Policy Making with Reputation Concerns*

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Abstract

We study in a model of political economy the policy choice of an incumbent politician when he is concerned with the public’s perception of his capability. The politician decides whether to maintain the status quo or to conduct a risky reform. The success of any reform critically depends on the ability of the politician in office, while the true ability of the politician is privately known to himself only. The public observes both his policy choice and the outcome of the reform, and forms a posterior on the true ability of the politician. We show that politicians may engage in socially detrimental reform in order to be perceived as more capable. Conservative institutions that thwart reform may potentially improve social welfare.

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1 Introduction

She was not much deceived as to her own skill either as an artist or a musician, but she was not unwilling to have others deceived, or sorry to know her reputation for accomplishment often higher than it deserved.

*Emma*, vol. 1, ch. 6, by Jane Austen

In making decision and taking actions, we are often concerned about inferences that people draw about us from our choices and/or their consequences. Positive assessment from others not only generates psychological satisfaction and improves one’s social status, but also could lead to various tangible gains such as opportunities in career development. To a large extent, our success, professional or otherwise, is determined by such inferences. The concerns on reputation form one important dimension of the informal incentives that motivate economic agents to make relevant decisions in various contexts.

Economic agents’ reputation concerns play a particularly prominent role in public sectors or nonprofit organizations. Formal contracts based on explicit performance-based incentives are usually rare in these environments, and actions chosen by decision makers to enhance their reputation are widespread. For instance, a politician in office can be worried about his chance of being reelected or how the public evaluates his legacy. A bureaucrat in SEC may desire a promotion or a generous job offer from private financial sectors after his term of service. Reputation concerns may loom large in private sectors as well. A long list of CEO recognition prizes are awarded every year, which could tremendously contribute to the career development of highlighted business leaders. Both anecdotal and empirical evidence are available to testify the nontrivial role of recognition awards on executives’ behaviors. The incentives of financial analysts provide another salient example. They have to strategically report the information they have received as the ex post realization of market activities tremendously affect the perception of investors on their capabilities, which critically influences their future career.

An enormous amount of economic literature has been devoted to analyzing economic agents’ behavior in a wide array of environments where they are subject to reputation or career concerns. In this paper, we identify one particular way in which such concerns affect individuals’ behavior – they may take on risky or innovative initiatives whose success depends on their capability to improve the public’s perception on their talent, even though they know their own capability is low, and they have a lesser chance of making success. Throughout our paper, we refer to the decision maker in our model as a “politician”, though our analysis may encompass a variety of environments: a CEO who has to decide whether to implement an expansion plan or not, a doctoral candidate who has to decide to pursue a cutting-edge
project or not, or a potential mate who has to decide whether to perform an innovative courting ritual or not, etc.

In our model, an office-holding politician decides whether to drop the status quo policy in favor of a reform proposal. The ex post performance of the reform depends on two factors: the inherent merit of the available reform proposal (its potential value) and implementation by the politician. The potential value of a reform proposal follows certain continuous distribution, which can be costlessly discovered by the politician in office before he decides to undertake the reform. We assume that the potential value of the available reform proposal is a piece of “hard” information and therefore is verifiable, while publicizing this information is in the discretion of the politician. There exist two types of politicians with differing abilities. The ability of a politician in office can be either high or low and is privately known only to the politician himself.

If the status quo is retained, the performance of the politician does not vary across differing types, and it would not be subject to perturbation. Arguably, a continuing policy reduces uncertainty and allows the politician in office to gather reliable information from past. By contrast, uncertain situations arise once the status quo is abandoned, and the politician has to manage the resulted uncertainty. He receives a private signal regarding the true state of the world, and has to choose his action in response to his conjectured underlying state of the world. The reform could succeed only if the politician chooses the ex post optimal response to the state of the world, while it fails otherwise. The ability of the politician plays a critical role in the outcome of a reform. A high-type politician receives a more precise signal about the true state of the world, which enables him to choose responsive action and conduct socially beneficial reform given sufficiently promising reform proposal; while the low type receives only a noisy signal, which causes him to make more mistakes. We assume that the reform implemented by a low-type politician is ex ante socially inefficient regardless of the merit of the reform proposal. His ability is inadequate for managing the uncertainty after the status quo is overthrown.

We posit that the politician in office is concerned about the public’s perception of his type. He therefore makes his policy choice (reform or not reform) to maximize his reputation payoff. The public observes the policy choice of the politician as well as the resulted performance. The public then updates their belief on the type of the politician based on the two pieces of information. Two possibilities may lie beneath when status quo is maintained. Firstly, a high-type politician may not reform if a rosy proposal is available; Secondly, a low-type may abstain from reform even if considerably valuable proposal appears as he is afraid of failure. If a reform has been implemented, a politician is more likely to be regarded as being capable if the reform succeeds, while he suffers from a more pessimistic assessment if a miserable outcome results. Hence, reform function as a costly signal that conveys the politician’s...
private information; while the posterior is formed based on not only the politician’s action of reform (or no reform) but also his (random) performance.

We show that there exist a continuum of Perfect Bayesian equilibria in this game. Each equilibrium is characterized by a distinct cutoff, such that the high-type politician reforms if and only if the available reform proposal carries a potential value that exceeds the cutoff. We find that there exists no fully-separating equilibrium. We show that a high-type politician is always “eager” to reveal more information by undertaking reform: he reforms with probability one once the value of available proposal exceeds the cutoff of prevailing equilibrium. The low type always mimics his high-type counterpart with a positive probability. Although the reform undertaken by a low type fails with a higher probability, his reputation concerns “force” him to do so, because he would otherwise suffer from more unfavorable assessment.

Our analysis allows us to make a number of interesting observations.

- **Pressure to prove oneself.** The probability of the low type undertaking reform strictly decreases with the public’s assessment of the likelihood that the politician is capable. When the public holds a more pessimistic prior, the low-type politician would expect a greater gain if his reform turns out to succeed. Because of this effect, we predict that reform will be observed less often when the public holds a more favorable prior on the type of the politician, or a higher portion of high-type politicians exist in the population. Furthermore, though our model is static, this result points towards the following conjecture about the dynamic behavior of a politician: a politician who has failed in the past is more likely to take radical action in the future. Past failure lowers his rating among the public, which therefore makes more lucrative an accidental success in the future.

- **Tough act to follow.** The higher is the capability differential between the high type and the low type, the less likely the low type undertakes reform. On the one hand, it could lead high type to reform more, which forces the low type to follow suit. On the other hand, it makes successful mimicry more difficult. We show that the latter effect always prevails and the low type in equilibrium must reform less often to avoid failure.

- **Thwarted good reforms.** Our model has multiple-equilibria, with different thresholds for reform. Suppose a legislative body can set the threshold, i.e., a limit of the freedom of the politician, and require the politician not to conduct reform unless the value of available reform proposal exceeds the threshold. Thus, what is the optimal threshold that maximizes expected social welfare? Competing effects again result. A higher threshold reduces the gambling of the low type on the one hand, while it reduces the efficient reform undertaken by the high type on the other. We derive a fairly general conclusion about the socially optimal threshold, and we find that moderate
“conservatism” can be optimal in this context, despite that it must thwart ex ante beneficial reform.

In the rest of this section, we discuss the link between our paper and the relevant literature. In Section 2, we set up the model. We carry out our analysis in Section 3, which establishes equilibria of the model and present comparative statics of relevant environmental factors. We discuss the welfare implications of our equilibrium results and the issue of institution design in Section 4. Section 5 provides a concluding remark.

**Relation to Literature**

In our paper, the action of reform serves as a signalling device for the office-holding politician. A reform is more costly to a low-type politician as he would be more likely to fail and therefore suffer an unfavorable assessment. In the current setting, the politician takes action to maximize his reputation payoff. Hence, our paper also belongs to the extensive literature on career concerns.

The notion of career or reputation concern is not novel. Ever since the pathbreaking work of Holmstrom (1999) and Dewtripont, Jewitt, and Tirole (1999), the incentive effect of career (repuation) concerns has been the subject of a vast amount of scholarly effort in a wide array of contexts. For instance, Alesina and Tabellini (2007) discuss the appropriate task allocation to bureaucrats when they take action in fulfilling their duty to maximize their reputation payoff.

Within this literature, our paper is related to studies on managerial herding, such as Scharfstein and Stein (1990), Zwiebel (1995) and Ottaviani and Sørensen (2006). These papers concern themselves with the decision making of managers with varying abilities when the public observe their performance and update their belief on the manager’s ability. Scharfstein and Stein (1990) show that a manager tends to ignore the market information available to him but to follow the decision made by the leader. Ottaviani and Sørensen (2006) show that, if an expert cares about being perceived more capable, he may make a report that conforms to the decision maker’s priors even though he has observed different information.

Zwiebel (1995) also explores how reputation concerns moderate one’s incentive to undertake innovative but risky action. He shows that, in a setup where innovation rarely occurs and the act of innovation is not observable, managers with intermediate capability may not want to innovate even if it is beneficial to the firm. The main difference between our model and his is that the act of reform is observable in our model. Hence, the setting of Zwiebel (1995), as well as those in the managerial herding literature, does not involve costly signaling action on the part of the decision maker. We also arrive at the opposite conclusion that there can be too much reform when the politician cares about his reputation.
A handful of studies include the flavors from both the literature of signaling and career concerns. Prendergast and Stole (1996) argue that career concerns impact the behavior of young and old investors differently. They show that young investors may overreact to the new information they receive, which allow them to signal that they are fast learners. Majumder and Mukand (2004) studies governments’ dynamic incentive of policy experimentation and persistence when the government’s payoff depends on not only the performance of the economy but also its chance of being re-elected, which is determined by voters’ perception of its ability. They show that the government can be either too radical or too conservative at early stage of its term. Our work is also closely related to that of Chung and Esö (2008). They build a model in which a worker only has an imperfect signal about his own capability. He wants to choose a task to both signal his capability to potential employers and learn information about his own capability. They also assume that the more difficult task is a worse (less informative) device for evaluating the capability of the worker. They show that workers with very high and low capability choose the more difficult task while those with intermediate capability choose the easier task. Their result is similar in spirit to that of Feltovich, Harbaugh, and To (2002), who show that a worker with very high capability may in fact not engage in costly signalling (in other words, they “countersignal”) but a worker with intermediate capability does. In the setting of Chung and Esö (2008), “labor market” forms posterior upon observing the choice of action. By contrast, in our setting the public’s posterior is formed by not only the action itself but also the realized performance of the politician. In another closely related paper, Levy (2007) looks into voting behaviors when economic agents, motivated by career concerns make collective decision in a committee. She finds that radical action is more likely to be accepted when the committee voting process is transparent, and the public is able to infer a voter’s ability by observed vote.

Our paper is also conceptually related to Hermalin (2005) and Dominguez-Martinez, Swank and Visser (2008). Hermalin (2005) construct a model where board members engage in costly effort to monitor CEO. Higher effort allows them to learn better about the type of the CEO, and they retain the CEO only if the posterior is sufficiently optimistic. However, the model of Hermalin (2005) does not involve strategic action of the CEO. Dominguez-Martinez, Swank and Visser (2008) extends Hermalin (2005) but allows the CEO to design and implement “projects”. Board members design efficient contract, infer the CEO’s ability by monitoring his performance, and make retention decision. This study also mainly focuses on the behavior of board. Our paper therefore also complements this literature.
2 Setup

A risk-neutral politician makes a policy choice between two alternatives: maintaining the status quo or implementing a reform. If the politician retains the status quo, the outcome of this polity, \( y \), is deterministic, which we normalize it to 0. By contrast, if the politician chooses to undertake the reform, uncertainty will arise that affect the outcome and the politician must take an action to address it. The outcome is given by

\[
y = \theta - (a - \omega)^2.
\]

where \( \theta \) measures the value of reform, \( \omega \) is the true state of the world, and \( a \) is the action taken by the politician in response to his assessment of \( \omega \). The politician observes the value of the reform, \( \theta \), before choosing whether to implement it. We assume that the signal \( \theta \) is a piece of “hard” and verifiable information. It is common knowledge that \( \theta \) is continuously distributed on \([-\tilde{\theta}, \tilde{\theta}]\) with distribution function \( F \) and density function \( f \), where \( \tilde{\theta} \in (1, 2) \). The state of the world, \( \omega \), may take two values, \(-1\) or \(1\), each with probability \(1/2\). Neither the politician nor the public observe the true state. The action \( a \) is chosen from \(\{-1, 1\}\). Thus, when a reform is implemented, the best outcome is achieved when the politician takes an action that turns out to match the state of the world.

