Allocative Efficiency, Mark-ups, and the Welfare Gains from Trade

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Abstract

This paper examines the welfare effects of trade, decomposing effects into an index of average productivity and an index of allocative efficiency across firms. We show that movements in the productivity index exactly correspond to a condition derived in Arkolakis, Costinot, and Rodriguez-Clare (2012). We derive a formula for the allocative efficiency component of welfare change that depends upon differences in the cost shares of price between domestic firms and foreign firms. We obtain conditions under which reductions in trade frictions within a sector raise allocative efficiency within the sector. If the average mark-up within the sector is higher than the economy-wide average, allocative efficiency across sectors also increases. In addition to evaluating the direction of the effect, we also consider the magnitude. In the special case where firms in different countries draw from symmetric productivity distributions, small trade frictions have negligible effects on allocative efficiency. However, large trade frictions can have relatively significant effects.

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1 Introduction

When some goods are monopolized and others are not, monopoly mark-ups distort the allocation of resources across goods. If trade barriers fall and firms that would otherwise have monopoly power are forced to compete with foreign firms, decreases in monopoly mark-ups affect welfare by reallocating resources across goods.

In an influential recent paper, Arkolakis, Costinot, and Rodriguez-Clare (2012), hereafter ACR, derive a condition summarizing the welfare gains from trade that depends upon the aggregate share of expenditure on imported goods, as well as an elasticity parameter. The condition is valid across a variety of different models of the underlying sources of gains from trade, including the Ricardian framework of Eaton and Kortum (2002), as well as other frameworks like Armington (1969), Krugman (1980), and Melitz (2003). While these models differ in underlying microeconomic structure, in terms of aggregate welfare they look alike, given the same observed aggregate trade flows. Simply put, in the general ACR framework, to get welfare benefits from trade, we need to have trade, and measured trade flows provide a summary statistic for the benefits of trade.

The starting point of this paper is the observation that a reduction in trade frictions can have impact, even if no trade is observed, if domestic firms are forced to change mark-ups to meet increased competition from foreign competitors. The threat of imports can affect resource allocation across domestic firms and these effects will not get picked up in trade statistics. It is an old idea that trade can matter even when unseen, e.g., Markusen (1981). Salvo (2010) has recently highlighted that the threat of imports cut mark-ups in the cement industry in Brazil, despite the near absence of imports in terms of volume. Schmitz (2005) shows how the threat of iron ore imports into the U.S. led to huge changes in the industry.

The objective of this paper is to integrate an analysis of the welfare channel considered in ACR, with an analysis of how trade affects welfare through its impact on the distribution of mark-ups. The ACR channel takes the form of increased productivity, from Ricardian gains from trade as in Eaton and Kortum (2002). We call the second channel the allocative efficiency channel, as changes in the distribution of mark-ups affect how resources are allocated across firms. We derive a measure of welfare that is separable in the productive efficiency component and the allocative efficiency component. Focusing on the Cobb-Douglas case, we show that the ACR statistic in our framework exactly tracts the productive efficiency component of the welfare measure, leaving the allocative efficiency component as an unaccounted-for residual. While productivity and allocative efficiency can be separated out in the welfare formula, it is interesting to consider them jointly in an integrated analysis because the same underlying model parameters that affect one can also affect the other.
If there is more heterogeneity across firms in productivity, it affects the productivity gains from trade by increasing the importance of selection. But the distribution of mark-ups and hence allocative efficiency is also affected by the degree of firm heterogeneity.

One thing to emphasize up front is that while the productivity gain from trade can only be positive—increasing the choice set over which firms produce can only result in a more productive firm—the allocative efficiency effect of trade is in general ambiguous. Consider an example where in autarky every firm is a monopolist and suppose for simplicity that the monopoly mark-up is constant across firms. In this case there is a first-best allocation of resources across firms; the equal monopoly mark-up across firms cancels out in the marginal conditions for efficiency. If this economy is opened up to trade, forcing some firms to lower mark-ups, while other firms continue with the original high mark-ups, the increased dispersion in mark-ups will reduce allocative efficiency. The point that a constant monopoly mark-up across all goods achieves allocative efficiency has long been understood. (See Robinson (Ch 27, 1934).) The large literature on the theory of second best, e.g., Lipsey and Lancaster (1956-1957), has long emphasized that lowering distortions on some goods while maintaining distortions on other goods can lower overall welfare. A contribution of this paper is to connect these earlier ideas to the more recent trade literature focusing on selection over heterogenous firms.

We derive a formula for the effect of a change in the trade friction on allocative efficiency. The formula depends upon how mark-ups by domestic firms differ on average from mark-ups by foreign firms. We show that, taking into account the trade friction, if domestic firms have an overall cost advantage over foreign firms in the domestic market, then a reduction in the trade friction within a particular sector raises allocative efficiency within the sector. It increases overall allocative efficiency across all sectors is if the average mark-up in the affected sector is at least as high as the average mark-up in the economy as a whole.

We examine the magnitude of the allocative efficiency component of welfare relative to the productive efficiency component. Suppose that firms in different countries draw from the same productivity distributions. Starting from a trade friction of zero, introducing a small friction has no first-order effect on allocative efficiency. Hence, to a first approximation, the effect of a small trade friction operates entirely through the productivity channel.

For large frictions, there tends to be wedge between the mark-ups on domestic-sourced goods and mark-ups on foreign-sourced goods, and reductions in trade frictions will generally have a first-order effect on allocative efficiency. We consider the baseline numerical example considered in ACR, and discuss assumptions under which the allocative efficiency effect of trade is of the same order of magnitude as the productivity effect of trade. This turns out to be true for all four productivity distribution functional forms that we consider, assuming
there is only one firm per good in each country. This continues to be true when we allow for more firms using a bounded productivity distribution or the log-normal. However for the fat-tailed distributions of the Fréchet and Pareto, the importance of allocative efficiency drops off dramatically if there is more than one firm in each country. In fact, for the Pareto, with two or more firms in a country, allocative efficiency is a constant, invariant to trade frictions.

We conclude that the shape of the right tail of the productivity distribution matters for the role of allocative efficiency in trade liberalizations. When the tail is not too fat, if there are many firms, the top firms tend to be relatively similar. As trade liberalizations increase the number of competing firms, mark-ups are depressed. But with a fat tail, as market size increases and the number of firms increases, there is still relatively high probability weight that a very high productivity firm will emerge that is able set high mark-ups, being insulated from competition because it is such an outlier in productivity relative to its rivals. In such cases, trade liberalizations have less force in compressing mark-ups.

Our model follows the Bertrand oligopoly approach of Bernard, Eaton, Jensen, and Kortum (2003), hereafter BEJK, in which there are multiple head-to-head competitors for each good. If firms draw productivity from the Fréchet or Pareto in our model, in the limit as the number of firms gets large, our model converges to BEJK. A well-known result of BEJK is that trade liberalizations have no effect on the distribution of mark-ups. In such a case the allocative efficiency component of welfare is a constant. The ACR general framework encompasses BEJK because trade liberalizations only affect the productivity efficiency component of welfare, not allocative efficiency. Away from the BEJK limit, where there are a finite number of firms, trade can affect allocative efficiency. Atkeson and Burstein (2008) consider a finite number of firms for each good, and our model follows their approach. They focus on “pricing to market” behavior and exchange rate pass-through issues. de Blas and Russ (2010) have worked through a version of BEJK with finite Fréchet draws and have emphasized how opening to trade shifts down the distribution of mark-ups. Our paper is different from these papers in that we focus on allocative efficiency, and in particular, derive the connection to the welfare analysis of ACR.

There has recently been broad interest in models where changes in trade regime affect the distribution of mark-ups. Melitz and Ottaviano (2008) develop a framework that captures the selection and reallocation effects of trade, highlighted in Melitz (2003), while at the same time allowing for mark-ups to be endogenous. Our modeling structure of head-to-head Bertrand competitors of the same good is very different from the Melitz and Ottaviano monopolistic competition structure. And our focus on allocative efficiency is also different. Behrens and Murata (forthcoming) develop a model of trade and efficiency, using a model
of monopolistic competition with variable elasticity of substitution, in which larger markets have higher elasticities and lower mark-ups. Like us, they consider how an expansion of trade affects mark-ups across sectors and hence resource allocation. Our modeling approach is very different, but more broadly we are different in the way that heterogeneity in firm productivity plays a central role in our paper, contributing both productivity gains from trade and allocative efficiency effects. We are also different in the way we tie the analysis to ACR. Devereux and Lee (2001) is an early model of how trade affects endogenous mark-ups with a particular focus on how mark-ups affect entry. Finally, the recent working paper by Edmond, Midrigan, and Xu (2011) is closely related to our paper. They highlight the allocative efficiency effects of trade and take the Atkeson and Burstein (2008) model and apply it to data on Taiwanese manufacturing establishments. The papers are very different in that we focus on deriving theoretical results, while their paper takes a quantitative approach.

