D_1$ at a price equal to $\bar{c}$. Hence, $D_1$ buys the input from $U_1$ at $\underline{c}$ if $c_1 = \underline{c}$ and at $\bar{c}$ if $c_1 = c_2 = \bar{c}$, and buys it from $U_2$ at $\bar{c}$ if $c_1 = \bar{c}$ and $c_2 = \underline{c}$. Note that in all cases, $w_1 = c_1$.

As in OSS, we assume that when $U_1$ and $D_1$ integrate, $U_1$ commits not to sell to $D_2$. Hence $U_2$ becomes the sole supplier to $D_2$ and sets the input price, $w_2$, to maximize its profit

$$(w_2 - c_2) q_2 (c_1, w_2).$$

We will assume that this profit is concave in $w_2$. This assumption holds for example in the linear demand example shown below. The profit maximizing value of $w_2$ is defined implicitly by the following first-order condition:

$$q_2 (c_1, w_2) + (w_2 - c_2) \frac{\partial q_2 (c_1, w_2)}{\partial w_2} = 0. \quad (28)$$

The solution for the equation, $w_2 (c_1, c_2)$, determines $D_2$’s marginal cost. Clearly, $w_2 (c_1, c_2) > c_2$ for all $c_2$. Moreover, $w_2 (\bar{c}, \underline{c}) \geq \bar{c}$ under the following assumption:

Assumption 1: $q_2 (\bar{c}, \bar{c}) + (\bar{c} - \underline{c}) \frac{\partial q_2 (\bar{c}, \underline{c})}{\partial w_2} \geq 0.$

Since $\frac{\partial q_2 (\bar{c}, \underline{c})}{\partial w_2}$ is bounded from above and $q_2 (\bar{c}, \bar{c}) > 0$, Assumption 1 holds when $\bar{c} - \underline{c}$ is sufficiently small.

The expected profit of $D_1$ under integration is

$$V_1^D = \frac{1}{4} \pi_1 (\underline{c}, w_2 (\underline{c}, \bar{c})) + \frac{1}{4} \pi_1 (\bar{c}, w_2 (\underline{c}, \bar{c})) + \frac{1}{4} \pi_1 (\bar{c}, w_2 (\bar{c}, \underline{c})) + \frac{1}{4} \pi_1 (\bar{c}, w_2 (\bar{c}, \bar{c})). \quad (29)$$

Notice that since $\pi_i (w_i, \bar{w}_j) > \pi_i (w_i, w_j)$ for $\bar{w}_j > w_j$ and since $w_2 (\bar{c}, \bar{c}) > w_2 (\bar{c}, \underline{c}) \geq \bar{c}$ and $w_2 (\underline{c}, \underline{c}) > \underline{c}$,

$$V_1^D = \frac{1}{4} \pi_1 (\underline{c}, w_2 (\underline{c}, \bar{c})) + \frac{1}{4} \times \pi_1 (\underline{c}, w_2 (\underline{c}, \bar{c})) + \frac{1}{4} \pi_1 (\bar{c}, w_2 (\bar{c}, \underline{c})) + \frac{1}{4} \pi_1 (\bar{c}, w_2 (\bar{c}, \bar{c}))
> \frac{1}{4} \pi_1 (\underline{c}, \underline{c}) + \frac{1}{4} \pi_1 (\bar{c}, \underline{c}) + \frac{1}{4} \pi_1 (\bar{c}, \bar{c}) + \frac{1}{4} \pi_1 (\bar{c}, \bar{c})
> \frac{1}{4} \pi_1 (\underline{c}, \underline{c}) + \frac{1}{4} \pi_1 (\bar{c}, \underline{c}) + \frac{1}{4} \pi_1 (\bar{c}, \bar{c}) + \frac{1}{4} \pi_1 (\bar{c}, \bar{c})
= V_0^D.$$