The distinction between policies (status quo or reform) and actions is important in our model. Policies are macro-level or “strategic” decisions such as whether to introduce a new product or whether to start a war. By contrast, actions are micro-level or “tactical” decisions such as which technology to use in the new product or how many troops to deploy in the war. Though there may be general agreement about how desirable a reform is (\(\theta\)), there may well be disagreement over the optimal way to implement the reform (\(a\)). The true nature of the problem (\(\omega\)) determines which action is ex post suitable for implementing the reform.

The politician’s talent, \(t\), which affects the success of the reform, can be high (\(t = H\)) or low (\(t = L\)). The talent of the politician is his private information. A high-talent politician receives an informative signal \(\sigma \in \{-1, 1\}\), which matches the true state with probability

\[
q = \Pr(\sigma = \omega) > \frac{3}{4}.
\]

By contrast, a low-talent politician’s signal is completely uninformative.\(^1\) Let \(\alpha\) be the probability of \(t = H\), which is commonly known. We assume that the proportion of “good”

\(^1\)Though we do not model how the politician obtains his signal, one may interpret the politician’s talent in our model as the ability to gather information from various sources. The US presidential historian, Erwin C. Hargrove, paints two completely different pictures of Franklin D. Roosevelt and Herbert Hoover with respect to information gathering. Roosevelt brought together experts who held a great variety of views and balanced them off against each other while Hoover did not enjoy critical advice from anyone. See pp 70-73 and pp 114-116, Hargrove, E. C. (1966): Presidential Leadership, MacMillan Company, New York.
politicians in the population is small:\(^2\)

\[ \alpha < \frac{1}{2}. \]

Upon receiving \( \sigma \) (either informative or uninformative), the politician takes an action.

The public observes the politician’s policy choice (status quo or reform) and the final outcome.\(^3\) Their updated belief, or the reputation of the politician, can be written as

\[ \mu_i(y) \equiv \Pr(t = H | y, i) \]

by Baye’s rule, where \( i = 0 \) indicates status quo and \( i = 1 \) indicates reform. We use \( \mu_0 \) to denote the politician’s reputation when no reform is implemented as the outcome is always zero. We also use \( \mu_{1H}(\theta) \) and \( \mu_{1L}(\theta) \) to denote the expected reputation payoff of the high type and the low type from choosing to reform when the value of the reform proposal is \( \theta \). Similar to Chung and Esö (2008) and Ottaviani and Sørensen (2002), we assume that the politician’s payoff purely depends on his reputation. The politician therefore chooses the action that maximizes his reputation.

We adopt the concept of Perfect Bayesian Equilibrium to analyze the game.

### 3 The Analysis

When the status quo is abandoned, and an action \( a \) is taken, the expected output of the reform is given by

\[ E(y) = \theta - E_{\omega \in \{-1,1\}} (a - \omega)^2 \geq 0. \] (2)

Apparently, the action taken by the low-type politician is irrelevant as he receives completely noisy signal regardlessly. A high-type politician, however, would follow his signal to maximize the likelihood of success.

In the first-best situation, a politician would abandon the status quo if and only if the expected outcome \( E(y) \) is nonnegative. Recall that the upper bound of the value of reform, \( \hat{\theta} \), is in the interval \((1, 2)\). A low-type politician should never reform regardless of \( \theta \) as the expected loss from wrong actions always exceeds the benefit of reform, that is,

\[ E(y) = \frac{1}{2} \theta + \frac{1}{2} (\theta - 4) \leq \hat{\theta} - 2 < 0. \]

\(^2\)This regularity assumption guarantees that the low type has an incentive to undertake reform and mimic the high type when the high type takes perfectly informed action when he implements reform (see the proof of Lemma 1).

\(^3\)In our setup, whether or not the public observe the action is inconsequential. Once the politician chooses reform, the belief of the public is determined only by whether the outcome is a “failure” or “success.”
By contrast, the expected outcome for a high-type politician is given by

\[ E(y) = \theta - 4(1 - q). \]

Thus, the high type should undertake reform if and only if he receives a reform proposal that is sufficiently valuable:

\[ \theta \geq 4(1 - q). \]

When the value of a reform falls below \( 4(1 - q) \), a reform is socially destructive regardless of the type of the politician.

We now formally analyze the politician’s policy choice. We assume that the politician is subject to a minimum level of “accountability” constraint such that no reform with a value \( \theta < 4(1 - q) \) is acceptable. Clearly, when a reform with \( \theta < 4(1 - q) \) is undertaken, the reform must be ex ante socially detrimental even if it is implemented by a high-talent politician. Given the accountability obligation, we assume that a politician in office is not allowed by public to undertake obviously socially destructive activities. We then focus on our attention on equilibria where the politician reforms only when \( \theta \geq 4(1 - q) \).

Let a type-\( t \) politician choose reform with probability \( \rho_t(\theta) \) for each realized value \( \theta \). We focus on monotonic equilibria, where the politician’s probability of undertaking reform is nondecreasing with \( \theta \), the potential value of the reform. Define \( \bar{t}_t = \inf\{ \theta | \rho_t(\theta) > 0 \} \). Thus, a type-\( t \) politician undertakes reform with a positive probability only if the value \( \theta \) exceeds a cutoff \( \bar{t}_t \).

When the politician maintains the status quo, the public forms a posterior

\[ \mu_0 = \frac{\alpha F(\bar{t}_H) + \alpha \int_{\bar{t}_H}^{\theta} [1 - \rho_H(\theta)] f(\theta) d\theta}{\alpha F(\bar{t}_H) + \alpha \int_{\bar{t}_H}^{\theta} [1 - \rho_H(\theta)] f(\theta) d\theta + (1 - \alpha) F(\bar{t}_L) + (1 - \alpha) \int_{\bar{t}_L}^{\theta} [1 - \rho_L(\theta)] f(\theta) d\theta}. \]

Clearly, when no reform occurs, the politician’s payoff does not depend on his true type \( t \), because \( \mu_0 \) is uniform for all \( \theta \in \{ \theta | \rho_t(\theta) < 1 \} \).

When the politician implements a reform of value \( \theta \), the public, upon observing the performance \( y \), forms the posterior

\[ \mu_1(\theta) = \frac{\alpha q \rho_H(\theta) f(\theta)}{\alpha q \rho_H(\theta) f(\theta) + (1 - \alpha) \frac{1}{2} \rho_L(\theta) f(\theta)}. \]

when the reform succeeds and

\[ \mu_1(\theta - 4) = \frac{\alpha q \rho_H(\theta) f(\theta)}{\alpha q \rho_H(\theta) f(\theta) + (1 - \alpha) \frac{1}{2} \rho_L(\theta) f(\theta)}. \]

If he implements a reform with value \( \theta \), a high-talent politician receives an expected payoff

\[ \mu_{1H}(\theta) = q \mu_1(\theta) + (1 - q) \mu_1(\theta - 4), \]
while a low-talent politician

$$\mu_{1L}(\theta) = \frac{1}{2} \mu_1(\theta) + \frac{1}{2} \mu_1(\theta - 4).$$

In any equilibrium, the low type does not reform when the realized value of reform satisfies $\theta < \bar{\theta}_H$. If he did in equilibrium, since the high type does not reform for $\theta < \bar{\theta}_H$, the public must assign probability one to him being the low type regardless of success or failure, which thereby leaves him worse off than if he does not reform. Hence, we must have $\bar{\theta}_L \geq \bar{\theta}_H$ in any equilibrium. In the following lemma, we show that their strategies follow the same cutoff $\bar{\theta} = \bar{\theta}_L = \bar{\theta}_H$.

**Lemma 1** In any equilibrium that involves a positive probability of reform, (1) the cutoffs for reform must be the same for the low type and the high type, i.e., $\bar{\theta}_L = \bar{\theta}_H = \bar{\theta}$; and (2) the high-type politician plays a pure strategy $\rho_H(\theta) = 1$ for any $\theta \in [\bar{\theta}, \bar{\theta}]$.

**Proof.** First, observe that $q > 1/2$ implies that $\mu_1(\theta) > \mu_1(\theta - 4)$ as long as $\rho_H(\theta) > 0$ and $\rho_L(\theta) > 0$. But, this implies that $\mu_{1H}(\theta) > \mu_{1L}(\theta)$. Thus, whenever both types choose reform with a positive probability, the high type must choose it with probability one.

Second, we claim that whenever the high type chooses reform with a positive probability, the low type must do so as well. We have shown that whenever both types choose reform with positive probability, the high type’s probability of reform is one and therefore at least as high as the low type’s. Therefore, the overall probability for the low type to choose the status quo, $P_{0L}$, is weakly higher than that for the high type, $P_{0H}$. Thus, if the low type chooses the status quo, his reputation is $\mu_0 = \frac{\alpha P_{0H}}{\alpha P_{0H} + (1 - \alpha) P_{0L}} \leq \alpha$.

However, if he deviates and undertakes reform, he is believed to be a high type with probability one if $q < 1$. If $q = 1$, his payoff depends on the public’s off-equilibrium belief when reform fails. However, he succeeds with probability $\frac{1}{2}$, and the resulting expected payoff still exceeds $\alpha$. Therefore, it cannot be that the low type always chooses the status quo when the high type chooses reform. This completes our proof.

Q.E.D. 

The above lemma states that there is no full separation of the two types regardless of the value of the reform proposal. The same cutoff level $\bar{\theta} = \bar{\theta}_L = \bar{\theta}_H$ applies to both types of the politician. Above this threshold, the high type always undertakes reform and the low type mixes between reform and no reform.

We now determine the low-type politician’s probability of reform for a proposal with value $\theta$, which we denote by $\rho(\theta)$ to economize on notation. By (3), if the politician maintains the
status quo, his payoff is

\[
\mu_0 = \frac{\alpha F(\bar{\theta})}{\alpha F(\bar{\theta}) + (1 - \alpha)F(\bar{\theta}) + (1 - \alpha)\int_{\bar{\theta}}^{\theta} [1 - \rho(\theta)]f(\theta)d\theta}
\]

\[
= \frac{\alpha}{\alpha + (1 - \alpha)\frac{F(\bar{\theta}) + \int_{\bar{\theta}}^{\theta} [1 - \rho(\theta)]f(\theta)d\theta}{F(\bar{\theta})}}.
\]

(4)

Note that it does not depend on \(\theta\).