Our paper is connected to the recent literature relating distortions to aggregate productivity. (See, for examples, Rustucia and Rogerson, 2008, and Hsieh and Klenow, 2009). This literature has focused on policy distortions, such as taxes on some firms and subsidies on others, that distort the allocation of resources across firms. Differential mark-ups across goods, arising through differences in market power, operate like tax differentials in terms of the distortions they create. More generally, there has been great interest in recent years in incorporating heterogeneity into economic models of industry, such as heterogeneity in productivity, as in Melitz (2003), or heterogeneity in distortions faced, as in the distortions literature. This paper highlights a different kind of heterogeneity—heterogeneity in mark-ups—and with such heterogeneity, opening up to trade delivers welfare effects not captured in standard models, and not picked up in trade flows.

The rest of the paper is organized as follows. Section 2 presents the model and develops the indexes of allocative and productive efficiency. Section 3 presents our results for how trade affects allocative efficiency. Section 4 derives the link in our model between productive efficiency and the ACR statistic. Section 5 concludes.

2 Model

2.1 Description of the Model

There are a continuum of goods on the unit interval, each good indexed by \( \omega \in [0, 1] \). Utility is Cobb-Douglas,

\[
U = \exp \left( \int_0^1 \ln q_\omega d\omega \right),
\]
where $q_\omega$ indicates the quantity of consumption good $\omega$.

For simplicity of exposition, assume there are two countries, $i = 1, 2$. Each country has a measure $L_i$ workers. Labor is the only factor of production. Let $w_i$ be the wage at $i$.

The different goods potentially vary in: the number of firms capable of producing the good, the productivity of the different firms, and the trade cost of shipping between countries. In particular, there are $n_{\omega,i}$ different firms capable of producing good $\omega$ at $i$. The total number of firms for good $\omega$ across the two countries is $n_\omega = n_{\omega,1} + n_{\omega,2}$. Assume that $n_\omega \geq 2$. Let $k \in \{1, 2, \ldots, n_{\omega,i}\}$ index a particular firm located at $i$ capable of producing good $\omega$ at $i$.

Define $x_{k,\omega,i}$ as the productivity of firm $k$ located at $i$ for good $\omega$. This is the output per unit of labor employed. Suppose we rank all $k$ firms at $i$ for good $\omega$ in terms of descending productivity. Let $x_{\omega,i}^*$ and $x_{\omega,i}^{**}$ be the productivity of the first highest and second highest. (If there is only one firm for good $\omega$ at $i$ then set $x_{\omega,i}^{**} = 0$.) The third highest and beyond will be not be relevant for pricing or production, following the standard logic of Bertrand competition, which will be used here.

Let $\tau_{\omega,ij}$ be the iceberg trade cost of shipping good $\omega$ from location $i$ to location $j$, so that to deliver one unit, $\tau_{\omega,ij} \geq 1$ units must be shipped. Assume $\tau_{\omega,ii} = 1$, so there is no cost to ship locally.

Define a concept of cost-adjusted delivered productivity to take into account wages and trade cost. This is what will be relevant for pricing. For good $\omega$, the cost-adjusted delivered productivity of firm $k$ located at $i$ and selling to destination $j$ is defined as

$$\bar{x}_{k,\omega,ij} \equiv \frac{x_{k,\omega,i}}{\tau_{\omega,ij} w_i}.$$ 

This is the quantity of output the firm can deliver to country $j$, per dollar of expenditure on input at location $i$.

We gather together the cost adjusted productivities of the top two firms at each location for delivering to country $j$ into the following set,

$$\left\{ \frac{x_{\omega,1}^*}{\tau_{\omega,1j} w_1}, \frac{x_{\omega,1}^{**}}{\tau_{\omega,1j} w_1}, \frac{x_{\omega,2}^*}{\tau_{\omega,2j} w_2}, \frac{x_{\omega,2}^{**}}{\tau_{\omega,2j} w_2} \right\},$$

where the first and second elements are the top two firms at country 1, and the third and four elements are the top two at country 2. Define $y_{\omega,j}$ to be the maximum element of this set and $z_{\omega,j}$ to be the second highest. These are the cost-adjusted delivered productivities of the first and second most efficient producers for serving $j$, across all firms from both countries.
Firms compete in price in a Bertrand fashion, market by market. Following standard reasoning, e.g., Grossman and Helpman (1991), the most efficient producer will get the sale and the Bertrand equilibrium price will equal the marginal cost of the second most efficient producer. The second-most efficient producer can deliver \( z_{\omega,j} \) units to \( j \) per dollar of expenditure, so its marginal cost is \( \frac{1}{z_{\omega,j}} \). Thus, the price of good \( \omega \) delivered to \( j \) equals

\[
p_{\omega,j} = \frac{1}{z_{\omega,j}}.
\]

The mark-up of price over the most-efficient firm’s cost is

\[
m_{\omega,j} = \frac{p_{\omega,j}}{y_{\omega,j}} = \frac{y_{\omega,j}}{z_{\omega,j}},
\]

its delivered, cost-adjusted productivity advantage. For subsequent calculations, rather than work with the mark-up, it is convenient to work with its inverse

\[
c_{\omega,j} = \frac{1}{m_{\omega,j}} = \frac{z_{\omega,j}}{y_{\omega,j}}.
\]

This is the marginal cost share of the price.

Assume profits of firms located in country \( i \) are distributed to consumers in country \( i \) in a lump-sum fashion. Given the pricing rules described above, the wages at each location adjust so the markets clear.

### 2.2 Productivity Distributions

We now specify the particulars about productivity distributions. Assume there are a finite number \( S \) of sectors indexed by \( s \). Let \( \alpha_s \) be the fraction of goods in sector \( s \). Label the goods so that \( \omega \in [0, \alpha_1) \) are the sector 1 goods, \( \omega \in [\alpha_1, \alpha_1 + \alpha_2) \) are the sector 2 goods, and so on.

For each sector \( s \) good, there are \( n_{s,1} \) firms in country 1 and \( n_{s,2} \) firms in country 2. But as discussed above, only the top two most efficient producers at each location will potentially be relevant for the equilibrium. For sector \( s \), let \( H_s(x_1^*, x_1^{**}, x_2^*, x_2^{**}) \) be the joint distribution of the first and second highest productivity at 1, and the first and second highest productivity at 2. Let \( h_s(x_1^*, x_1^{**}, x_2^*, x_2^{**}) \) be the joint probability density. For our main results, we do not impose the assumption that the productivity draws are independent across firms.

In addition to our general results, we will examine several examples and for these examples we will assume that firms make independent draws from a given productivity distribution.
We consider four different example productivity distributions.

\begin{align*}
\text{Fréchet} & : \quad \Pr\{X_{k,s,i} \leq x\} = e^{-\xi_{s,i} x^{-\theta}}, \quad x \geq 0, \\
\text{Pareto} & : \quad \Pr\{X_{k,s,i} \leq x\} = 1 - \left(\frac{x}{\xi_{s,i}}\right)^{-\theta} , \quad x \geq \xi_{s,i}, \\
\text{Power Function} & : \quad \Pr\{X_{k,s,i} \leq x\} = \left(\frac{x}{\xi_{s,i}}\right)^{\theta} , \quad x \in [0, \xi_{s,i}], \quad \theta \geq 1, \\
\text{Log-Normal} & : \quad \Pr\{X_{k,s,i} \leq x\} = N(\ln(x);\xi_{s,i},\frac{\xi_{s,i}}{\theta}), \quad x > 0,
\end{align*}

where $N(x;\mu,\sigma)$ is the normal cumulative distribution function given mean $\mu$ and standard deviation $\sigma$.

In these distributions the parameter $\xi_{s,i}$ shifts the scale of the distribution proportionately, while $\theta$ governs the curvature or shape. The higher is $\theta$, the more similar the draws.

The Fréchet and the Pareto both have fat-right tails and are closely related.\footnote{They are \textit{tail equivalent} in the sense that the tail probabilities on the right are proportional to each other in the limit.} The Fréchet is an extreme value distribution and if we take $n$ independent draws from the Fréchet, the distribution of the maximum is also distributed Fréchet. If we take the maximum of $n$ independent draws from the Pareto, the distribution of the maximum goes to the Fréchet for large $n$.\footnote{See Eaton and Kortum (2010) for further discussion about obtaining the Fréchet based on draws from the Pareto.}

Like the Fréchet, the power function has the property that the maximum of $n$ draws from the distribution remains in the same distribution family. This makes calculations with the power case tractable, like they are for the Fréchet case. A key difference between the power function and the Fréchet is that the power function distribution is bounded above, while the Fréchet is unbounded above, with a fat right tail. Thus, considering both the Fréchet and power cases illustrates two very different possibilities.

Finally, the log-normal distribution is unbounded above, but the right tail is not as fat as for the Fréchet or Pareto. The log-normal is useful to include in the analysis, given its prominence in empirical studies of productivity distributions.