\footnote{There is a debate about $U_1$’s ability to make this commitment: see Hart and Tirole (1990) and Reiffen (1992), and see Ordover, Salop, and Saloner (1992) for a response. Several papers have proposed models that are immune to this criticism, including Ma (1997), Chen (2001), Choi and Yi (2001), and Church and Gandal (2000). We will follow OSS in assuming that $U_1$ can commit not to sell to $D_2$ because our purpose here is to show that a variant of the OSS model predicts that there are cases in which $G > L$ and there are cases in which the opposite holds.}
That is, vertical integration and the foreclosure of \( D_2 \) boost the profit of \( D_1 \). Since \( U_1 \) commits not to sell to \( D_2 \) and since it transfers the input to \( D_1 \) at cost, its profit is \( V_{U_1}^D = 0 \). Given that its pre-merger profit is \( V_{U_1}^U > 0 \), it follows that integration and the foreclosure of \( D_2 \) involve a transfer of profits from \( U_1 \) to \( D_1 \).

Vertical integration is profitable if the downstream gain exceeds the upstream loss:

\[
\frac{V_D^1 - V_U^1}{G} > \frac{V_U^0 - V_{U_1}^U}{L} = V_{U_1}^U, 
\]

where \( G \) is the downstream benefit from vertical integration and the foreclosure of \( D_2 \) and \( L \) is the associated upstream loss. The next example shows that \( G > L \) for a broad range of parameters.

### 7.2.3 Example

Assume that \( q_i = A - p_i + \gamma p_j \), where \( \gamma \in [0, 1] \) is the degree of product differentiation. The profit of each downstream firm \( i \) is \( \pi_i = q_i (p_i - w_i) \). The Nash equilibrium when both firms choose their prices simultaneously is

\[
p_1(w_1, w_2) = \frac{(2 + \gamma) A + 2w_1 + \gamma w_2}{4 - \gamma^2}, \quad p_2(w_1, w_2) = \frac{(2 + \gamma) A + 2w_2 + \gamma w_1}{4 - \gamma^2}.
\]

The resulting quantities are

\[
q_1(w_1, w_2) = \frac{(2 + \gamma) A - (2 - \gamma^2) w_1 + \gamma w_2}{4 - \gamma^2},
\]

and

\[
q_2(w_1, w_2) = \frac{(2 + \gamma) A - (2 - \gamma^2) w_2 + \gamma w_1}{4 - \gamma^2}.
\]

The equilibrium profits are \( \pi_1(w_1, w_2) = q_1(w_1, w_2)^2 \) and \( \pi_2(w_1, w_2) = q_2(w_1, w_2)^2 \). Notice that \( \pi_i(w_i, w_j) \) decreases with \( w_i \) and increases with \( w_j \), as Lemma 1 above states.

Given these expressions, the expected pre-merger profits of \( D \) and \( U_1 \) are:

\[
V_D^0 = \frac{3}{4} \pi_1(\overline{c}, \overline{c}) + \frac{1}{4} \pi_1(\overline{c}, \overline{c}) = \frac{3 (A - (1 - \gamma) \overline{c})^2 + (A - (1 - \gamma) \overline{c})^2}{4 (2 - \gamma^2)}, \tag{31}
\]

and

\[
V_U^0 = \frac{(\overline{c} - \overline{c})}{2} \times \frac{(2 + \gamma) A - (2 - \gamma^2) \overline{c} + \gamma \overline{c}}{4 - \gamma^2}. \tag{32}
\]

\[
= \frac{(\overline{c} - \overline{c}) (A - (1 - \gamma) \overline{c})}{2 (2 - \gamma)}.
\]
To calculate the price at which $U_2$ sells to $D_2$ after $U_1$ and $D_1$ integrate, recall that after integration, $w_1 = c_1$. Substituting $q_2(c_1, w_2)$ into (28) and solving for $w_2$ yields

$$w_2(c_1, c_2) = \frac{(2 + \gamma) A + (2 - \gamma^2) c_2 + \gamma c_1}{2(2 - \gamma^2)}.$$  