On the other hand, if the low-type politician undertakes the reform, his payoff is given by

\[
\mu_{1L}(\theta) = \frac{1}{2} \cdot \frac{q\alpha f(\theta)}{q\alpha f(\theta) + (1 - \alpha)\frac{1}{2}\rho(\theta)f(\theta)} + \frac{1}{2} \cdot \frac{(1 - q)\alpha f(\theta)}{(1 - q)\alpha f(\theta) + \frac{1}{2}(1 - \alpha)\rho(\theta)f(\theta)}
\]

\[
= \frac{1}{2} \cdot \frac{\alpha}{\alpha + (1 - \alpha)\frac{\rho(\theta)}{q}} + \frac{1}{2} \cdot \frac{\alpha}{\alpha + \frac{1}{2}(1 - \alpha)\rho(\theta)}.
\]

(5)

If the low-type plays a completely mixed strategy, \(\rho(\theta) \in (0, 1)\), we need to equate (4) and (5), which implies that \(\rho(\theta)\) must be a constant \(\rho\) regardless of the value \(\theta\). Consequently, in equilibrium,

\[
\frac{\alpha}{\alpha + (1 - \alpha)\frac{F(\bar{\theta}) + \int_{\bar{\theta}}^{\theta} [1 - \rho(\theta)]f(\theta)d\theta}{F(\bar{\theta})}} = \frac{1}{2} \cdot \frac{\alpha}{\alpha + (1 - \alpha)\frac{\rho}{q}} + \frac{1}{2} \cdot \frac{\alpha}{\alpha + \frac{1}{2}(1 - \alpha)\rho},
\]

(6)

which we may rewrite as

\[
\frac{1}{1 + \lambda(\alpha)A} = \frac{1}{2} \cdot \frac{1}{1 + \lambda(\alpha)B} + \frac{1}{2} \cdot \frac{1}{1 + \lambda(\alpha)C},
\]

(7)

where

\[
\lambda(\alpha) = \frac{1 - \alpha}{\alpha}, \ A = 1 + (1 - \rho) \cdot \frac{1 - F(\bar{\theta})}{F(\theta)}, \ B = \frac{\rho}{q}, \ C = \frac{\rho}{1-q}.
\]

The expression \(\lambda(\alpha)\) is the likelihood ratio of the low type versus the high type, while \(A, B,\) and \(C\) are the likelihood ratios of the low type achieving a particular outcome versus the high type doing so.

**Theorem 1** There exist a continuum of Perfect Bayesian Equilibria with cutoffs \(\bar{\theta} \in [4(1 - q), \bar{\theta}]\). For any \(\bar{\theta}\), there exists a unique equilibrium probability \(\rho^* \in (0, 1)\), which solves Equation (7), such that the low-type politician undertakes reform with the probability \(\rho^*\) whenever he receives a signal \(\theta \geq \bar{\theta}\).

**Proof.** Consider the equilibrium condition (7). Note that its LHS is \(\mu_0\) and its RHS is \(\mu_{1L}\). When \(\rho = 0\), LHS \(\leq \alpha\), while RHS = 1 as \(B = C = 0\). Therefore, LHS < RHS. By
contrast, when $\rho = 1$, $LHS = \alpha$ as $A = 1$, and $RHS < \alpha$. This can be shown by observing that when $\rho = 1$
\[ \alpha \mu_{1H} + (1 - \alpha) \mu_{1L} = \alpha, \]
where $RHS$ of (7) is $\mu_{1L}$.

Both the $RHS$ and $LHS$ of (7) are continuous in $\rho$. Furthermore, it is straightforward to show that the $LHS$ strictly increases with $\rho$, while the $RHS$ strictly decreases with $\rho$. Hence, we conclude that there must exist a unique $\rho^* \in (0, 1)$ that solves (7).

Q.E.D. 

Consider the limiting case where the high-type politician receives a perfect signal, i.e. $\Pr(\sigma = \omega|\omega) = 1$. Equation (7) can be rewritten as
\[ \frac{1}{1 + \lambda(\alpha) A} = \frac{1}{2\alpha + (1 - \alpha) \rho}, \]
which yields the solution
\[ \rho^* = \frac{1 - \alpha[1 + F(\theta)]}{1 - \alpha}. \]

Observe that the high-type politician reforms with probability one whenever the prospect of reform is sufficiently good, i.e., $\theta \geq \bar{\theta}$. Hence, it allows us to focus on the equilibrium behavior of the low type. The low type mimics the high type with a constant positive probability $\rho^*$ in any given environment.

**Comparative Statics**

We now study the properties of every equilibrium with a given cutoff $\bar{\theta}$ with respect to other environmental parameters. We first examine the comparative statics of $\alpha$, i.e., the prior of the public, or the proportion of high-type politicians. We investigate how an increasing $\alpha$ would affect the probability of a low-type politician conducts reform.

The answer to this question is not straightforward. Recall the equilibrium condition, i.e., Equation (7). When $\alpha$ increases, both $LHS$ and $RHS$ increase. A more favorable prior improves a low-type politician’s expected payoff under either policy choice. Formal analysis allows us to conclude the following

**Theorem 2** Fixing a cutoff for reform, $\bar{\theta}$, the probability of reform by the low type, $\rho^*$, is strictly decreasing in $\alpha$, the probability of high type.

**Proof.** Consider the equilibrium condition (7). We have shown above that the left hand side of (7) is increasing in $\rho$ and the right hand side decreasing in $\rho$. Note that $A$, $B$, and $C$ do not contain $\alpha$ in their expressions. Thus, we may write
\[
\frac{\partial (LHS - RHS) \text{ of } (7)}{\partial \alpha} = -\frac{1}{\alpha^2} \left[ -\frac{A}{(1 + \lambda(\alpha) A)^2} + \frac{1}{2} \cdot \frac{B}{(1 + \lambda(\alpha) B)^2} + \frac{1}{2} \cdot \frac{C}{(1 + \lambda(\alpha) C)^2} \right].
\]
We want to evaluate the above derivative at the value of $\rho$ that satisfies (7). Observe that $0 < B < C$ as $q \geq 3/4$, we may conclude then $B < A < C$ based on (7). From (7), we can also see that

\[ 1 - \frac{A}{1 + \lambda(\alpha)A} = 1 - \left[ \frac{1}{2} \cdot \frac{B}{1 + \lambda(\alpha)B} + \frac{1}{2} \cdot \frac{B}{1 + \lambda(\alpha)C} \right] = 1 - \left[ \frac{1}{2} \cdot \frac{B}{1 + \lambda(\alpha)B} + \frac{1}{2} \cdot \frac{B}{1 + \lambda(\alpha)C} \right], \]

which yields

\[ \frac{A}{1 + \lambda(\alpha)A} = 1 - \frac{B}{1 + \lambda(\alpha)B} + \frac{1}{2} \cdot \frac{C}{1 + \lambda(\alpha)C}. \]

Therefore,

\[ \frac{1}{2} \cdot \frac{B}{(1 + \lambda(\alpha)B)^2} + \frac{1}{2} \cdot \frac{C}{(1 + \lambda(\alpha)C)^2} = \frac{A}{1 + \lambda(\alpha)A} \left[ \frac{B}{1 + \lambda(\alpha)B} + \frac{C}{1 + \lambda(\alpha)C} \cdot \frac{1}{1 + \lambda(\alpha)B} + \frac{C}{1 + \lambda(\alpha)C} \cdot \frac{1}{1 + \lambda(\alpha)C} \right]. \]

The expression in the brackets is a convex combination of $\frac{1}{1 + \lambda(\alpha)B}$ and $\frac{1}{1 + \lambda(\alpha)C}$. Since $0 < B < C$, the former is larger, but the coefficient on the former is smaller than $\frac{1}{2}$. Using Equation (7), we have

\[ \frac{1}{2} \cdot \frac{B}{(1 + \lambda(\alpha)B)^2} + \frac{1}{2} \cdot \frac{C}{(1 + \lambda(\alpha)C)^2} < \frac{A}{(1 + \lambda(\alpha)A)^2}. \]

Hence, at the value of $\rho$ that satisfies (7),

\[ \frac{\partial (LHS - RHS) of (7)}{\partial \alpha} > 0. \]

Thus, by the implicit function theorem, the probability of reform by the low type, $\rho$, is decreasing in $\alpha$, the probability of high type.

Q.E.D. ■

Theorem 2 states that the less favorable the public’s prior assessment, the more likely the low type conducts reform. The analysis that is laid out above reveals its logic. A greater $\alpha$ increases the payoff of a low-type politician when he maintains the status quo relative to when he takes reform: On the one hand, the public would be more likely to attribute his choice to the lack of opportunities (when a lower $\theta$ is realized) rather than the lack of talent; on the other hand, a greater $\alpha$ enlarges his loss if the reform fails, which consequently weakens his incentive to take the risky action. By contrast, a less favorable prior would strengthen his incentive to risk, because a smaller $\alpha$ implies a lesser loss if he fails but more gain if he succeeds. We then interpret this as a “pressure to prove oneself” phenomenon.

The result of Theorem 2 allows us to investigate another property of the equilibrium. In this game, reform would take place with a probability

\[ \phi(\bar{\theta}; \alpha) = [1 - F(\bar{\theta})][\alpha + (1 - \alpha)\rho^*]. \]
When $\alpha$ increases, i.e., the public holds a more favorable prior on the politician’s type or a higher proportion of capable politicians exist in the population, would more or less reform be expected ex ante? Taking first order derivative of (XX) with respect to $\alpha$ yields

$$\frac{\partial \phi(\tilde{\theta}; \alpha)}{\partial \alpha} = [1 - F(\tilde{\theta})][1 - \rho^* + (1 - \alpha) \frac{\partial \rho^*}{\partial \alpha}].$$

(11)

Apparently, competing forces must come into play when $\alpha$ is raised. On the one hand, a high-type politician always engage in reform as long as a $\theta \geq \tilde{\theta}$ is realized, while a low type reforms only with a probability $\rho^*$, which tends to have $\sigma(\tilde{\theta}; \alpha)$ increase with $\alpha$ by a rate $[1 - F(\tilde{\theta})][1 - \rho^*].$ On the other hand, a greater $\alpha$ reduces the desire of a low-type politician to conduct reform, which tends to decrease $\sigma(\tilde{\theta}; \alpha)$ by a rate $[1 - F(\tilde{\theta})](1 - \alpha) \frac{\partial \rho^*}{\partial \alpha}.$ We now formally investigate this issue.

Define $m \equiv \frac{1 - F(\tilde{\theta})}{F(\tilde{\theta})}.$ The equilibrium condition can be rewritten as

$$G(\rho^*, \alpha) = [1 + (1 - \rho)m] - \frac{\rho[\lambda(\alpha)\rho + 1]}{4q(1 - q) + \lambda(\alpha)\rho} = 0.$$

(12)

We have

$$\frac{\partial G(\rho^*, \alpha)}{\partial \rho^*} = [m + 1 + \frac{4q(1 - q)[1 - [4q(1 - q)]^2]}{[4q(1 - q) + \lambda(\alpha)\rho^*]^2}],$$

(13)

and

$$\frac{\partial G(\rho^*, \alpha)}{\partial \alpha} = -\frac{1}{\alpha^2} \cdot \frac{\rho^2[1 - 4q(1 - q)]}{[4q(1 - q) + \lambda(\alpha)\rho^*]^2}.$$

(14)

Applying implicit theorem yields

$$\frac{\partial \rho^*}{\partial \alpha} = \frac{\rho^2[1 - [4q(1 - q)]^2]}{[4q(1 - q) + \lambda(\alpha)\rho^*]^2}.$$

(15)

Our analysis leads to the following prediction.

**Theorem 3** The overall likelihood of reform $\phi(\tilde{\theta}; \alpha)$ strictly decreases with $\alpha$.

**Proof.** See Appendix. □

The analysis shows that the overall likelihood of reform would be unambiguously reduced when $\alpha$ increases. The negative effect (i.e., a greater $\alpha$ makes the low type reform less) always dominates the positive effect. Theorem 3 yields empirically testable hypothesis. One may predict that when $\alpha$ decreases, i.e., when a smaller proportion of capable politicians exist in the population or when the public holds a less optimistic prior, more reform would be expected, without having to know the true type of the politician in office (which is private information and cannot be verified).

Next, we explore the comparative statics of $\rho^*$, the low type’s frequency of reform, with respect to $q$, the ability measure of the high-type politician.
Theorem 4  Fix any equilibrium with cutoff \( \theta \), the probability of reform by the low type, \( \rho^* \), is strictly decreasing in \( q \).

**Proof.** Recall the equation (12) that defines the equilibrium condition:

\[
G(\rho^*, q) = [1 + (1 - \rho)m] - \frac{\rho(\lambda \rho + 1)}{4q(1-q) + \rho}.
\]

Because \( q \geq \frac{3}{4} \), \( G(\rho^*, q) \) must be decreasing with \( q \). Further,

\[
\frac{\partial G(\rho^*, q)}{\partial \rho^*} = -\frac{\partial G(\rho^*, q)}{\partial \rho^*} = \frac{[m + 1 + 4q(1-q)](1 - [4q(1-q)]^2)}{[4q(1-q) + \rho]^2} < 0.
\]

We then obtain \( \frac{d\rho^*}{dq} = -\frac{\partial G(\rho^*, q)}{\partial \rho^*} < 0 \).