For the case of $n$ firms drawing from the Fréchet, de Blas and Russ (2010) have derived the distribution of mark-ups. We report their formula in terms of the cost share of price (the inverse of the mark-up). The distribution will in general depend upon the source country. Let $\chi^* \in \{1, 2\}$ denote whether the source country is 1 or 2. (This is the location of the firm with the highest, cost-adjusted delivered productivity.) The formula for the distribution of the cost share conditional on the source is (letting $C$ denote the random variable of the cost
share)\)

\[
\Pr(C \leq c|\chi^* = i) = \left[ c^{-\theta} - \frac{\phi_i}{n_1 \xi_1 w_1^{-\theta} + n_2 \xi_2 w_2^{-\theta}} \tau^{-\theta} (c^{-\theta} - 1) \right]^{-1}.
\]

(1)

for

\[
\phi_1 = \xi_1 w_1^{-\theta},
\phi_2 = \xi_2 w_2^{-\theta} \tau^{-\theta}.
\]

From inspection of this formula, we can see that if the source is the home country, \(\chi^* = 1\), then an increase in the friction \(\tau\) shifts the distribution down in a first-order stochastic dominant fashion. (Or equivalently mark-ups increase.) This is intuitive, because a domestic firm limit pricing against a foreign firm will raise its price when \(\tau\) increases. If the source is the foreign country, \(\chi^* = 2\), an increase in \(\tau\) shifts the distribution up. This is intuitive because a foreign firm limit pricing against a domestic firm will see cost increase relative to price, when the friction increases.

Next consider taking the limit where \(n_1\) and \(n_2\) are made arbitrarily large, but at the same time the scaling parameters \(\xi_1\) and \(\xi_2\) are reduced so the distribution of the maximum over the \(n_1\) and \(n_2\) draws does not explode. In particular, let \(n_1 = vn_1^0, n_2 = vn_2^0, \xi_1 = \xi_1^0/v,\) and \(\xi_2 = \xi_2^0/v\) for constants \(n_1^0, n_2^0, \xi_1^0,\) and \(\xi_2^0,\) and scaling \(v\). The limiting distribution of productivities as \(v\) goes to infinity results in the probability structure assumed in BEJK. As firm counts get large, the second term on the right of (1) goes to zero, and the limit distribution is

\[
\lim_{v \to \infty} \Pr(C \leq c|\chi^* = i) = c^\theta.
\]

This is the well-known result in the BEJK model that the cost share has the Pareto distribution. Note the distribution does not depend upon the source country and does not vary with the trade friction \(\tau\). This goes back to a point mentioned in the introduction, that in the BEJK model, a change in \(\tau\) has no effect on the distribution of mark-ups.

### 2.3 Welfare

Let \(R_i\) be total revenue of firms located at \(i\) across all goods, including domestic sales as well as exports. This will equal the total income at \(i\). Income is divided between labor and profits

\[
R_i = w_i L_i + \Pi_i.
\]

Given Cobb-Douglas utility and a unit measure of goods, \(R_i\) will be equal spending at \(i\) on each good \(\omega\).
Define $E c^\text{sell}_i$ to be the revenue-weighted mean share of variable cost in revenue across goods with source at location $i$. This equals

$$E c^\text{sell}_i = \frac{w_i L_i}{R_i} = \frac{\int_{\{\omega: \chi_j(\omega) = 1\}} c^{\text{buy}}_{\omega,1} R_1 d\omega + \int_{\{\omega: \chi_j(\omega) = 2\}} c^{\text{buy}}_{\omega,2} R_2 d\omega}{\int_{\{\omega: \chi_j(\omega) = 1\}} R_1 d\omega + \int_{\{\omega: \chi_j(\omega) = 2\}} R_2 d\omega},$$

(2)

where $\chi_j(\omega) \in \{1, 2\}$ denotes the source country for any particular good $\omega$ at destination $j$. We include the superscript “sell” in $E c^\text{sell}_i$ to distinguish it from $c^{\text{buy}}_{\omega,j}$ which is notation for a good $\omega$ “bought” at $j$ or with destination $j$. To understand (2), observe that by definition, the spending on labor to produce good $\omega$ for destination 1 must be $c^{\text{buy}}_{\omega,1} R_1$. In the numerator of the term above on the right, we first integrate labor spending for goods produced at $i$ and sold at 1, and then account for goods produced at $i$ and sold at 2. The denominator is the analogous integral for revenues.

With a Cobb-Douglas utility function, the price index takes a simple form. The price index at location $i$ equals

$$P_i = \exp \left( \int_0^1 \ln p_{\omega,i} d\omega \right) = \exp (E \ln p_i),$$

(3)

where $p_{\omega,i}$ is the price of $\omega$ at $i$, and $E \ln p_i$ is simpler notation for the expectation from integrating over the $\omega$ at $i$. Notice that the price index $P_i$, in fact, the geometric mean of prices. Our welfare measure, real income at $i$, equals

$$W_i^{\text{Total}} = \frac{R_i}{P_i} = \frac{w_i L_i}{E c^\text{sell}_i P_i},$$

(4)

where we use equation (2) to substitute in for the nominal income $R_i$.

It is convenient to introduce notation for what prices would be under marginal cost pricing. Let

$$p^{\text{mc}}_{\omega,i} = \frac{1}{y_{\omega,i}}$$

be the marginal cost of the most efficient producer of $\omega$ at $i$ and let $P_i^{\text{mc}}$ be the price index at $i$, with $p^{\text{mc}}_{\omega,i}$ sustained in for $p_{\omega,i}$ for each good $\omega$ in formula (3).

With this notation, we can write total welfare (4) at $i$ as

$$W_i^{\text{Total}} = w_i L_i \times W_i^{\text{Prod}} \times \frac{E c_i^{\text{buy}}}{E c^\text{sell}_i} \times W_i^{\text{A}}$$

(5)
for the productive efficiency index $W^{Prod}_i$,  

$$
W^{Prod}_i \equiv \frac{1}{P_{mc}^i} = \exp (E \ln y_i),
$$

(6)

and the allocative efficiency index $W^A_i$  

$$
W^A_i \equiv \frac{P_{mc}^i}{P_i \times E_{buy}^i} = \frac{\exp (-E \ln z_i) \cdot 1}{\exp (-E \ln z_i) E_{buy}^i} = \frac{\exp \left(E \ln c^{buy}_i \right)}{E_{buy}^i}.
$$

To see the second line of the expression for $W^A_i$, recall that $c_\omega, i = z_\omega, i / y_\omega, i$, so $\ln c_\omega, i = \ln z_\omega, i - \ln y_\omega, i$.

The manipulation results in a decomposition (5) of welfare into four terms. Without loss of generality we will focus on the welfare of country 1, and we will set the wage at 1 to be the numeraire, $w_1 = 1$. As the labor supply $L_i$ will be fixed in the analysis, the first term in the welfare decomposition is a constant that we will henceforth ignore.

The second term, the productive efficiency index $W^{Prod}_i$, is the geometric mean of first highest delivered, cost-adjusted productivity. It is what the price index would be with no mark-up, i.e., if the cost share in the price were one for all goods. This index is affected if there is technical change determining the underlying levels of productivity. It is also affected if trade cost declines, increasing the relative productivity of foreign firms to deliver goods locally. Terms of trade effects also show up in $W^{Prod}_i$, because a lower wage from a country that is a source of goods will raise the index.

The third and fourth terms depend upon mark-ups, or equivalently, the inverse of the mark-ups which are the cost shares. In ACR, and in the broader literature that it encompasses, trade has no effect on the distribution of mark-ups, and so it has no effect on the third and fourth terms. Thus in ACR, the welfare effects of trade operate entirely through the effects on the productive efficiency index $W^{Prod}_i$.

The third term is a “terms of trade” effect on mark-ups. Holding fixed the other components of welfare, total welfare is higher in country $i$ if the cost share of price in the goods that it purchases tends to be high relative to the cost share of price in the goods that it sells. In a symmetric version of the model where the two countries are mirror images of each other, the “buy” cost share will equal to the “sell” cost share, and the third term will drop out.
The fourth term, the allocative efficiency index $W_i^A$, depends upon the distribution of the buy cost shares. It is, in fact, the ratio of the geometric mean to the arithmetic mean of cost shares. Observe first that if the cost-share $c_{\omega,i}^{buy}$ were some constant $c'$ across all goods, the numerator and denominator would both equal $c'$, so the $c'$ would cancel out and $W_i^A$ would equal one. Next note that if there is any variation in $c_{\omega,i}^{buy}$ across $\omega$, then the allocative efficiency index is strictly less than one, $W_i^A < 1$. To see this, take the log of $W_i^A$ and observe that $E \ln c_i^{buy} < \ln \left( E c_i^{buy} \right)$ because of Jensen’s inequality. Thus, we see that an increase in dispersion of the cost shares lowers the index.