Hence, the profit of $D_1$, given $c_1$ and $c_2$, is

$$\pi_1(c_1, w_2(c_1, c_2)) = q_1(c_1, w_2(c_1, c_2))^2 = \left(\frac{\beta A - c_1 (8 - 9\gamma^2 + 2\gamma^4) + \gamma c_2 (2 - \gamma^2)}{2 (2 - \gamma^2)(4 - \gamma^2)}\right)^2,$$

where

$$\beta \equiv 8 + 6\gamma - 3\gamma^2 - 2\gamma^3.$$  

Substituting into (29) and rearranging,

$$V^D_1 = \frac{1}{4} \pi_1(\xi, w_2(\xi, \xi)) + \frac{1}{4} \pi_1(\xi, w_2(\xi, \bar{\xi})) + \frac{1}{4} \pi_1(\bar{\xi}, w_2(\bar{\xi}, \xi)) + \frac{1}{4} \pi_1(\bar{\xi}, w_2(\bar{\xi}, \bar{\xi}))$$

$$= \frac{2\beta^2 A^2 + \phi (\xi^2 + \bar{\xi}^2) - \gamma (32 - 52\gamma^2 + 26\gamma^4 - 4\gamma^6) \bar{\xi} - 2 (1 - \gamma) A \beta^2 (\xi + \bar{\xi})}{8 (2 + \gamma)^2 (4 - 2\gamma - 2\gamma^2 + \gamma^3)^2},$$

where

$$\phi \equiv 64 - 16\gamma - 140\gamma^2 + 26\gamma^3 + 109\gamma^4 - 13\gamma^5 - 35\gamma^6 + 2\gamma^7 + 4\gamma^8.$$  

To simplify the computations, we will use the normalizations $A = 1$ and $\xi = 0$. To ensure that $w_2(\bar{\xi}, \xi) \geq \bar{\xi}$, we will also

$$\bar{\xi} \leq \frac{2 + \gamma}{4 - \gamma - 2\gamma^2}. \quad (34)$$

Substituting from (31), (32), and (33) into (30) and using the normalizations, we get

$$\frac{V^D_1 - V^0_D}{G} - \frac{V^U_0}{L} = \frac{2\beta^2 + \phi \bar{\xi}^2 - 2 (1 - \gamma) \beta^2 \bar{\xi}^2}{8 (2 + \gamma)^2 (4 - 2\gamma - 2\gamma^2 + \gamma^3)^2} - \frac{4 - 2 (1 - 2\gamma) \bar{\xi} - (1 - \gamma^2) \bar{\xi}^2}{4 (2 - \gamma)^2}. \quad (35)$$

This expression depends only on the degree of product differentiation, $\gamma$, and on $\bar{\xi}$. Figure 1 shows that the combinations of $\gamma$ and $\bar{\xi}$ for which (35) holds. The relevant range of parameters which satisfy (34) are those below the $\frac{2 + \gamma}{4 - \gamma - 2\gamma^2}$ curve. The figure shows that the downstream benefit from vertical integration and the foreclosure of $D_2$, $G$, exceeds the associated upstream loss, $L$, when $\gamma$ is sufficiently large, i.e., the downstream products are sufficiently close substitutes. When $\gamma$ is low, $L$ exceeds $G$ (note in particular that when $\gamma = 0$, $D_1$ and $D_2$ do not compete with each other, so $G = 0$, implying that $L > G$; by continuity this is also true when $\gamma$ is positive but small).
7.3 A variant of Salinger (1988)

This example shows that our basic setup is also consistent with Salinger (1988). In his model, there are \( N \geq 2 \) symmetric upstream suppliers \( U_1, \ldots, U_N \), which is produce a homogenous input at a cost \( c \) per unit. The upstream firms compete by setting quantities and the input price, \( w \), clears the input market. For simplicity, we will assume here that there are only two downstream firms, \( D_1 \) and \( D_2 \), which convert the input to a final product on a 1:1 basis at no additional cost. The two downstream firms also compete by setting quantities. The demand for the final good is given by \( p = A - Q \), where \( Q \) is the sum of the quantities of \( D_1 \) and \( D_2 \).