Q.E.D.  

Theorem 3 states that the low-type politician would mimic his high-type counterpart less often when his hightype counterpart is able to receive a more precise signal. It is not difficult to grasp the logic that underlies this result. A higher \( q \) leads the public to believe that an unsuccessful reform is more likely to be implemented by a low-type politician, which unambiguously reduces the expected payoff of the low type to undertake reform. This logic can be easily verified by evaluating \( \frac{\partial \mu_{1L}(\theta)}{\partial q} \) with \( q \) for a fixed \( \rho \), which yields

\[
\frac{\partial \mu_{1L}(\theta)}{\partial q} = \frac{1}{2} \lambda(\alpha) \rho \left[ \frac{1}{q + \frac{1}{2} \lambda(\alpha) \rho} - \frac{1}{(1-q) + \frac{1}{2} \lambda(\alpha) \rho} \right].
\]

It must be negative because \( q > 1 - q \). A greater ability differential makes it more difficult for a low type to mimic his high-type counterpart, and therefore leads to a lower \( \rho^* \). We then interpret this result as a “tougher action to follow” phenomenon.

As implied by the analysis laid out above, the distribution of \( \theta \) does not qualitatively alter the main prediction of our analysis. We now examine how the equilibrium probability of low type undertaking reform would quantitatively depends on the properties of \( F(\theta) \).

Theorem 5  Let \( \rho \) and \( \rho' \), respectively, denote the equilibrium probabilities of the low type undertaking reform associated with distributions \( F(\cdot) \) and \( G(\cdot) \). For a fixed \( \theta \), then, \( \rho > \rho' \) if and only if \( F(\cdot) \) first order stochastically dominates \( G(\cdot) \).

**Proof.** Rewrite the equilibrium condition as the following

\[
\frac{1}{1 + (1 - \alpha)(1 - \rho) \left[ \frac{1 - F(\theta)}{F(\theta)} \right]} = \frac{q \alpha}{q \alpha + \frac{1}{2} (1 - \alpha) \rho} + \frac{1}{2} \cdot \frac{(1 - q) \alpha}{(1 - q) \alpha + \frac{1}{2} (1 - \alpha) \rho}.
\]
If \( F(\cdot) \) first order stochastically dominates \( G(\cdot) \), we must have \( F(\bar{\theta}) < G(\bar{\theta}) \) because \( \bar{\theta} \geq 4(1-q) > 0 \). This implies that for any given \( \rho \),

\[
\frac{1}{1 + (1 - \alpha)(1 - \rho) \frac{[1 - F(\bar{\theta})]}{F(\bar{\theta})}} < \frac{1}{1 + (1 - \alpha)(1 - \rho) \frac{[1 - G(\bar{\theta})]}{G(\bar{\theta})}}
\]

because \( \frac{[1 - F(\bar{\theta})]}{F(\bar{\theta})} \) strictly decreases with \( F(\bar{\theta}) \). Furthermore, LHS of (18) strictly increases with \( \rho \), while RHS strictly decreases. Thus, only a smaller \( \rho' \) can balance the two sides. ■

A stochastically dominant distribution of \( \theta \) implies that a bigger amount of probability mass rests over the portion of support above any given equilibrium cutoff \( \bar{\theta} \). The intuition is as follows: when more probability mass is shifted upward, the public would then believe a no-reform outcome is more likely to be caused by the politician’s lack of talent, instead of the lack of opportunities (a lower \( \theta \) is realized). It therefore lowers the public’s rating of the politician when they observe no reform, and further entices the low type to reform more often.

**Comparison across Equilibria**

Analogous to standard signaling game, our analysis yields multiple equilibria, which are characterized by differing cutoffs. One may interpret a higher cutoff \( \bar{\theta} \) in the prevailing equilibrium as a proxy for escalating conservatism or more resistance to reform. Then we first investigate how the equilibrium strategy of a low-type politician \( \rho^* \) would differ across differing equilibria, i.e., how it responds to differing level of “conservatism”.

**Theorem 6**  The equilibrium probability \( \rho^*(\bar{\theta}) \) strictly decreases with the cutoff \( \bar{\theta} \).

**Proof.** Recall the equilibrium condition (13). When \( \bar{\theta} \) increases, \( m \equiv \frac{1 - F(\bar{\theta})}{F(\bar{\theta})} \) must decrease, which cause \( G(\rho^*, \bar{\theta}) \) to decrease. Further, as we have shown in the proof for previous results, \( G(\rho^*, \bar{\theta}) \) strictly decreases with \( \rho^* \). Hence, we establish that when \( \bar{\theta} \) increases, \( \rho^* \) must decrease.

Q.E.D. ■

The intuition is in line with that of Theorem 5. A higher cutoff \( \bar{\theta} \) increases the size of \( F(\bar{\theta}) \), which makes the public less able to infer the type of the politician when no reform is implemented. The low type therefore expects a more favorable assessment by not undertaking reform when \( \theta \) exceeds \( \bar{\theta} \), which then leads to our result.

Theorem 6 allows us to further explore a politician’s inclination or preference for “conservatism”. We now consider the equilibrium payoff of the politician in office. We ask the following questions: Do politicians prefer more reform?
Recall that the equilibrium is defined by the equation

\[ \frac{\alpha}{1 + \frac{(1-\alpha)(1-\rho^*)}{F(\bar{\rho})}} \mu_0 = \frac{1}{2} \left[ \frac{\alpha}{\mu_1^*} + \frac{(1-\alpha)\rho^*}{q} \right] - \frac{1}{2} \left[ \frac{\alpha}{\mu''_1} + \frac{(1-\alpha)\rho^*}{1-q} \right]. \tag{19} \]

The politician in office receives a payoff \( \mu_0 \) when he maintains the status quo. He has a payoff \( \mu_1^* \) when he successfully implements a reform, while has \( \mu''_1 \) if a failure is realized. Define

\[ q_t = \begin{cases} 
q & \text{for } t = H; \\
\frac{1}{2} & \text{for } t = L.
\end{cases} \]

In any equilibrium with a given \( \bar{\rho} \), the type-\( t \) politician receives a payoff

\[ u_t = \begin{cases} 
q_t \mu_1^* + (1 - q_t)\mu_1', & \text{for } \theta \geq \bar{\rho}; \\
\mu_0 & \text{for } \theta < \bar{\rho}.
\end{cases} \tag{20} \]

Hence, in this equilibrium, the expected payoff of a type-\( t \) politician is given by

\[ E(u_H) = \mu_0 F(\theta) + [q_t \mu_1^* + (1 - q_t)\mu_1'][1 - F(\theta)]. \tag{21} \]

Taking first order derivative of (3) with respect to \( \bar{\rho} \) yields

\[ \frac{dE(u_t)}{d\bar{\rho}} = \frac{\partial}{\partial \bar{\rho}} \left[ \mu_0 f(\bar{\rho}) - [q_t \mu_1^* + (1 - q)\mu_1''][1 - F(\bar{\rho})] \right] + \left\{ d[q_t \mu_1^* + (1 - q_t)\mu_1'']/d\bar{\rho} \right\} [1 - F(\theta)]. \]

**Theorem 7** The low-type politician always prefers an equilibrium with a higher cutoff \( \bar{\rho} \); while the high-type politician always benefits from an equilibrium with a lower \( \bar{\rho} \).

**Proof.** For a low-type politician, \( E(u_t) = \mu_0 \) because \( \mu_0 = \frac{1}{2} \mu_1^* + \frac{1}{2} \mu''_1 \). Hence, we only need to verify \( \frac{d\mu_0}{d\bar{\rho}} > 0 \). Define \( H(\rho^*, \bar{\rho}) = \frac{\alpha}{1 + \frac{(1-\alpha)(1-\rho^*)}{F(\bar{\rho})}} - \left\{ 1 - \frac{1}{2} \left[ \frac{\alpha}{\mu_1^*} + \frac{(1-\alpha)\rho^*}{q} \right] \right\} \).

Because

\[ \frac{\partial}{\partial \bar{\rho}} \left[ \frac{\alpha}{\mu_1^*} \right] = \frac{\alpha}{\mu_1^*} \left[ \frac{\mu_1'' - \mu_1'^*}{(1-q \rho^*)} \right], \quad \frac{\partial}{\partial \bar{\rho}} \left[ \frac{\alpha}{\mu''_1} \right] = \frac{\alpha}{\mu''_1} \left( \frac{\mu''_1 - \mu''_1'}{1-q \rho^*} \right), \]

we then have \( \frac{d\mu_0}{d\bar{\rho}} = \frac{\partial}{\partial \bar{\rho}} [1 - \frac{\alpha}{\mu_1^*} + \frac{(1-\alpha)\rho^*}{q}] \). We must have \( 1 - \frac{\mu_0}{\partial \rho^*} + \frac{\partial G(\rho^*, \bar{\rho})}{\partial \rho^*} \) because \( \frac{\partial}{\partial \rho^*} \left[ \frac{\alpha}{\mu''_1} \right] = \frac{\alpha}{\mu''_1} \left( \frac{\mu''_1 - \mu''_1'}{1-q \rho^*} \right), \) while \( \frac{\partial}{\partial \rho^*} > 0, \frac{\partial}{\partial \rho^*_1} > 0, \frac{\partial}{\partial \rho^*_1} < 0. \)

We then investigate the payoff of the high type by resorting to the fact \( \alpha E(u_H) + (1 - \alpha)E(u_L) = \alpha \). Rewrite it as

\[ \alpha \mu_0 F(\bar{\rho}) + \alpha (q \mu_1' + (1 - q)\mu_1'') [1 - F(\bar{\rho})] + (1 - \alpha)\mu_0 = \alpha, \]
which leads to

\[
[q\mu' + (1 - q)\mu''][1 - F(\bar{\theta})] = \frac{\alpha - \alpha\mu_0 F(\bar{\theta}) - (1 - \alpha)\mu_0}{\alpha}.
\] (22)

The expected payoff of the high-type politician therefore boils down to

\[
E(u_{H}) = \mu_0 F(\bar{\theta}) + [q\mu' + (1 - q)\mu'\prime'][1 - F(\bar{\theta})]
\]
\[
= \mu_0 F(\bar{\theta}) + \frac{\alpha - \alpha\mu_0 F(\bar{\theta}) - (1 - \alpha)\mu_0}{\alpha}
\]
\[
= \frac{\alpha - (1 - \alpha)\mu_0}{\alpha}
\]
\[
= 1 - \frac{(1 - \alpha)\mu_0}{\alpha}.
\] (23)

Clearly, \(E(u_{H})\) is strictly decreasing in the cutoff \(\bar{\theta}\) because \(\mu_0\) is strictly increasing in it. Hence, we obtain that the high-type politician always prefers to reform as much as possible.

Q.E.D. ■

Theorem 7 states that the low type always prefers more conservative equilibria, while the high type prefers equilibria with more reform. The low type’s aversion to reform embodies the logic that explains Theorem 5. On one hand, when \(\bar{\theta}\) increases, a no-reform outcome reveals less information to the public, which allows the low type to receive a higher payoff from maintaining status quo. On the other hand, when the low type reforms less often, the public would believe a reform is increasingly likely to be implemented by the high type, which further reduces the damage to the low type when an unsuccessful reform realizes. Both effects contribute to the result.

However, the logic is less explicit as to the preference of high-type politician for a more aggressive equilibrium. The high-type politician benefits from more reform, as it allows the public to infer his type more often from successful reform. However, because \(\rho^*(\bar{\theta})\) decreases with \(\bar{\theta}\), an equilibrium with a lower cutoff \(\bar{\theta}\) encourages his low-type counterpart to conduct reform, which then makes his reform less informative and tends to offsets the gain he may have by undertaking reform. Our analysis nevertheless indicates that the former positive effect strictly dominates the latter negative one, and predicts that the high type always prefers to reform as much as possible!