When evaluating the effect of a change in the trade friction, it will be convenient to take logs and conduct the analysis in elasticity terms,

$$ \eta^{Total} \equiv \frac{d \ln W^{Total}}{d \ln \tau} = \eta^{Prod} + \eta^{c-term\_trade} + \eta^A, $$

where $\eta^k$ is the elasticity for component $k$. (Again the first term $w_1 L_1$ of (5) is a constant given the normalization so the elasticity for this component is zero.) In Propositions 1 and 5, we derive formulas, respectively, for $\eta^A$ and $\eta^{Prod}$. All we have to say about $\eta^{c-term\_trade}$ is that it is zero in the symmetric country case.

While we focus on the Cobb-Douglas utility case in this paper, it is worth noting the welfare decomposition developed above holds for general utility functions. In particular, in the first line of the definitions of $W^{Prod}$ and $W^A$ in (6) and (7), we provide the general definition of the index and then in the second lines of these expressions substitute the value for the Cobb-Douglas case. Suppose for the general case, that price is a constant markup over cost. As the price index is homogeneous of degree one in the constant mark-up, it is immediate from the expression for $W^A$ that $W^A = 1$ in this case.

Throughout the rest of the paper we will repeatedly need to refer to the buy variant of the cost share $c^{buy}$. To simplify notation, we will leave the “buy” superscript implicit, i.e., $c$ will refer to the buy cost share.

### 3 Allocative Efficiency and Trade

This section presents our main results about the link between the size of the trade friction $\tau$ and allocative efficiency. The first subsection provides two motivating examples. The second subsection presents our results for how trade affects allocative efficiency within a sector. The third section considers the effects across sectors.
3.1 Motivating Examples

To highlight key forces, we begin with two simple examples of how trade can affect allocative efficiency. In both examples, the countries are symmetric, so we can normalize the wage to be one in both countries, \( w_1 = w_2 = 1 \).

In both examples, we assume all firms have identical productivity equal to one, \( x_{k,\omega,i} = 1 \). With wage equal to one, the maximum cost-adjusted delivered productivity equals one in both countries for all goods, \( y_{\omega,i} = 1 \). Therefore, the productive efficiency index \( W^{Prod} \) is a constant, \( W^{Prod} = \exp(\ln(1)) = 1 \).

3.1.1 Example 1

In the first example, goods can be classified into two sectors that vary in the number of firms. In both sectors, the trade cost is a constant \( \tau \) to ship from one country to the other. Define sector one to consist of goods \( \omega \in [0, \alpha) \) and assume that there is a one firm in each country for each such good, \( n_{\omega,1} = n_{\omega,2} = 1 \). For this sector, the second lowest cost firm for each good is the foreign firm, with a delivered cost of \( \tau \). Hence, in Bertrand competition, the cost share will be \( c_{\omega,i} = 1/\tau \) for all sector 1 goods.

Sector 2 consists of the remaining goods \( \omega \in [\alpha, 1] \). In this sector, there are two firms in each country, \( n_{\omega,1} = n_{\omega,2} = 2 \). For such goods, the second lowest cost and the first lowest cost are identical, so price equals marginal cost in Bertrand equilibrium, and the cost share of price is \( c_{\omega,i} = 1 \).

Now consider comparative statics with the trade friction \( \tau \). As already noted, the productive efficiency index \( W^{Prod} \) is constant for this case. The allocative efficiency index equals

\[
W^A = \exp \left( \frac{\alpha \cdot \ln \left( \frac{1}{\tau} \right) + (1 - \alpha) \cdot \ln (1)}{\alpha \cdot \frac{1}{\tau} + (1 - \alpha) \cdot 1} \right) = \frac{\tau^{1-\alpha}}{\alpha + (1 - \alpha) \tau}. \tag{8}
\]

It is immediate that this is maximized at \( \tau = 1 \), at which point \( W^A = 1 \), and otherwise \( W^A \) is strictly decreasing for \( \tau \geq 1 \). For example, if half the goods have a single firm in each country, so that \( \alpha = 1/2 \), and if \( \tau = 2 \), then allocative efficiency is \( W^A = .94 \) according to the formula. With the high mark-up in sector 1, insufficient resources are being allocated to sector 1 relative to sector 2. The smaller the friction \( \tau \), the more mark-ups are compressed together, and allocative efficiency improves.

3.1.2 Example 2

In the second example, all goods have a single firm in each country, \( n_{\omega,1} = n_{\omega,2} = 1 \), all \( \omega \). The two sectors differ in trade costs. Sector 1 goods \( \omega \in [0, \alpha) \) have trade cost \( \tau_1 > 1 \), while
sector 2 goods $\omega \in [\alpha, 1]$ have trade cost $\tau_2 > 1$. Hence, the cost shares are $1/\tau_1$ and $1/\tau_2$ for the two sectors. The allocative efficiency index equals

$$W^A = \frac{\exp \left( \alpha \cdot \ln \left( \frac{1}{\tau_1} \right) + (1 - \alpha) \cdot \ln \left( \frac{1}{\tau_2} \right) \right)}{\alpha \cdot \frac{1}{\tau_1} + (1 - \alpha) \cdot \frac{1}{\tau_2}}.$$ 

Hold the friction in sector 2 fixed at $\tau_2^* > 1$ and consider variations in $\tau_1$. The index is maximized at $\tau_1 = \tau_2^*$ where the first best allocative efficiency level of $W_i^A = 1$ is obtained. At this point with mark-ups identical across goods, the mark-ups cancel out in the marginal rate of substitution conditions for efficiency. The index $W_i^A$ decreases if $\tau_1$ is increased above $\tau_2^*$. It also decreases when $\tau_1$ is lowered below $\tau_2^*$. This is an illustration of a well understood point that welfare can decrease if frictions in one sector are decreased while maintained in a second sector. (See the literature on the theory of second best (Lipsey and Lancaster (1956-1957).)

In general, with symmetric countries, the productive efficiency index can only go up when trade frictions are lowered, because the minimum possible cost to deliver to any location can only decrease. By contrast, we see with this second example that the allocative efficiency index can in general increase or decrease when frictions are lowered. The focus of the rest of this section is to derive conditions determining the direction of the net effect.

Note that in both of these examples, changing trade barriers affects welfare, even though in equilibrium there is no trade. Recall that in ACR, welfare changes if and only if trade flows change. But ACR focuses on the productive efficiency component of welfare gain and this component indeed is constant in these examples. So in that sense ACR gets it right. Section 4 examines in more detail the extent to which the ACR condition tracts the cost-efficiency component of welfare gain.

Finally, in these examples we have shut down productivity heterogeneity across firms. Such heterogeneity is of inherent interest for mark-ups and allocative efficiency. We incorporate such heterogeneity in the general model considered in the rest of this section.

### 3.2 Allocative Efficiency within a Sector

Recall that all goods within a sector are the same in terms of the number of firms in each country and in the underlying distribution of productivities firms draw from. Goods within a sector in general differ in terms of the particular realization of draws. In this subsection, we analyze allocative efficiency within a sector. To do this, let the expectations in the definition

3. Note that the productivity index depends upon relative wages. With symmetric countries, a change in $\tau$ has no effect on relative wages.
of $W^A$ in (7) be taken conditional on the sector of goods. For notational simplicity, in this subsection we leave the sector $s$ subscript implicit. Also, without loss of generality we focus on country 1 as the destination market and leave the destination index $j = 1$ implicit. Finally, let $\tau$ be the trade cost of shipping between the countries and let $w$ be the wage at country 2. (Since the wage at country 1 is normalized to one, $w$ is the relative wage between country 2 and country 1.)

In the analysis it will be necessary to distinguish the location of the highest delivered, adjusted productivity firm, as well as the second highest. Let $\chi^{**} = (i, j)$ denote the event that the highest is at $i$ and the second highest is at $j$. Let $\pi_{i,j}$ be the probability of this event. Using the joint density $h(x_1^*, x_1^{**}, x_2^*, x_2^{**})$, we calculate the formula for $\pi_{1,2}$,

$$
\pi_{1,2} = \int_0^\infty \int_0^\infty \int_0^{x_2^*} \int_0^{x_2^*} h(x_1^*, x_1^{**}, x_2^*, x_2^{**}) dx_2^{**} dx_1^{**} dx_2^* dx_1^*.
$$

(9)

To derive this expression, we begin by integrating over all possible values of the highest productivity $x_1^*$ at 1. We are conditioning on the fact that across both countries, the highest adjusted, delivered productivity is at 1. Therefore $x_1^* \geq \frac{x_2^*}{\tau w}$ must hold, or equivalently $\tau w x_1^* \geq x_2^*$, accounting for the second range of integration. Next, we are conditioning on the second-highest overall being at 2. Therefore, the first highest at 2 beats the second highest at 1, $x_1^{**} \leq \frac{x_2^*}{\tau w}$, accounting for the third range of integration. Finally, by definition of $x_2^{**}$, it must be that $x_2^* \geq x_2^{**}$, accounting for the fourth range of integration. By similar arguments, we can construct $\pi_{1,1}$, $\pi_{2,2}$, and $\pi_{2,1}$.