7.3.1 Nonintegration

Since \( D_1 \) and \( D_2 \) convert the input to a final product on a 1:1 basis at no additional cost, their marginal costs are equal to the input price \( w \). Noting that \( D_1 \) and \( D_2 \) engage in Cournot competition, the output of each firm is \( \frac{4-A-w}{3} \). Hence, the total demand for the input is \( Q = \frac{2(A-w)}{3} \), so the inverse demand for the input is \( w = A - \frac{3Q}{2} \), where \( Q = q_1 + \ldots + q_N \) is the total output of the upstream suppliers.

The profit of each upstream supplier \( i \) is given by \( q_i(w - c) \). Each upstream supplier \( i \) chooses \( q_i \) to maximize his profit. The resulting Nash equilibrium output of each upstream firm is

\[
q^* = \frac{2(A-c)}{3(N+1)},
\]
and the equilibrium price of the input is 
\[ w^* = A - \frac{3Nq^*}{2} = \frac{A + Nc}{N + 1}. \]

The equilibrium profit of each upstream firm then is 
\[ V_0^U = q^* (w^* - c) = \frac{2}{3} \left( \frac{A - c}{N + 1} \right)^2, \] (36)

and the equilibrium profit of each downstream firm is 
\[ V_0^D = \left( \frac{A - w^*}{3} \right)^2 = \left( \frac{N (A - c)}{3(N + 1)} \right)^2. \] (37)

### 7.3.2 Integration

As Salinger argues, when \( U_1 \) and \( D_1 \) integrate, \( U_1 \) finds it optimal to withdraw from the input market and supply only \( D_1 \), who buys the input at a cost \( c \). Hence, \( V_1^U = 0 \), implying that the upstream loss from vertical integration is \( L = V_0^U \).

Now, \( D_2 \) buys the input at \( w \), while \( D_1 \) buys it at \( c \). In a Nash equilibrium in the downstream market, the output of \( D_1 \) is \( \frac{A - 2c + w}{3} \) and the output of \( D_2 \) is \( \frac{A - 2w + c}{3} \). Since only \( D_2 \) buys the input in the upstream market (\( D_1 \) is supplied by \( U_1 \) at marginal cost), the inverse demand for the input is \( w = \frac{A + c - 3Q}{2} \).

The profit of each nonintegrated upstream supplier \( i \) is given by \( q_i (w - c) \). Each upstream supplier \( i \) chooses \( q_i \) to maximize his profit. The resulting Nash equilibrium output of each upstream firm is 
\[ q^{**} = \frac{A - c}{3N}, \]
and the equilibrium price of the input is 
\[ w^{**} = \frac{A + c - 3(N - 1)q^*}{2} = \frac{A + (2N - 1)c}{2N}. \]

Consequently, the equilibrium profit of \( D_1 \) is 
\[ V_1^D = \left( \frac{A - 2c + w^{**}}{3} \right)^2 = \left( \frac{(2N + 1)(A - c)}{6N} \right)^2. \] (38)

Substituting from (37), (36), and (38) into (30) and using the normalizations, we get
\[ V_1^D - V_0^D - V_0^U = \left( \frac{2N + 1}{6N} \right)^2 \left( \frac{(A - c)}{3(N + 1)} \right)^2 - \frac{2}{3} \left( \frac{A - c}{N + 1} \right)^2 \] (39)
\[ = \left( \frac{A - c}{6N(N + 1)} \right)^2 \left( \frac{1}{6N} - 11N^2 + 12N^3 \right). \]

This expression positive for all \( N \) so vertical integration is always profitable in the Salinger model.
8 References