4 Social Welfare

In this part, we study the welfare implication of our model. For any equilibrium with a given equilibrium cutoff \(\bar{\theta}\), the social welfare can be written as a function

\[
W = \alpha \int_{\underline{\theta}}^{\bar{\theta}} [\theta - 4(1 - q)]f(\theta)d\theta + (1 - \alpha)\rho^* \int_{\underline{\theta}}^{\bar{\theta}} (\theta - 2)f(\theta)d\theta.
\] (24)
where the first term is the expected outcome from reform undertaken by the high-type politician, while the second that by the low type. Because \( \int_{\theta}^{0} \theta f(\theta) d\theta > \int_{\theta}^{0} (\theta - 2) f(\theta) d\theta \), and \( \frac{\partial \rho^*}{\partial \alpha} < 0 \), it is straightforward to see that the measure of social welfare strictly increases with \( \alpha \), regardless of the characteristic of the prevailing equilibrium.

The game, analogous to standard signaling models, yields multiple equilibria with differing cutoff \( \check{\theta} \). However, standard equilibrium refinement technique does not bite in the current setting. To gain more insight, we focus on our attention on the equilibrium that is subject to minimum “resistance” such that a cutoff \( \check{\theta} = 4(1 - q) \) applies. Such an equilibrium, we name it a “Minimum Accountability” equilibrium, simply reflects a belief that “a capable politician always prefers to reform as much as possible”, which has been formalized by Theorem 7.

In what follows in this section, we study two issues separately. First, we focus on a benchmark equilibrium, the “Minimum Accountability” equilibrium, and explore its relevant welfare implication. Second, We ask the question: how does the equilibrium cutoff affect the welfare of the society? Does more (less) resistance to reform necessarily lead to welfare gain? Then what is the most desirable level of resistance to reform from the view point of social welfare? We then explore the issue of institution design. We allow a legislative body to set the limit of the authority of the politician, which restricts the politician’s ability to initiate a reform. We then characterize the optimal authorization rule that maximizes social welfare.

### 4.1 “Minimum Accountability” Equilibrium

We now consider the equilibrium with the lowest acceptable cutoff \( \check{\theta} = 4(1 - q) \). We write down the social welfare function in this given context as follows,

\[
W(q) = \alpha \int_{4(1-q)}^{\check{\theta}} [\theta - 4(1-q)] f(\theta) d\theta + (1 - \alpha) \rho^* \int_{4(1-q)}^{\check{\theta}} (\theta - 2) f(\theta) d\theta. \tag{25}
\]

We now ask the following question: When such an equilibrium is always played, does the society necessarily benefit from a more capable high-talent politician, i.e., a higher \( q \)?

Differentiation of \( W \) with respect to \( q \) yields

\[
\frac{dW(q)}{dq} = 4\alpha [1 - F(\check{\theta})] + (1 - \alpha) \frac{d\rho^*}{dq} \int_{4(1-q)}^{\check{\theta}} (\theta - 2) f(\theta) d\theta + 4(1 - \alpha) \rho^* [4(1-q) - 2] f(\check{\theta}). \tag{26}
\]

The overall effect of the high type’s ability on social welfare is decomposed into three components. The answer is ambiguous. An increase in \( q \) brings about global changes to the “minimum accountability” equilibrium, because it lows the prevailing cutoff \( 4(1 - q) \).
When the high type’s ability further improves, the high type would reform more often and more gain from efficient reform can be expected, which unambiguously improves social welfare. This positive effect is thus reflected by $W_1$. Secondly, a lower cutoff tends to increase the loss from the inefficient reform implemented by the low type, fixing the low type’s probability of reform. This unambiguously negative effect is reflected by the term $W_3$. Thirdly, as the ability of high type improves, its effect on the low type’s equilibrium probability of reform remains less than explicit. On the one hand, $\rho^*$ tends to decrease in any equilibrium with a given threshold $\bar{\theta}$, under the “disciplinary” force verified by Theorem 4. On the other hand, the prevailing threshold for reform $\bar{\theta} \equiv 4(1-q)$ decreases, which nevertheless forces the low-type to reform more often. The overall effect on $d\rho/dq$ remains ambiguous. The term $W_2$ therefore reflects the welfare effect of improvement in the high type’s ability through its impact on the equilibrium probability of reform. The overall welfare impact of improvement in the high type’s ability therefore depends on the balance of the three effects.

We now focus on the case of a uniform distribution with c.d.f. $F(\theta) = \frac{\theta - 4(1-q)}{2\theta}$. We then have

$$\frac{dW(q)}{dq} = 4\alpha \cdot \frac{\tilde{\theta} - 4(1-q)}{2\tilde{\theta}} + (1 - \alpha) \left( \frac{d\rho^*}{dq} \int_{\theta=4(1-q)}^{\tilde{\theta}} (\theta - 2) \cdot \frac{1}{2\theta} d\theta + \frac{4(1-q) - 2}{2\tilde{\theta}} \right)$$

(27)

Under the assumption of uniform distribution, we obtain $F(\bar{\theta}) = \frac{4(1-q) + \bar{\theta}}{2\bar{\theta}}$ and $1 - F(\bar{\theta}) = 4(1-q) + \bar{\theta}$. The equilibrium condition is then rewritten as

$$1 + (1 - \alpha)(1 - \rho) \frac{\tilde{\theta} - 4(1-q)}{4(1-q) + \tilde{\theta}} = \frac{4\alpha q(1-q) + (1 - \alpha)\rho}{[2\alpha q + (1 - \alpha)\rho][2\alpha(1-q) + (1 - \alpha)\rho]}.$$ (28)

We now formally investigate in this case how an increase in $q$ leverages on the equilibrium probability of reform $q$ when $\bar{\theta} \equiv 4(1-q)$ always prevails. Define $b \equiv \{[4(1-q) + \tilde{\theta}](1 - 2\alpha) + [\tilde{\theta} - 4(1-q)][(1 - \alpha) - 4\alpha q(1-q)]\}$, and $c \equiv 8\alpha\tilde{\theta}(1 - \alpha)q(1-q)$. $\rho$ is then uniquely solved for as

$$\rho = \frac{b + \sqrt{b^2 + 8\tilde{\theta}c}}{4(1 - \alpha)\tilde{\theta}}.$$ (29)

We then have

$$\frac{d\rho}{dq} = \frac{1}{4(1 - \alpha)\tilde{\theta}} \cdot \left[ \frac{db}{dq} + \frac{2\tilde{\theta} - 8\tilde{\theta}d\tilde{\theta}}{2\sqrt{b^2 + 8\tilde{\theta}c}} \right].$$ (30)

**Proposition 1** When $\theta$ is uniformly distributed on the support $[-\hat{\theta}, \hat{\theta}]$ and the “Minimum Accountability” equilibrium always prevails, the equilibrium probability $\rho^*$ of the low type undertaking the reform strictly decreases with $q$. 
Proof. See Appendix. ■

The result shows that that the “disciplinary” force embodied through Theorem 4 dominates the indirect effect of $q$ that functions through $\bar{\theta}$. Hence, the effect that is represented by $W_2$ must be positive given the uniform distribution. Despite that, the overall effect $\frac{dW(q)}{dq}$ still remains ambiguous, and obfuscates theoretical prediction. We conduct numerical exercise to obtain further insights because of the complexity. We show by Figure 1 that the negative effect that is depicted from term $W_3$ could still dominate positive effects under a wide set of parameterizations. In Figure 1, we set $\bar{\theta}$ to 1.6, and $\alpha = 0.3$. Under this parameterization, we observe that $W(q)$ consistently decreases with $q$. Under this circumstance, the improved ability of the high type nevertheless leads to socially detrimental outcome as it forces the low type to reform more (because of a lower prevailing cutoff).

4.2 Institution Design: Welfare Maximizing Cutoff

In our baseline analysis, we have assumed that the politician in office is subject to virtually no institutional constraint except the accountability constraint, such that he is empowered to undertake any reform as he will. In this part, we investigate the issue of institution design. We assume that there exists a social-welfare maximizing legislative body. The legislative body sets the limit of authority that restricts the action of the politician. In particular,
it sets a threshold $\widetilde{\theta}$, and a politician is allowed to undertake a reform only if the value of the available reform proposal exceeds the cutoff $\widetilde{\theta}$. As we have assumed that the least conservative equilibrium is always played, such an authorization rule leads to an equilibrium with $\theta = \widetilde{\theta}$ in the subsequent game. Hence, does there exist an optimal cutoff $\theta^*$ that maximizes social welfare?

To explore this issue, we now compare the social welfare that could result from differing equilibria. The social welfare is written as a function

$$W(\theta) = \alpha \int_{\theta}^{\theta'} [\theta - 4(1-q)] f(\theta) d\theta + (1 - \alpha) \rho^* \int_{\theta}^{\theta'} (\theta - 2) f(\theta) d\theta. \quad (31)$$

Suppose a cutoff $\theta \in [4(1-q),\widetilde{\theta}]$ is enforced. Then when a reform with a value $\theta \in [\theta, \widetilde{\theta}]$ is implemented, it generates an expected output $E(y) = \alpha[\theta - 4(1-q)] + (1 - \alpha) \rho^*(\theta - 2)$. Define define $\rho \equiv \lim_{\theta \to 0} \rho^*$. We have the following.

**Lemma 2** Whenever $\frac{(1-\alpha)\rho}{\alpha} < \frac{\theta - 4(1-q)}{2-\theta}$, there must exists a unique $\widetilde{\theta} \in (4(1-q), \widetilde{\theta})$, which unique solves $\alpha[\widetilde{\theta} - 4(1-q)] + (1 - \alpha) \rho^* = \theta - 2 = 0$.

**Proof.** Consider the value of $\alpha[\widetilde{\theta} - 4(1-q)] + (1 - \alpha) \rho^*(\theta - 2)$. For all $\theta' \in [4(1-q),\widetilde{\theta}]$, it must strictly increase with $\theta'$, because $\theta < 2$, and $\rho^*(\theta - 2)$ strictly decreases when $\theta'$ increases. When $\theta' = 4(1-q)$, it must be negative. When $\theta'$ approaches $\theta$, we have its value approach $\alpha[\theta - 4(1-q)] + (1 - \alpha) \rho(\theta - 2)$, which is positive if and only if $\frac{(1-\alpha)\rho}{\alpha} < \frac{\theta - 4(1-q)}{2-\theta}$. Under this condition, there must exist a unique $\theta$ that solves the equation.

Q.E.D. ■

Suppose that $\theta$ indeed exists (i.e., the condition $\frac{(1-\alpha)\rho}{\alpha} < \frac{\theta - 4(1-q)}{2-\theta}$ is met.) and that the legislative body enforces a cutoff $\theta$. In that case, only reform with $\theta \geq \theta$ is allowed to be implemented. Unde this rule, all admissible reform must create nonnegative expected output: $E(y) = \alpha[\theta - 4(1-q)] + (1 - \alpha) \rho^*(\theta - 2) \geq 0$ if and only if $\theta \geq \theta$ by the definition of $\theta$. Any less conservative authorization rule (with $\theta' < \theta$) must admits “bad” reform, while any more conservative rule must eliminates “good” reform. Then is it the optimal cutoff $\theta^*$ that maximizes social welfare? If not, then is the the optimal institution more conservative or less conservative?