We can write the expected cost share as

$$
Ec = (1,2) + (2,1) + (1,1) + (2,2)
$$

(10)

For each line, we indicate the event $\chi^{**} = (i, j)$ that is being accounted for. Consider event $(1,2)$, the first line. We are integrating over the same set of events in (9) used to calculate $\pi_{1,2}$, except now we include in the integrand the cost share, here equal to $c = x_2^*/(\tau w x_1^*)$, the ratio of delivered productivity at 2 to delivered productivity at 1. Note the $\tau w$ term
appears in the denominator of \( c \) to account for the relative differences between a firm at 2 and a firm at 1 in the trade friction and wages. The second line accounts for event (2,1). Here \( \tau w \) appears in the numerator of the \( c \) term, inverting what happens in the previous case. The third and fourth lines above account for events (1,1) and (2,2). Note that \( \tau w \) does not appear in the \( c \) term in either case. In both of these cases, the first and second highest are located in the same country, so there is no difference in the relative trade frictions or wages faced by top two firms.

The following result plays a key role in our analysis.

**Lemma 1** Let \( \mathcal{E}_{i,j} \) be the expected value of \( c \) conditional on event \( \chi^{**} = (i,j) \). We have

\[
(i): \quad \frac{d\mathcal{E}_c}{d\tau} = -\frac{1}{\tau w} (\pi_{1,2} E_{C_{1,2}} - \pi_{2,1} E_{C_{2,1}}) \frac{d(\tau w)}{d\tau},
\]

\[
(ii): \quad \frac{d\ln c}{d\tau} = -\frac{1}{\tau w} (\pi_{1,2} - \pi_{2,1}) \frac{d(\tau w)}{d\tau}.
\]

**Proof.** See appendix. ■

Note that we write the term \( d(\tau w) / d\tau \) to allow for a general equilibrium effect of \( \tau \) on the relative wage. Assume \( d(\tau w) / d\tau > 0 \) so the direct effect of the increase in trade cost dominates any potentially offsetting effect on the relative wage. In the symmetric country case, for example, the relative wage \( w = 1 \), so \( d(\tau w) / d\tau = 1 \).

To see the intuition for Lemma 1, consider the symmetric country case where the relative wage is constant, \( w = 1 \). Consider a particular set of productivity draws yielding event (1,2). In this event, \( c = \frac{x_{2}}{x_{1}} \). If we increase \( \tau \) for this case, the slope is \( \frac{dc}{d\tau} = -\frac{1}{\tau w} \frac{x_{2}}{x_{1}} = -\frac{1}{\tau} c \). Therefore, on average the slope for event (1,2) is \( -\frac{1}{\tau} E_{C_{1,2}} \). Analogously, the average slope for event (2,1) is \( \frac{1}{\tau} E_{C_{1,2}} \). Taking into account the probability of each event, the expected slope is \( -\frac{1}{\tau} (\pi_{1,2} E_{C_{1,2}} - \pi_{2,1} E_{C_{2,1}}) \), which matches the equation given in Lemma 1. This discussion has ignored two things. First, we didn’t say anything about events (1,1) and (2,2) where the first and second highest are from the same country. A change in \( \tau \) has no direct effect on \( c \) in these cases. Second, we didn’t say anything about how a change in \( \tau \) shifts the margins of when events (1,2), (2,1), etc., occur. An increase in \( \tau \) can flip the event the occurs from (2,1) to (1,2). However, these changes at the margin have no first-order effect on \( \mathcal{E}_c \), because at these margins, adjusted productivities are the same for both firms involved with the flip. On account of Bertrand competition, price equals marginal cost, i.e., \( c = 1 \) at these margins. The flips do not change \( c \), to a first order. More formally, taking the derivative of \( \mathcal{E}_c \) in equation (10), all the terms involving changes in the limits of integration net out to zero, and the only terms that remain are where \( \tau \) appears in the integrand, which is true for events (1,2) and (2,1).
Part (ii) of Lemma 1 provides the formula for the log case, \( dE \ln c/d\tau \). For a productivity
draw yielding event (1,2), \( \ln c = \ln \left( \frac{x^2}{x^2 + \tau} \right) \), so here \( \frac{d\ln c}{d\tau} = -\frac{1}{\tau} \). The rest of the argument is
the same as in the previous paragraph.

With Lemma 1 in hand, we can state and prove Proposition 1.

**Proposition 1** (i) The elasticity of the allocative efficiency index with respect to to changes
in the trade friction has the following formula

\[
\eta^A \equiv \frac{d \ln W^A}{d \ln \tau} = -\pi_{1,2} \left( \frac{E_c - E_{c1,2}}{E_c} \right) - \pi_{2,1} \left( \frac{E_{c2,1} - E_c}{E_c} \right) \left( 1 + \frac{d \ln w}{d \ln \tau} \right).
\]  

(11)

(ii) If

\[
E_{c1,2} < E_c < E_{c2,1},
\]  

then an increase in the trade friction \( \tau \) strictly decreases allocative efficiency \( W^A \).

**Proof.** Noting that \( \ln W^A = E \ln c - \ln E_c \), we have

\[
\frac{d \ln W^A}{d \ln \tau} = \frac{\tau d \ln c}{d \tau} - \frac{\tau}{E_c} \frac{d E_c}{d \tau} = -\left( \pi_{1,2} - \pi_{2,1} \right) \frac{d (\tau w)}{wd \tau} - \frac{\pi_{1,2} E_{c1,2} - \pi_{2,1} E_{c2,1}}{E_c} \frac{d (\tau w)}{wd \tau}
\]

where the second line substitutes in the results from Lemma 1. Rearranging terms yields
(11). To prove claim (ii), note our earlier assumption that \( \tau \) is equivalent to
(1 + \frac{d \ln w}{d \ln \tau}) > 0 \), so (11) and (12) imply \( \eta^A < 0 \).

The formula for the elasticity of \( W^A \) depends upon the cost shares of price for domestic
firms limit pricing against foreign firms, and the cost shares for foreign firms limiting pricing
against domestic firms, in both cases relative to the average cost shares in the economy. If
condition (12) holds, where the domestic cost shares are lower than the average, and the
foreign cost shares higher than average, then an increase in the trade friction necessarily
decreases allocative efficiency. The formula also depends upon the probability weight on
domestic firms limit pricing against foreign firms, and vice versa.

It is intuitive to expect condition (12) to hold. Suppose event \( \chi^{**} = (1,2) \) is realized
where a domestic firm gets the sale and is limit pricing against a foreign firm. The foreign
firm has to bear the trade friction, while the domestic firm does not, and this tends to increase
the price the domestic firm can charge, reducing the cost share of price. Analogously, for
event \( \chi^{**} = (2,1) \), a foreign firm gets the sale and is limit pricing against a domestic firm.
The trade friction incurred by the foreign firm tends to increase the cost share of price.
To examine more formally the circumstances under which condition (12) holds, we begin with the case where firms draw productivity from the Fréchet distribution. Earlier, in equation (1) we reported the de Blas and Russ (2010) formula for the distribution of the cost share conditional on the source location $\chi^* = i$. In a separate appendix, we show for the Fréchet that the distribution of cost share conditional on $\chi^{**} = (i,j)$ is the same across $j$ and is equal to distribution (1) for $\chi^* = i$. That is, after conditioning on the location of the first highest firm, the location of the second highest firm makes no difference for the distribution of cost shares. Therefore, using (1), the distribution of $c$ conditional on $\chi^{**} = (2,1)$ stochastically dominates the distribution conditional on $\chi^{**} = (1,2)$, if and only if

$$\xi_2 w_2^{-\theta} \tau^{-\theta} < \xi_1 w_1^{-\theta}. \quad (13)$$

Thus, condition (12) holds if and only if (13) holds. Condition (13) implies that costs, including any trade friction, tend to be lower for domestic firms to serve the local market compared with foreign firms. In particular, in the symmetric case with identical productivity scaling coefficients, $\xi_1 = \xi_2$, and identical wages, $w_1 = w_2$, then (13) holds if $\tau > 1$.

Recall that if we scale up the firm counts $n_1$ and $n_2$ as discussed in Section 2.2, in the limit the model goes to BEJK, where the cost share distribution is invariant to the source $\chi^* = i$, and to $\chi^{**} = (i,j)$ as well. Hence in this limit, $E_c_{2,1} = E_c_{1,2} = Ec$, implying the elasticity $\eta^A$ is zero. That is, changes in the trade friction have no effect on allocative efficiency in the BEJK limit. However, when we are away from the limit, changes in the trade friction do affect allocative efficiency.

We consider alternative functional forms for the productivity distributions, focusing on the symmetric case where $\xi_1 = \xi_2$ and $w_1 = w_2$. For the power distribution case, and $n_1 = n_2 = 1$, we show in the separate appendix that if $\tau > 1$, then (12) holds, in which case the elasticity is strictly negative, $\eta^A < 0$. For general $n_1$ and $n_2$ for the power function case, as well as for the Pareto and lognormal cases, we use numerical methods to establish that (12) holds under symmetry and $\tau > 1$. We conclude that the condition under which an increase in the trade friction lowers allocative efficiency holds in a wide class of cases.

As discussed in the introduction, the ACR statistic is calculated with trade flow data. As the ACR statistic equals $\eta^{Prod}$ (this is Proposition 5 below), $\eta^{Prod}$ is calculated with trade flow data. In contrast, the formula for $\eta^A$ is calculated with data on the cost share of price. The data requirements for $\eta^A$ are more challenging than for $\eta^{Prod}$. To calculate $\eta^A$, we need to determine the variable cost share of price, and condition not only on whether the source (the most efficient firm) is domestic or foreign, but also on the location of the second most efficient firm. It is interesting to note that for the Fréchet case, the data requirements are less burdensome, because conditioned upon the source, the distribution of $c$ is invariant
to the second-most efficient firm’s location. Thus, to evaluate the formula (11) we can substitute in expectations for \( c \) conditional only on the source. To evaluate formula (11), we also need to know \( \pi_{1,2} \), the share of goods where the first highest is domestic and the second highest is foreign, as well as the analogous \( \pi_{2,1} \). For the Fréchet, there are relatively straightforward expressions for \( \pi_{1,2} \) and \( \pi_{2,1} \). Section 4 will revisit the issue of the data requirements under the Fréchet for calculating \( \eta^A \). Section 4 will also introduce the idea of using what works for the Fréchet as an approximation for more general cases.

The next issue we consider is the magnitude of \( \eta^A \). In one interesting case, \( \eta^A \) is negligible. This is the case where the friction is small and firms in the different countries draw from the same productivity distribution. For this result, suppose each firm in country \( i \) draws from a productivity distribution \( F_i(x) \) and suppose the draws are i.i.d. across firms.

**Proposition 2** Suppose the firms at 1 and 2 draw from the same productivity distributions after adjusting for any wage differences, i.e., \( F_1(x) = F_2(wx) \). Then at the limit of \( \tau = 1 \),

\[
\frac{dEc}{d\tau} = 0, \quad \frac{dE\ln c}{d\tau} = 0, \quad \text{and} \quad \eta^A = 0.
\]

yet,

\( \eta^{Prod} < 0 \).

**Proof.** In the limit of \( \tau = 1 \), each of the \( n_1 + n_2 \) firms in the economy is equally likely to the highest or second highest most efficient firm. This implies \( \pi_{1,2} = \pi_{2,1} \). Also, \( Ec_{1,2} = Ec_{2,1} = Ec \). Hence, from Lemma 1, \( dEc/d\tau = 0 \) and \( dE\ln c/d\tau = 0 \). Following the proof of Proposition 1, \( \eta^A = 0 \).

To show that \( \ln W^{Prod} \) strictly decreases, note that \( \ln W^{Prod} = E \ln y \). The cumulative distribution function for maximum adjusted productivity \( y \) is

\[
G(y|\tau) = F_1(y)^{n_1} F_2(\tau wy)^{n_2}.
\]

Observe that

\[
\frac{dG(y|\tau)}{d\tau} = n_2 F_1(y)^{n_1} F_2(\tau wy)^{n_2-1} f_2(\tau wy)y \frac{d(\tau w)}{d\tau} \geq 0,
\]

with the inequality holding strictly for the sub-domain of \( y \) in which \( f_2(\tau wy) > 0 \). Thus, for any \( \tau' > \tau \), \( G(y|\tau') \geq G(y|\tau) \) for all \( y > 0 \), and the inequality holds strictly for some \( y \), implying that \( E(\ln y|\tau') < E(\ln y|\tau) \). The result follows.

When all firms draw from the same productivity distributions, not taking into account trade frictions, imposing a small friction on some firms but not others has no first order effect
on the expected cost shares. And no first-order effect on allocative efficiency. In contrast, there is a first-order effect on the productive efficiency. Hence the welfare effect of small frictions around symmetry is 100% productive efficiency and 0% allocative efficiency, to a first approximation.

Some intuition for Proposition 2 can be obtained by considering the hypothetical possibility of negative trade costs, i.e., a $\tau < 1$, such that when a good is shipped it expands to $1/\tau > 1$ of its original quantity, rather than contracts. An export subsidy is an example. With $\tau < 1$, the local firms and foreign firms switch places, with the foreign firms having the advantage in the local market. In fact, for the power and Fréchet functional forms, we have found the roles are completely symmetric. In particular, for these cases the distribution of $c$ given $\tau^o > 1$ is exactly the same as it is for $\tau' = 1/\tau^o < 1$ (assuming also $n_1 = n_2$ and $w_1 = w_2$). Hence $W^A(\tau) = W^A(1/\tau)$. Given this symmetry at $\tau = 1$, it is immediate that the slope $dW^A(\tau)/d\tau = 0$. Allocative efficiency is maximized at $\tau = 1$ where rival firms are evenly matched, in an ex ante sense, and any local deviation from $\tau = 1$ has no first-order effect on $W^A$.

We note that the assumption of symmetric cost distributions in Proposition 2 is not just a convenient simplifying assumption, but is crucial for the results. If instead the firms in different countries draw from different distributions, then in general we expect a small friction will have a first-order effect on allocative efficiency.

### 3.3 Allocative Efficiency Across Sectors

The previous subsection considers how allocative efficiency varies with trade frictions within a sector, where the number of firms in each country and the underlying productivity distributions are held fixed across goods. This subsection considers overall allocative efficiency across goods from all sectors, where firm counts or productivity distributions can vary. Let $\tau_s$ denote the trade friction in sector $s$. We consider liberalizations that either lower $\tau_s$ for a particular sector, or lower the friction for all sectors together.

We first note that if productivity distributions within a sector are symmetric across the two countries and if wages are the same, then Proposition 2 implies any change in the sector frictions $\tau_s$ around $\tau = 1$ have no first-order effect on overall allocative efficiency. This follows because such changes have no first-order effects on $Ec_s$ and $E\ln c_s$ within a given sector $s$, and therefore no first-order effect overall.

Now consider an increase in $\tau_s$ for a particular sector away from the limit where $\tau_s = 1$. Assuming the condition for Proposition 1 holds, this decreases allocative efficiency $W^A_s$ within sector $s$. The following result states a sufficient condition for overall allocative efficiency
$W^A$ to decrease.

**Proposition 3** Suppose $E_{c_1,s} < E_{c_s} < E_{c_2,s}$ holds within sector $s$, so that $\eta_s^A < 0$ holds from Proposition 1. Suppose wage ratios are not affected by a change in $\tau_s$, as is the case with symmetric counties. If

$$E_{c_s} \leq E_c \equiv \sum_{k=1}^{S} \alpha_k E_{c_k}, \quad (14)$$

and

$$\pi_{1,2,s} \geq \pi_{2,1,s}, \quad (15)$$

then overall allocative efficiency strictly decreases in $\tau_s$, i.e., $\eta^A < 0$.

**Proof.** Note first that cost shares are not affected in sectors other than sector $s$, because relative wages stay the same and only $\tau_s$ changes. The log of overall allocative efficiency equals

$$\ln W^A = \sum_{k=1}^{S} \alpha_k E \ln c_k - \ln \left( \sum_{k=1}^{S} \alpha_k E_{c_k} \right).$$

The slope has the sign of

$$Ec \frac{d \ln W^A}{d \tau_s} = \alpha_s \left[ Ec \frac{d E \ln c_s}{d \tau_s} - \frac{d E_{c_s}}{d \tau_s} \right] = \frac{\alpha_s}{\tau_s} \left[ -Ec \left( \pi_{1,2,s} - \pi_{2,1,s} \right) + \pi_{1,2,s} E_{c_1,s} - \pi_{2,1,s} E_{c_2,s} \right]$$

$$\leq \frac{\alpha_s}{\tau_s} \left[ -Ec_s \left( \pi_{1,2,s} - \pi_{2,1,s} \right) + \pi_{1,2,s} E_{c_1,s} - \pi_{2,1,s} E_{c_2,s} \right]$$

$$< 0.$$  

The first line is immediate. The second line uses Lemma 1 to substitute in for $dE \ln c_s/d\tau_s$ and $dE_{c_s}/d\tau_s$. The third line uses assumptions (14) and (15). The fourth line follows from $E_{c_1,s} < E_{c_s} < E_{c_2,s}$.  

Thus, if the trade friction declines in a sector where the cost share of price is less than the economy wide average (condition (14)), overall allocative efficiency increases. The overall benefit has two sources. First, from the previous subsection, we know that the decrease in $\tau_s$ increases allocative efficiency within the sector. Second, since this particular sector has a lower cost share of price relative to the economy as a whole, reducing the sector $s$ trade friction increases allocative efficiency across sectors, as labor resources are shifted into sector $s$, which is under-utilized.