Taking first order derivative of (31) with respect to $\theta$ yields

$$\frac{dW(\theta)}{d\theta} = -\alpha[\theta - 4(1-q)] f(\theta) - (1 - \alpha) \rho^*(\theta - 2) f(\theta)$$

$$+ (1 - \alpha) \frac{d\rho^*}{d\theta} \int_{\theta}^{\theta'} (\theta - 2) f(\theta) d\theta. \quad (32)$$

We now decompose $\frac{dW(\theta)}{d\theta}$ to find out the underlying trade-offs when $\theta$ varies. An increasing threshold $\theta$ affects $W(\theta)$ through three venues. Firstly, it reduces the beneficial
reform undertaken by the high type, and therefore decreases the gain from the productivity of the high-type politician. This loss is given by the term $-\alpha \overline{\theta} f(\overline{\theta})$, which is obviously negative. Secondly, the rising cutoff $\overline{\theta}$ reduces the ex ante inefficient reform undertaken by the low type. This effect is embodied through the term $-(1 - \alpha) \rho^* (\overline{\theta} - 2) f(\overline{\theta})$, which is unambiguously positive, because $\overline{\theta} < \hat{\theta} < 2$. Thirdly, it allows the low-type politician to refrain from reforming for any $\theta \geq \overline{\theta}$, which further reduces the loss from the inefficient reform undertaken. This positive effect is depicted by the term $(1 - \alpha) \frac{d\rho^*}{d\theta} \int_{\overline{\theta}}^{\hat{\theta}} (\theta - 2) f(\theta) d\theta$.

The decomposition of $\frac{dW(\overline{\theta})}{d\theta}$ unambiguously point out that $\tilde{\theta}$ is never the optimum, despite that it leads to only reform with positive ex pected output. When $\tilde{\theta}$ is enforced, we see that $\frac{dW(\overline{\theta})}{d\theta}$ must remain positive, because $-\alpha [\tilde{\theta} - 4 (1 - q)] f(\tilde{\theta}) - (1 - \alpha) \rho^* (\overline{\theta} - 2) f(\overline{\theta}) = 0$ by the definition of $\tilde{\theta}$, but the term $(1 - \alpha) \frac{d\rho^*}{d\theta} \int_{\overline{\theta}}^{\tilde{\theta}} (\theta - 2) f(\theta) d\theta$ is positive. Hence, we conclude that the optimal cutoff $\overline{\theta}^*$ tends to exceed $\tilde{\theta}$: when a more conservative cutoff is enforced, although it would deter a positive amount of productive reform, it would further deter the detrimental reform undertaken by the low type by decreasing $\rho^*$. We then learn that the optimum must require proper "conservatism" towards potential reform.

We then continue to explore the existence and properties of the optimal $\overline{\theta}^*$. Because $f(\overline{\theta}) > 0$ for all $\overline{\theta} \in [-\hat{\theta}, \tilde{\theta}]$, the sign of (32) is the same as that of $\frac{dW(\overline{\theta})}{d\theta} / f(\overline{\theta})$. For our purpose, it suffices to explore $\frac{dW(\overline{\theta})}{d\theta} / f(\overline{\theta})$. We have

$$\frac{dW(\overline{\theta})}{d\theta} / f(\overline{\theta}) = -\alpha [\theta - 4 (1 - q)] f(\overline{\theta}) - (1 - \alpha) \rho^* (\overline{\theta} - 2) f(\overline{\theta}) + (1 - \alpha) \frac{d\rho^*}{d\theta} \int_{\overline{\theta}}^{\tilde{\theta}} (\theta - 2) f(\theta) d\theta. \quad (33)$$

We first establish the following.

**Lemma 3** $\frac{dW(\overline{\theta})}{d\theta} / f(\overline{\theta})$ strictly decreases with $\overline{\theta}$.

**Proof.** See Appendix.

This property of $\frac{dW(\overline{\theta})}{d\theta}$ allows us to further explore the optimal cutoff $\overline{\theta}^*$.

**Theorem 8** The public prefers no reform, i.e., $\overline{\theta} = \hat{\theta}$, if and only if $\frac{(1 - \alpha) \rho}{\alpha} \geq \frac{\tilde{\theta} - 4 (1 - q)}{2 - \overline{\theta}}$; while a unique socially optimal cutoff $\overline{\theta}^* \in (\tilde{\theta}, \hat{\theta})$ exists if and only if $\frac{(1 - \alpha) \rho}{\alpha} < \frac{\tilde{\theta} - 4 (1 - q)}{2 - \overline{\theta}}$.

**Proof.** If $\frac{(1 - \alpha) \rho}{\alpha} \geq \frac{\tilde{\theta} - 4 (1 - q)}{2 - \overline{\theta}}$, then $\tilde{\theta}$ does not exist. Any reform with a value $\theta < \tilde{\theta}$ must lead to negative expected output. Hence, no reform is ex ante beneficial, which implies $\overline{\theta}^* = \hat{\theta}$.

If $\frac{(1 - \alpha) \rho}{\alpha} < \frac{\tilde{\theta} - 4 (1 - q)}{2 - \overline{\theta}}$, then $\tilde{\theta}$ exists. $\frac{dW(\overline{\theta})}{d\theta} / f(\overline{\theta}) \bigg|_{\overline{\theta} = \tilde{\theta}} > 0$, but $\frac{dW(\overline{\theta})}{d\theta} / f(\overline{\theta}) \bigg|_{\overline{\theta} = \hat{\theta}} < 0$ (because $\frac{(1 - \alpha) \rho}{\alpha} < \frac{\tilde{\theta} - 4 (1 - q)}{2 - \overline{\theta}}$), then there must exist a unique $\overline{\theta}^* \in (\tilde{\theta}, \hat{\theta})$ that solves $\frac{dW(\overline{\theta})}{d\theta} / f(\overline{\theta}) = 0$.

Q.E.D. 

We examine the condition under which a $\overline{\theta}^* \in (\tilde{\theta}, \hat{\theta})$ exists, i.e., $\frac{(1 - \alpha) \rho}{\alpha} < \frac{\tilde{\theta} - 4 (1 - q)}{2 - \overline{\theta}}$. Because $\rho$ decreases with $\alpha$ (by Theorem 2), LHS must strictly decreases with $\alpha$. Hence, this condition
is more likely to be met when a larger $\alpha$ prevails, i.e., high-talent politicians represent a larger fraction of the population. When the talent required for successful reform is excessively scarce, the public may prefer not to allow for any reform. Under this circumstance, the gain from the efficient reform undertaken by the high type cannot offset the loss from increased inefficient reform.

Similarly, its RHS increases with $q$. Hence, positive amount of reform can be desirable only if high-talent politicians, who conduct socially beneficial reform, are sufficiently capable, because a higher $q$ implies that the high type makes more success when he reforms.

These arguments further lead to more general conclusions on the impact of $\alpha$ and $q$ on the properties of $\theta^* \in (\overline{\theta}, \bar{\overline{\theta}})$.

**Theorem 9** The socially optimal cutoff $\theta^*$ decreases with $\alpha$ and $q$.

**Proof.** See Appendix. ■

### 4.3 Example: Uniform Distribution and Perfect Signal

In this part, we restrict our attention to the case of uniform distribution with $F(\theta) = \frac{\theta + \theta}{2\theta}$ to gain more insights on the optimal institution $\theta^*$. For the sake of simplicity and tractability, we consider the limiting case where a high-type politician receives prefect signal with $q = 1$ in order to obtain further insights.

Theorem 5 demonstrates that the equilibrium behavior depends on the properties of the distribution of $\theta$. We now discuss its impact on $\theta^*$. Given a uniform distribution, the property of the distribution $F(\theta)$ is therefore reflected by the size of its supper support $\overline{\theta}$. Is it necessary that a greater $\overline{\theta}$ would raise $\theta^*$? Additional uncertainty arises when $\overline{\theta}$ increases, as it results in a more dispersed distribution and implies that more high-valued reform proposals are more likely to be realized. Mere intuition suggests competing effects on $\theta^*$. On the one hand, it tends to have the cutoff $\theta^*$ fall in order to realize more gain form efficient reform. On the other hand, for any given cutoff, inefficient reform increases as the low type is forced to reform more (see Theorem 5), which increases social loss and tends to lift $\theta^*$. The overall effect remains obscure.

To illustrate this point, let us consider the welfare implication of an increasing $\overline{\theta}$ in an arbitrary equilibrium with a fixed $\overline{\theta}$. The following figure testifies to such nonmonotonicity and ambiguity, where $\theta$ is assumed to follow a uniform distribution.

Now we examine how a higher upper support $\overline{\theta}$ could affect $\frac{dW(\overline{\theta})}{d\overline{\theta}}$ for any given $\overline{\theta}$. When the high-type politician is perfectly informed, any positive $\theta$ would imply an efficient reform.
Figure 2: The nonmonotonic effect of $\bar{\theta}$ on social welfare

In any equilibrium with a given cutoff $\bar{\theta}$, the equilibrium strategy $\rho^*$ is given by

\[
\rho^* = \frac{1 - \alpha[1 + F(\bar{\theta})]}{1 - \alpha} = \frac{2\hat{\theta} - 3\alpha\bar{\theta} - \alpha\hat{\theta}}{2\hat{\theta}(1 - \alpha)}.
\] (34)

The first-order derivative of the welfare function is as follows

\[
\frac{dW(\bar{\theta})}{d\bar{\theta}} = \frac{1}{2\hat{\theta}}[-\alpha \int^{\hat{\theta}}_{\bar{\theta}} \frac{1}{2\theta} d\theta - \frac{1}{2\theta} \bar{\theta} \frac{\bar{\theta} + \hat{\theta}}{2\theta} - (\bar{\theta} - 2) + \alpha \frac{\bar{\theta} + \hat{\theta}}{2\theta} \cdot \bar{\theta}]
\]

\[
= \frac{1}{2\theta^2} \left[ \frac{3\alpha}{4\theta^2} - \left(1 + \frac{2\alpha}{\bar{\theta}} - \frac{\alpha}{2}\right) \bar{\theta} + \left(2 - 2\alpha - \frac{\alpha\hat{\theta}}{4}\right) \right]
\]

Because $\frac{1}{2\theta} > 0$, we only need to look at the sign of the term $w$ in the bracket learn the property of $\frac{dW(\bar{\theta})}{d\bar{\theta}}$. It is straightforward to verify in this scenario $\rho = \frac{1 - 2\alpha}{1 - \alpha}$. The condition for $\bar{\theta}^* < \hat{\theta}$, i.e., $\frac{1 - \alpha}{\alpha} < \frac{\hat{\theta} - 4(1 - \alpha)}{2 - \bar{\theta}}$, then boils down to $\alpha > \frac{2 - \hat{\theta}}{4 - \bar{\theta}}$. Assume that this condition is satisfied and a $\bar{\theta}^* < \hat{\theta}$ exists. Then the optimal cutoff $\bar{\theta}^*$ is determined by the equation

\[
W = \frac{3\alpha}{4\theta} \frac{\theta^2}{2} \left(1 + \frac{2\alpha}{\bar{\theta}} - \frac{\alpha}{2}\right) \bar{\theta} + \left(2 - 2\alpha - \frac{\alpha\hat{\theta}}{4}\right) = 0
\] (35)
We then obtain the following.

**Proposition 2** The socially optimal cutoff point for reform, $\bar{\theta}^*$ strictly increases with in $\hat{\theta}$. The probability of reform, $1 - F(\bar{\theta}^*)$, is strictly increasing in $\hat{\theta}$; so is the overall likelihood of reform $[\alpha + (1 - \alpha)\rho][1 - F(\bar{\theta}^*)]$.

**Proof.** We prove the first part by the implicit function theorem. It suffices to show

$$\frac{\partial w}{\partial \hat{\theta}} = -\frac{3\alpha \hat{\theta}^2}{4\hat{\theta}^2} + \frac{2\alpha \hat{\theta}}{\hat{\theta}^2} - \frac{\alpha}{4} > 0,$$

which is equivalent to

$$\frac{4 - \sqrt{16 - 3\hat{\theta}^2}}{3} < \bar{\theta}^* < \frac{4 + \sqrt{16 - 3\hat{\theta}^2}}{3}.$$ 

Since $\hat{\theta} \leq 2$, the right boundary is greater than or equal to $\hat{\theta}$. So, we just need to check the first half of the inequality. By Part 1, $\bar{\theta}^*$ is decreasing in $\alpha$. Therefore, it is sufficient to check the condition for $\alpha = 1/2$. Solving the first order condition, we obtain $\bar{\theta}^* = \left(3\hat{\theta} + 4 - \sqrt{12\hat{\theta}^2 + 16}\right)/3$, which indeed satisfies the above inequality as $\hat{\theta} \leq 2$. This concludes the proof of Part 2.