Note the result also imposes a regularity condition (15) that it is at least as likely for a domestic firm to be first and a foreign firm second, as have the roles be reversed. Given the
trade friction advantage of a domestic firm, it is intuitive that this condition would hold. For the Fréchet case, we can show that (15) holds if and only if (the weak inequality version of) condition (13) holds, which is equivalent to (12), as mentioned above. Thus, for the Fréchet case, we can drop condition (15) in the statement of Proposition 3, because it is implied by the assumption that $E_{c_{1,2,s}} < E_{c_s} < E_{c_{2,1,s}}$.

If a liberalization tends to raise the cost share of a sector that already is above the average share in the economy, then it is possible the liberalization can decrease overall allocative efficiency.\footnote{This scenario might be relevant if the sectors with small markups (high cost shares) are usually those unable to lobby against trade liberalization.} Suppose, for example, there are three sectors. Sector 1 has $n_1 = 2$ and $n_2 = 0$. Sector 2 has $n_1 = 0$ and $n_2 = 2$. That is, for both sectors, there is duopoly for each good, but the duopolists for a given good are in the same country. Hence for these sectors, a change in $\tau$ has no effect on cost shares of price, because it has no effect on the relative cost advantages of the two firms. In sector 3 there are four firms for each good, two in each country, $n_1 = n_2 = 2$. Because there are four firms, cost shares will tend to be higher than in sectors 1 and 2. In addition, reducing $\tau$ will make cost shares in sector 3 higher still. Reducing $\tau$ overall in the economy raises allocative efficiency within sector 3, but inefficiently reallocates labor away from sectors 1 and 2 to sector 3. For the power function case, we have constructed examples where the net effect on overall allocative efficiency of decreasing $\tau$ is negative.\footnote{For example, let $\theta = 1.5$, $\alpha_1 = .25$, $\alpha_2 = .25$, and $\alpha_3 = .5$. $W^A$ increases in $\tau$ on the range $\tau \in [1, 1.4]$.}

In some ways this is an extreme example. For those goods for which there are only two firms, the two firms are in the same country, so a change in $\tau$ has no effect on competition. Consider instead the symmetric setup where the number of firms in each country is the same in each sector, i.e., $n_{s,1} = n_{s,2}$. Suppose the friction $\tau$ is the same across all sectors, and $w_1 = w_2$.

Example 1 in Section 3.1.1 is a special case of this symmetric setup. There, the productivity distribution is degenerate at $x = 1$. As cost shares are constant within a sector, allocative efficiency within each sector equals one, the maximum level. Overall allocative efficiency is less than one, given by the formula (8). An increase in overall $\tau$ reduces allocative efficiency across sectors, as labor moves out of the $s = 1$ that is already distorted. The following proposition generalizes this example to the class of bounded productivity distributions (which includes the power function case).

**Proposition 4** Assume firms draw from a bounded productivity distribution, and suppose countries are symmetric with two sectors, $s^o$ and $s'$. In sector $s^o$ there is one firm in each country. Suppose conditions (12) and (15) hold for sector $s^o$, i.e., $E_{c_{1,2,s}} < E_{c_s} < E_{c_{2,1,s}}$.
and \( \pi_{1,2,s^0} \geq \pi_{2,1,s^0} \). In sector \( s' \) there are \( n_{s'}/2 \) firms in each country. Suppose the friction \( \tau > 1 \) is the same across all sectors. For large \( n_{s'} \), \( \eta^A < 0 \), i.e., overall allocative efficiency \( W^A \) strictly decreases in \( \tau \).

**Proof.** To prove the result, we take the limit of large \( n_{s'} \). For bounded distributions, denote the upper bound as \( M \). As \( n_{s'} \to \infty \), the highest productivity in each country converges to \( M \) almost surely, and this implies that the highest delivered productivity from country 2 converges to \( M/\tau \). But, since the second highest productivity also converges to \( M \), for \( \tau > 1 \), the highest delivered productivity from country 2 is almost surely lower than the second highest productivity in country 1. So, the magnitude of \( \tau \) has no effect on \( E(\ln c_{s'}|n_{s'}) \) or \( E(c_{s'}|n_{s'}) \) in the limit. Since \( c_{s'} \to 1 \) almost surely as \( n_{s'} \to \infty \), \( E(\ln c_{s'}|n_{s'}) \to 0 \) and \( E(c_{s'}|n_{s'}) \to 1 \). Hence, for large \( n_{s'} \), \( E(c_{s'}|n_{s'}) < E(c_{s'}|n_{s'}) \) and (14) holds for sector \( s' \).

With (12), (15) and the fact that \( \tau \) has little effect on expected cost shares of sector \( s' \), all conditions of Proposition 3 hold for large \( n_{s'} \), and the result follows.

We use numerical methods to show that Proposition 4 generalizes to any value of \( n_{s'} \geq 2 \) and for the four classes of distributions that we have considered, with symmetric countries. Proposition 4 formalizes an intuitive idea. A uniform reduction in trade frictions across all sectors will have the biggest effect in raising cost shares in highly concentrated sectors, because cost shares in unconcentrated sectors would tend to be close to one to begin with. Thus, cost shares in concentrated and unconcentrated markets get compressed closer together, improving allocative efficiency across sectors.

### 4 The Link to the ACR Welfare Condition

In this section, we connect the analysis to the condition for welfare change derived in ACR. We show that the ACR condition exactly tracts the productive efficiency component of welfare, under general assumptions on the distribution of productivity draws, leaving the effect on allocative efficiency as a residual. We also consider an illustrative numerical example.

Following the notation in ACR, let \( \lambda \) denote country 1’s spending share on domestic goods. On account of the Cobb-Douglas utility assumption with equal weight on each good, here \( \lambda \) will also equal the fraction of all goods consumed in country 1 that originate in country 1. This equals the probability that the most efficient firm at 1 has a higher cost-adjusted productivity than the most efficient firm at 2, i.e., that \( x^*_1 \geq x^*_2/(w\tau) \). The probability of this event is

\[
\lambda = \int_0^\infty \int_0^{\tau w x_1} h^\star(x^*_1, x^*_2) dx_2 dx_1^*,
\]

(16)
where $h^*(\cdot, \cdot)$ is the joint distribution of the highest productivities at each location. (We allow this to be fully general here, in particular $x_1^*$ and $x_2^*$ need not be independent.)

A key component of the ACR analysis is the trade elasticity $\varepsilon$. They define this to be

$$\varepsilon \equiv \frac{\partial \ln \left( \frac{R_{21}}{R_{11}} \right)}{\partial \ln \tau}. $$

This partial derivative measures how the ratio (in logs) of import spending to domestic spending changes with changes in the log of the trade friction $\tau$, holding fixed wages in each country. Using the Cobb-Douglas assumption to substitute in $R_{21}/R_{11} = (1 - \lambda)/\lambda$, along with minor manipulations yields\(^6\)

$$\varepsilon = -\frac{\tau}{(1 - \lambda)} \frac{\partial \lambda}{\partial \tau}. \quad (17)$$

The contribution of ACR is to show that the proportional effect on welfare of a trade liberalization across a wide variety of different models equals the proportional effect on $\lambda^{1/\varepsilon}$. Taking logs to express the statistic as an elasticity, we define the ACR statistic to be

$$Stat^{ACR} \equiv \frac{1}{\varepsilon} \frac{d \ln \lambda}{d \ln \tau}. \quad (18)$$

Note that in defining this statistic, we do not take into account how changes in $\tau$ affect $\varepsilon$. Also note that in computing this statistic, we must take into account the general equilibrium effect of a change in the relative wage $w$ between the foreign country and the domestic country, i.e., the effect on $\lambda$ from a change in $\tau$ includes the direct effect of the change in $\tau$ as well as the indirect effect through any change in $w$.