Let $p \equiv F(\bar{\theta}) = 1/2 + \hat{\theta}/2\hat{\theta}$. We may rewrite the first order condition into

$$v(p) = 3p^2 \hat{\theta} \alpha + 2p \hat{\theta} \left(-1 - \alpha - \frac{2\alpha}{\hat{\theta}}\right) + 2 \left(\frac{1}{2} + \frac{1}{\hat{\theta}}\right) \hat{\theta}.$$ 

We have

$$\frac{\partial v}{\partial p} = 6p \hat{\theta} \alpha + 2 \hat{\theta} \left(-1 - \alpha - \frac{2\alpha}{\hat{\theta}}\right) \leq (4\alpha - 2) \hat{\theta} - 4\alpha < 0,$$

as $p \leq 1$ and $\alpha \leq 1/2$. Note that when we keep $p$ fixed and vary $\hat{\theta}$, $\bar{\theta}$ has to be adjusted. Thus, the derivative of $v$ with respect to $\hat{\theta}$ is different from that of $w$. Observe that

$$\frac{\partial v}{\partial \hat{\theta}} = \left[\frac{3\alpha}{4}(2p - 1) - \frac{\alpha}{2}\right] (2p - 1) - \frac{\alpha}{4} \leq \left[\frac{3\alpha}{2} - \frac{\alpha}{4} - 1\right] (2p - 1) - \frac{\alpha}{4} < 0,$$

as $1/2 \leq p \leq 1$ and $\alpha \leq 1/2$. Hence, $F(\bar{\theta}^*)$ is decreasing in $\hat{\theta}$. Therefore, the probability of reform, $1 - F(\bar{\theta}^*)$ is increasing in $\hat{\theta}$. It is straightforward to see that the total probability of reform, $[\alpha + (1 - \alpha)\rho][1 - F(\bar{\theta}^*)]$, also increases with $\hat{\theta}$ as $\rho$ increases when $F(\bar{\theta}^*)$ decreases.

Q.E.D. 

Our comparative static results yields unambiguous results. We find that when the uniform distribution of $\theta$ is more dispersed, more reform can always be expected, and it unambiguously lifts the optimal cutoff $\bar{\theta}^*$. A less conservative social optimum is then expected.
5 Concluding Remarks

In this paper, we study a politician’s incentive to implement reform when his true ability is privately known but he is concerned about the public’s perception of his ability. The politician then chooses his action to maximize his reputational payoff. We find that a high-type politician always attempts to reform as much as possible, which “forces” his low-type counterpart to mimic with positive probability. Socially inefficient reform may therefore result. We further explore the socially optimal level of empowerment, and we find that both radicalism and conservatism may find their support depending on the specific parameterization.

Our paper sets forth a simple theoretical framework to investigate politician’s incentive to undertake innovative but risky action when he has reputational concerns. Our paper leaves open plenty possibilities of extensions and variations. For instance, one may extend the model to allow a larger strategy space, or to allow the payoff of the politician to depend on realized outcome of his policy choice. Although we believe extensions in these directions would not yield predictions that fundamentally depart from those out of the current setting, these more comprehensive settings may still spawn richer comparative statics that further add to our understanding on this issue.

6 Appendix: Proofs

6.1 Proof of Theorem 4

Proof. Because \(\frac{\partial \rho^*}{\partial \alpha} < 0\), we only need to show \(|(1 - \alpha)\frac{\partial \rho^*}{\partial \alpha} + \rho^*| > 1\). We have

\[
(1 - \alpha) \frac{\partial \rho^*}{\partial \alpha} + \rho^* = \frac{(1 - \alpha)^2 \alpha^2}{\alpha^2} \left[ m + 1 + \frac{4q(1-q)[1-4q(1-q)]}{4q(1-q)+\lambda(\alpha)\rho^*} \right] + \rho^*
\]

By the equilibrium, \((1 - \rho^*)m = \frac{\rho^* - \lambda(\alpha)\rho^*}{4q(1-q)+\lambda(\alpha)\rho^*} - 1 = \frac{\rho^* - \lambda(\alpha)\rho^*}{4q(1-q)+\lambda(\alpha)\rho^*} = \frac{\lambda(\alpha)\rho^*}{4q(1-q)+\lambda(\alpha)\rho^*}\), which yields

\[
m = \frac{\lambda(\alpha)\rho^* + \rho^* - \lambda(\alpha)\rho^* - 4q(1-q)}{4q(1-q)+\lambda(\alpha)\rho^*}[1 - \rho^*],
\]

and therefore

\[
m + 1 = \frac{\lambda(\alpha)\rho^* + \rho^* - 4q(1-q) + [4q(1-q)+\lambda(\alpha)\rho^*][1 - \rho^*]}{[4q(1-q)+\lambda(\alpha)\rho^*][1 - \rho^*]}
\]

\[
= \frac{\rho^*[1 - 4q(1-q)]}{[4q(1-q)+\lambda(\alpha)\rho^*][1 - \rho^*]}.
\]
Hence,
\[
[m + 1 + \frac{4q(1-q)[1 - [4q(1-q)]]}{[4q(1-q) + \lambda(\alpha)\rho^2]^2}] \\
= \frac{\rho[1 - 4q(1-q)]}{[4q(1-q) + \lambda(\alpha)\rho^2][1 - \rho^2]} + \frac{4q(1-q)[1 - [4q(1-q)]}{[4q(1-q) + \lambda(\alpha)\rho^2]^2} \\
= \frac{1 - 4q(1-q)}{[4q(1-q) + \lambda(\alpha)\rho^2]^2} 4q(1-q) + \lambda(\alpha)\rho^2].
\]

We then obtain
\[
\left| (1 - \alpha) \frac{\partial \rho^*}{\partial \alpha} \right| + \rho^* \\
= \left( 1 - \alpha \right) \cdot \frac{\rho^2 [1 - [4q(1-q)]}{[4q(1-q) + \lambda(\alpha)\rho^2]^2} + \rho^* \\
= \frac{1 - \rho^*}{\alpha} \cdot \frac{1 - \rho^*}{[4q(1-q) + \lambda(\alpha)\rho^2] + \rho^*}.
\]

To our purpose, we only need to show \( \frac{1 - \alpha}{\alpha} \cdot \frac{\rho^2}{[4q(1-q) + \lambda(\alpha)\rho^2]} > 1 \). Rewrite it as \( \frac{1 - \alpha}{\alpha} \cdot \frac{\rho^2}{[4q(1-q) + \lambda(\alpha)\rho^2]} > 1 \). Hence, it suffices to show \( \frac{1}{[4q(1-q) + \lambda(\alpha)\rho^2]} > \alpha \).

We claim \( \frac{1}{[4q(1-q) + \lambda(\alpha)\rho^2]} > \alpha \), i.e., \( 4q(1-q) > \lambda(\alpha)\rho^2 \). To show that, recall the equilibrium condition \( 1 + (1 - \rho^*)m = \frac{\rho^* (\lambda(\alpha)\rho^* + 1)}{4q(1-q) + \lambda(\alpha)\rho^2} \), which implies \( \frac{\rho^* (\lambda(\alpha)\rho^* + 1)}{4q(1-q) + \lambda(\alpha)\rho^2} > 1 \Leftrightarrow \lambda(\alpha)\rho^* > 4q(1-q) + \lambda(\alpha)\rho^2 \). Because \( \lambda(\alpha) > 1 \), \( \lambda(\alpha)\rho^2 > 4q(1-q) \) must hold.

Q.E.D

6.2 Proof of Proposition 1

Proof. Evaluate \( x \) with respect to \( q \) gives
\[
\frac{\partial x}{\partial q} = \frac{\partial b}{\partial q} + \frac{1}{2} \frac{2b \frac{\partial \theta}{\partial q} + 8\theta \frac{\partial c}{\partial q}}{\sqrt{b^2 + 8\theta c}} \\
= \frac{\partial b}{\partial q} (b + \sqrt{b^2 + 8\theta c}) + \frac{4\theta \frac{\partial c}{\partial q}}{4\theta \sqrt{b^2 + 8\theta c}}.
\]

We first find out \( \frac{\partial b}{\partial q} \), which is given by
\[
\frac{\partial b}{\partial q} = -4(1 - 2\alpha) + 4[(1 - \alpha) - 4aq(1-q)] + 4\alpha[\theta - 4(1-q)](2q-1) \\
= 4\alpha [1 - q(1-q) + \theta - 4(1-q)](2q-1).\n\]
Its sign is indefinite depending on the size of $q$. We then turn to $\frac{\partial c}{\partial q}$, which is given by

$$\frac{\partial c}{\partial q} = 8\alpha \hat{\theta}(1 - \alpha)(1 - 2q), \quad (42)$$

which is apparently negative as $q > \frac{1}{2}$ by our assumption. We now claim $\frac{\partial b}{\partial q}(b + \sqrt{b^2 + 8c}) + 4\hat{\theta} \frac{\partial c}{\partial q} < 0$. If $\frac{\partial b}{\partial q} \leq 0$, the negativity is guaranteed. However, if $\frac{\partial b}{\partial q} > 0$, it suffice to show

$$(1 - \alpha) \frac{\partial b}{\partial q} + \frac{\partial c}{\partial q} < 0,$$

as $x = (1 - \alpha)\rho = \frac{b + \sqrt{b^2 + 8\hat{\theta}c}}{4\hat{\theta}} < 1 - \alpha$. Thus, we have

$$(1 - \alpha) \frac{\partial b}{\partial q} + \frac{\partial c}{\partial q} = 4\alpha(1 - \alpha)\{[1 - q(1 - q)] + [\hat{\theta} - 4(1 - q)](2q - 1)\} + 8\alpha \hat{\theta}(1 - \alpha)(1 - 2q) = 4\alpha(1 - \alpha)\{[1 - q(1 - q)] + [\hat{\theta} - 4(1 - q) - 2\hat{\theta}](2q - 1)\} = 4\alpha(1 - \alpha)\{[1 - q(1 - q)] - [\hat{\theta} + 4(1 - q)](2q - 1)\}. \quad (43)$$

Its sign is thus the same as that of $T(q) = [1 - q(1 - q)] - [\hat{\theta} + 4(1 - q)](2q - 1)$. We have

$$T(q) = [1 - q(1 - q)] - [\hat{\theta} + 4(1 - q)](2q - 1) = 9q^2 - (13 + 2\hat{\theta})q + (5 + \hat{\theta}). \quad (44)$$

$T(q) < 0$ if and only if the following holds:

$$\frac{(13 + 2\hat{\theta}) - \sqrt{4\hat{\theta}^2 + 16\hat{\theta} - 11}}{18} < q < \frac{(13 + 2\hat{\theta}) + \sqrt{4\hat{\theta}^2 + 16\hat{\theta} - 11}}{18}. \quad (45)$$

We claim $\frac{(13 + 2\hat{\theta}) + \sqrt{4\hat{\theta}^2 + 16\hat{\theta} - 11}}{18} > 1$. This condition holds if and only if $\sqrt{4\hat{\theta}^2 + 16\hat{\theta} - 11} > 5 - 2\hat{\theta}$, which is equivalent to $4\hat{\theta}^2 + 16\hat{\theta} - 11 > 25 + 4\hat{\theta}^2 - 20\hat{\theta}$. This is guaranteed because of the assumption $\hat{\theta} \geq 1$.

We then claim $\frac{(13 + 2\hat{\theta}) - \sqrt{4\hat{\theta}^2 + 16\hat{\theta} - 11}}{18} < \frac{4 - \hat{\theta}}{4}$. This condition holds if and only if $2\sqrt{4\hat{\theta}^2 + 16\hat{\theta} - 11} > 13\hat{\theta} - 10$. RHS is strictly less than 6 because $\hat{\theta} < 2$, while LHS is strictly greater than 6 because $4\hat{\theta}^2 + 16\hat{\theta} - 11 = 9$ when $\hat{\theta} = 1$, which makes LHS= 6, and it strictly increases with $\hat{\theta}$. By the assumption $q > \frac{4 - \hat{\theta}}{4}$, we then conclude $\frac{(13 + 2\hat{\theta}) - \sqrt{4\hat{\theta}^2 + 16\hat{\theta} - 11}}{18} < \frac{4 - \hat{\theta}}{4} < q$.

Q.E.D. ■

### 6.3 Proof of Lemma 2

**Proof.** Recall the equilibrium condition

$$G(\rho^*, m) = [1 + (1 - \rho^*)m] - \frac{\rho^*(\lambda\rho^* + 1)}{4q(1 - q) + \lambda\rho^*} = 0,$$
where \( m \equiv \frac{1-F(\bar{\theta})}{F(\bar{\theta})} \). Hence, we have \( \frac{\partial G(\rho^*,m)}{\partial m} = \lambda(1-\rho^*) \). Because \( \frac{\partial G(\rho^*,m)}{\partial \rho^*} = -[m + 1 + \frac{4q(1-q)[1-4q(1-q)^2]}{[4q(1-q)+\lambda \rho^*]^2}] < 0 \), we must have

\[
\frac{d\rho^*}{dm} = \frac{1-\rho^*}{m + 1 + \frac{4q(1-q)[1-4q(1-q)^2]}{[4q(1-q)+\lambda \rho^*]^2}}.
\]

and therefore

\[
\frac{d\rho^*}{d\theta}/f(\bar{\theta}) = -\frac{1-\rho^*}{[m + 1 + \frac{4q(1-q)[1-4q(1-q)^2]}{[4q(1-q)+\lambda \rho^*]^2}][F(\bar{\theta})]^2}.
\]

We now claim \(-\frac{d\rho^*}{d\theta}/f(\bar{\theta})\) strictly decreases with \( \bar{\theta} \). We have

\[
\frac{d[-\frac{d\rho^*}{d\theta}/f(\bar{\theta})]}{d\theta} = \left[\frac{-\frac{d\rho^*}{d\theta}[m + 1 + \frac{4q(1-q)[1-4q(1-q)^2]}{[4q(1-q)+\lambda \rho^*]^2}][F(\bar{\theta})]^2}{[m + 1 + \frac{4q(1-q)[1-4q(1-q)^2]}{[4q(1-q)+\lambda \rho^*]^2}][F(\bar{\theta})]^2} \right] - (1-\rho^*) \frac{d\rho^*}{d\theta}/f(\bar{\theta})^2.
\]

Note that \(-\frac{d\rho^*}{d\theta}[m + 1 + \frac{4q(1-q)[1-4q(1-q)^2]}{[4q(1-q)+\lambda \rho^*]^2}][F(\bar{\theta})]^2 = (1-\rho^*)f(\bar{\theta}) \). We then only need to prove \( \frac{d\rho^*}{d\theta}/f(\bar{\theta})^2 > f(\bar{\theta}) \). Rewrite \( [m + 1 + \frac{4q(1-q)[1-4q(1-q)^2]}{[4q(1-q)+\lambda \rho^*]^2}][F(\bar{\theta})]^2 \) as \( F(\bar{\theta}) + \frac{4q(1-q)[1-4q(1-q)^2]}{[4q(1-q)+\lambda \rho^*]^2} F(\bar{\theta})^2 \). When \( \bar{\theta} \) increases, both \( \frac{4q(1-q)[1-4q(1-q)^2]}{[4q(1-q)+\lambda \rho^*]^2} \) and \( F(\bar{\theta}) \) strictly increases. Hence, \( \frac{d\rho^*}{d\theta}/f(\bar{\theta})^2 > 0 \). Furthermore, \( \frac{dF(\bar{\theta})}{d\theta} = f(\bar{\theta}) \). We then establish our claim.

Q.E.D. ■

6.4 Proof of Theorem 8

Proof. Suppose that an interior optimum with \( \bar{\theta}^* \in (0, \bar{\theta}) \) exists. Define \( k \equiv [-\frac{d\rho^*}{d\theta}/f(\bar{\theta})] \). Then the optimal condition is

\[
v(\bar{\theta}, \alpha) = \alpha(\bar{\theta} - 4(1 - q)) + (1 - \alpha)\rho(\bar{\theta})(\bar{\theta} - 2) - (1 - \alpha)k \int_{\bar{\theta}}^{\bar{\theta}} (2 - \bar{\theta}) f(\theta) \, d\theta = 0. \tag{49}
\]

Apparently, \( \frac{dv(\bar{\theta}, \alpha)}{d\alpha} = -\frac{\partial v(\bar{\theta}, \alpha)}{\partial \alpha} > 0 \). We now claim \( \frac{dv(\bar{\theta}, \alpha)}{d\alpha} > 0 \). Taking first order derivative of \( v(\bar{\theta}, \alpha) \) yields

\[
\frac{dv(\bar{\theta}, \alpha)}{d\alpha} = \frac{[\bar{\theta} - 4(1 - q)] - \rho^*(\bar{\theta} - 2) + (1 - \alpha)\frac{d\rho^*}{d\alpha}(\bar{\theta} - 2)}{[\bar{\theta} - 4(1 - q)] - \rho^*(\bar{\theta} - 2) + (1 - \alpha)\frac{d\rho^*}{d\alpha}(\bar{\theta} - 2)} + k \int_{\bar{\theta}}^{\bar{\theta}} (2 - \bar{\theta}) f(\theta) \, d\theta - (1 - \alpha)\frac{\partial k}{\partial \alpha} \int_{\bar{\theta}}^{\bar{\theta}} (2 - \bar{\theta}) f(\theta) \, d\theta. \tag{50}
\]
It suffices to show $k$ strictly decreases with $\alpha$ and $q$. Recall by the proofs of previous results:

$$\frac{-d\rho^*}{d\alpha} = \frac{\rho^*^2[1-4q(1-q)]}{[4q(1-q)+\lambda\rho^*]^2} \cdot \frac{d\lambda}{d\alpha}.$$  

(51)

Note $-\frac{d\rho^*}{d\alpha} = -\frac{d\rho^*}{d\theta}$. Hence, we now evaluate $-\frac{d\rho^*}{d\alpha}$ with respect to $\theta$. We first rearrange it as

$$-\frac{d\rho^*}{d\alpha} = \frac{\rho^*^2[1-4q(1-q)]}{[m+1 + \frac{4q(1-q)[1-4q(1-q)]^2}{[4q(1-q)+\lambda\rho^*]^2}] \cdot \frac{d\lambda}{d\alpha}}.$$  

$$= \frac{(1-\rho)}{[m+1 + \frac{4q(1-q)[1-4q(1-q)]^2}{[4q(1-q)+\lambda\rho^*]^2}] \cdot [1 - [4q(1-q)]}.$$  

$$\cdot \frac{\rho^*^2}{1-\rho^*} \cdot \frac{1}{[4q(1-q)+\lambda\rho^*]^2}.$$  

(52)

We have established in the proof of Lemma 2 that $\frac{(1-\rho^*)}{[m+1 + \frac{4q(1-q)[1-4q(1-q)]^2}{[4q(1-q)+\lambda\rho^*]^2}]$ would strictly decrease with $\theta$. So we only need to show $\frac{\rho^*^2}{[4q(1-q)+\lambda\rho^*]^2}$ decreases with $\theta$ as well. Taking first order derivative of it with respect to $\theta$ yields

$$\frac{\rho^*}{(2-\rho^*)} \frac{d\rho^*}{d\theta} = \frac{1}{[4q(1-q)+\lambda\rho^*]^2}.$$  

$$+ \frac{\rho^*^2}{1-\rho^*} \cdot \frac{-2\lambda \frac{d\rho^*}{d\theta}}{[4q(1-q)+\lambda\rho^*]^2}.$$  

(53)

Because $\frac{d\rho^*}{d\theta} < 0$, we need to show $(2-\rho^*)[4q(1-q)+\lambda\rho^*] - 2\lambda\rho^*(1-\rho^*) > 0$, which is obvious because $(2-\rho^*)[4q(1-q)+\lambda\rho^*] - 2\lambda\rho^*(1-\rho^*) = (2-\rho^*)[4q(1-q)+\lambda\rho^*] - \lambda\rho^*(2-2\rho^*)$, and $2-\rho^* > 2-2\rho^*$. We further claim $\theta^*$ decreases with $q$. To show that, we have to prove \( \frac{d\nu(\theta,q)}{dq} > 0 \). We have

$$\frac{d\nu(\bar{\theta},q)}{dq} = 4\alpha + (1-\alpha) \frac{d\rho^*}{dq} (\bar{\theta} - 2) - (1-\alpha) \frac{dk}{dq} \int_{\bar{\theta}} \bar{\theta} (2 - \bar{\theta}) \cdot \nu(\theta) \cdot d\theta.$$  

(54)

It would suffice to show $\frac{dk}{dq} < 0$. We use the same technique as above. We have

$$\frac{-d\rho^*}{dq} = \frac{\frac{4(2\rho-1)^\rho^*[(\lambda\rho^*+1)]}{[4q(1-q)+\lambda\rho^*]^2}}{[m+1 + \frac{4q(1-q)[1-4q(1-q)]^2}{[4q(1-q)+\lambda\rho^*]^2}]}. $$  

(55)

We then claim $-\frac{\partial^2 \rho^*}{\partial q \partial \theta} < 0$. Rewrite $-\frac{d\rho^*}{dq}$ as

$$\frac{-d\rho^*}{dq} = \frac{\frac{1}{m+1 + \frac{4q(1-q)[1-4q(1-q)]^2}{[4q(1-q)+\lambda\rho^*]^2}} \cdot \frac{1}{1-\rho^*} \cdot \frac{4(2q-1)^\rho^*[(\lambda\rho^*+1)]}{[4q(1-q)+\lambda\rho^*]^2}}{[m+1 + \frac{4q(1-q)[1-4q(1-q)]^2}{[4q(1-q)+\lambda\rho^*]^2}]}. $$  

(56)
Because \( \frac{1 - \rho^*}{m + 1 + 4q(1 - q)(1 - \rho^*)} \) and \( \frac{1 - \rho^*}{1 - \rho^*} \) decreases with \( \bar{\theta} \), we only need to show \( \frac{\rho^*(\lambda \rho^* + 1)}{4q(1 - q) + \lambda \rho^*} \) decreases with \( \bar{\theta} \). Taking first order derivative of it with respect to \( \bar{\theta} \) yields

\[
\frac{d}{d\bar{\theta}} \frac{\rho^*(\lambda \rho^* + 1)}{4q(1 - q) + \lambda \rho^*} = \frac{\left(2 \lambda \rho^* + 1\right) \frac{d \rho^*}{d\bar{\theta}} [4q(1 - q) + \lambda \rho^*]^2 (1 - \rho^*) - 2 \rho^* (\lambda \rho^* + 1)(1 - \rho^*) [4q(1 - q) + \lambda \rho^*] \lambda \frac{d \rho^*}{d\bar{\theta}}}{(1 - \rho^*)^2 [4q(1 - q) + \lambda \rho^*]^4} + \rho^* (\lambda \rho^* + 1)[4q(1 - q) + \lambda \rho^*]^2 \frac{d \rho^*}{d\bar{\theta}}
\]

\[
= \frac{\rho^*}{4q(1 - q) + \lambda \rho^*} \left[ \frac{2 \lambda \rho^* + 1}{4q(1 - q) + \lambda \rho^*} (1 - \rho^*) - 2 \lambda \rho^* (\lambda \rho^* + 1)(1 - \rho^*) + \rho^* (\lambda \rho^* + 1)[4q(1 - q) + \lambda \rho^*] \right]
\]

The item in bracket is definitely positive because

\[
\left[ \frac{2 \lambda \rho^* + 1}{4q(1 - q) + \lambda \rho^*} (1 - \rho^*) - 2 \lambda \rho^* (\lambda \rho^* + 1)(1 - \rho^*) + \rho^* (\lambda \rho^* + 1)[4q(1 - q) + \lambda \rho^*] \right] > 0.
\]

Q.E.D. ■

References