We can write the log of the productive efficiency index (6) as

$$\ln W_i^{Prod} \equiv E \ln \left( \max \left\{ x_1^*, \frac{x_2^*}{w^*} \right\} \right). \quad (19)$$

Our result is

**Proposition 5** The ACR statistic equals the elasticity of the productive efficiency index with respect to changes in the trade friction, i.e.,

$$Stat^{ACR} = \eta_i^{Prod} \equiv \frac{d \ln W_i^{Prod}}{d \ln \tau}. $$

---

\(^6\)ACR assume the import demand system is such that $\varepsilon$ is a constant (Assumption R3 of that paper). This allows them to use variations in $\lambda^{1/\varepsilon}$ to examine welfare effects of large policy changes. Here we consider local policy changes, and for this purpose we need not assume $\varepsilon$ is constant.
Proof. Using (19), we can write the log of the productive efficiency index as
\[
\ln W_i^{\text{Prod}} = \int_0^\infty \int_0^{w\tau x_1^*} h^*(x_1^*, x_2^*) \ln(x_1^*)dx_2^*dx_1^* + \int_0^\infty \int_0^\infty h^*(x_1^*, x_2^*) \ln(x_2^*)dx_2^*dx_1^*
\]
\[
- (1 - \lambda) (\ln (w) + \ln (\tau))
\]
Straightforward differentiation yields
\[
\frac{d \ln W_i^{\text{Prod}}}{d \ln \tau} = - (\ln (w) + \ln (\tau)) \frac{d (w\tau)}{d \ln \tau} \int_0^\infty x_1^* h^*(x_1^*, w\tau x_1^*) dx_1^*
\]
\[
+ (\ln (w) + \ln (\tau)) \frac{d \lambda}{d \ln \tau} - (1 - \lambda) \frac{d (\ln (w) + \ln (\tau))}{d \ln \tau}
\]
\[
= - (1 - \lambda) \left( \frac{d \ln w}{d \ln \tau} + 1 \right).
\]
In the last equality, we have used \(d\lambda/d\tau = d (w\tau)/d\tau \times \int_0^\infty x_1^* h^*(x_1^*, w\tau x_1^*) dx_1^*\), which is straightforward from (16). Turning to the ACR statistic, substituting (17) into (18) yields
\[
\text{Stat}_{ACR} = - \frac{1 - \lambda \frac{d \lambda}{d \tau}}{d \tau}
\]
\[
= - \frac{d (w\tau)}{d \tau} \int_0^\infty x_1^* h^*(x_1^*, w\tau x_1^*) dx_1^* \times \int_0^\infty x_1^* h^*(x_1^*, w\tau x_1^*) dx_1^*
\]
\[
= - (1 - \lambda) \left( \frac{d \ln w}{d \ln \tau} + 1 \right).
\]

Recall the point made earlier that if firms draw from the Fréchet, then as the number of firms get large the model converges to BEJK. In that limit, changes in \(\tau\) have no effect on mark-up distributions; the entire effect of \(\tau\) on welfare operates through the effect on the productive efficiency. Since ACR analysis includes BEJK as a special case, we know the ACR statistic must tract the productive efficiency in the BEJK limit of our model. Proposition 5 shows that the ACR statistic tracts the productive efficiency more generally in our model.

Overall welfare depends upon both the productive and allocative efficiency. The analysis of this paper so far has shown the contributions of these two components to welfare can be widely different. Recall from Proposition 2 that when \(\tau\) is close to one and firms draw from symmetric productivity distributions, an increase in \(\tau\) has no first-order effect on allocative efficiency but does have a first-order effect on the productive efficiency. That is, to a first approximation, the total welfare effect is entirely the productive efficiency. The fact that
the ACR statistic does not pick up the allocative efficiency component here is not a problem, because the unaccounted-for effect is virtually zero.

Recall next the example in Section 3.1 where all firms have the same productivity, and some goods have one firm in each country and others have two. Here a change in $\tau$ has no effect on the productivity efficiency component of welfare, but does affect allocative efficiency, given in equation (8). Here the ACR statistic, tracking the productivity effect which is zero in this case, will miss all of the effect on welfare.

Keeping in mind that in general the range of possible outcomes is wide, it is nevertheless interesting to consider the numerical example highlighted in ACR as an illustrative case. In particular, appealing to the literature and U.S. data, ACR consider the example where the domestic spending share is $\lambda = .93$ and the trade elasticity is $\varepsilon = -5$. We consider the symmetric two country model, varying the total number of firms for each good between two, four, and six firms, with half of the firms in each country. We also vary the underlying productivity distribution across the four different functional form classes. We solve for the trade friction $\tau$ and the similarity parameter $\theta$ of the various distribution functions (draws are more similar the higher $\theta$) to match $\lambda = .93$ and $\varepsilon = -5$.

Table 1 reports the elasticities for components of welfare, with respect to changing the friction $\tau$. The expected cost share is also reported. First note that the elasticity of the productive efficiency index $W^{Prod}$ is essentially constant at $-.07$ throughout all the cases, i.e., a one percent increase in $\tau$ lowers $W^{Prod}$ by .07 percent. This component of welfare change is what is being captured by the ACR statistic.

We now turn our attention to the elasticity of the allocative efficiency index $W^A$. For two firms and Fréchet, it equals $-.052$, roughly the same order of magnitude as the elasticity of the productive efficiency index. Thus, the total effect on welfare, productive and allocative efficiency combined, is almost twice as large as predicted by the ACR index, which only picks up productive efficiency. But notice what happens when we go to four firms with the Fréchet. The elasticity for $W^A$ shrinks by an order of magnitude and is negligible.

Next consider the power function case. With duopoly, the elasticity for $W^A$ is $-.036$, half the value for $W^{Prod}$. The interesting result here is that increasing firm counts for the power function case leaves the $W^A$ elasticity roughly the same. To understand why the power and Fréchet cases are so different, it is important to note that for the power case the trade elasticity is increasing in the number of firms, while for Fréchet it is constant.\textsuperscript{7} Recall that the power function has an upper bound. When there are many draws, the highest draws get compressed against the upper bound, reducing the heterogeneity between the top two most efficient producers, increasing the trade elasticity $\varepsilon$. In contrast, with the fat right tail of

\textsuperscript{7}It is straightforward to derive that for the power case, $\varepsilon = -\theta n_1/\lambda$. 25
the Fréchet, there is no analogous compression between the top two producers. The upshot is that when the number of firms is increased in the power case, to keep the trade elasticity constant at $\varepsilon = -5$, the similarity parameter $\theta$ of the power function distribution must be reduced, and this increase in curvature sustains the importance of allocative efficiency.

The log-normal does not have as fat a right tail as the Fréchet. Analogous to the power function case, when the number of firms is increased for the log-normal case, the similarity parameter $\omega$ (equal to the coefficient of variation) must decrease to hold the trade elasticity fixed at $\varepsilon = -5$. With more firms, the elasticity for $W^A$ declines somewhat for the log-normal case. But the pace of decline is nothing like what it is for the Fréchet, and at six firms the $W^A$ elasticity is an economically significant 17 percent of the overall welfare elasticity.

For the Pareto case, when there is one firm in each country then the effect on $\tau$ on allocative efficiency is the same order of magnitude as productive efficiency. But, in an interesting pattern, once there is more than one firm in each country, allocative efficiency is essentially invariant to changes in $\tau$. We take a closer look at the Pareto and Fréchet cases by considering what happens when $\tau = 1$ and we vary the total number of firms $n = n_1 + n_2$. For the Fréchet, we calculate in the separate appendix that

$$E \ln c = \ln \left( \frac{n}{n-1} \right)^{-\frac{\omega}{\theta}} \quad \text{and} \quad E c = \frac{n}{n-1} \sum_{k=0}^{\infty} \frac{1 + k}{1 + k + \frac{1}{\theta}} \left( -1 \frac{1}{n-1} \right)^k.$$

Both variables increase in $n$, and as $n$ goes to infinity we obtain the values in the BEJ limit economy,

$$E^{BEJ} \ln c \equiv \lim_{n \to \infty} E \ln c = -\frac{1}{\theta} \quad \text{and} \quad E^{BEJ} c \equiv \lim_{n \to \infty} Ec = \frac{\theta}{1 + \theta}.$$

For the Pareto, for any $n \geq 2$ (and again $\tau = 1$), we calculate in the separate appendix that

$$E \ln c = -\frac{1}{\theta} \quad \text{and} \quad E c = \frac{\theta}{1 + \theta}.$$

Thus, the Pareto gets immediately to the BEJ limit with only two firms. In the BEJ limit, allocative efficiency is a constant. We conclude that if firms draw from the Pareto and if there are at least two head-to-head competitors in each country then an increase in allocative efficiency is not a source of gains from trade. As Table 1 makes clear, the situation is very different if firms draw from the log-normal or the power function.

We return to the issue of the data requirements for calculating $\eta^A$. Recall formula (11) depends upon the expectations of $c$, conditional on the source location, as well as the
location of the second most efficient firm. Things simplify, however, for the Fréchet, since it is sufficient to condition on the source location, which is something we can hope to observe in the data. Formula (11) also depends upon \( \pi_{1,2} \), the share of goods where the first highest is domestic and the second highest is foreign, as well as the analogous \( \pi_{2,1} \). As shown in the appendix, for the Fréchet, these equal

\[
\pi_{1,2} = \lambda (1 - \lambda) \frac{n_1 + n_2 \left( \frac{E_{x_2}}{\tau w E_{x_1}} \right)^{-\varepsilon}}{(n_1 - 1) + n_2 \left( \frac{E_{x_2}}{\tau w E_{x_1}} \right)^{-\varepsilon}}
\]

\[
\pi_{2,1} = \lambda (1 - \lambda) \frac{n_1 + n_2 \left( \frac{E_{x_2}}{\tau w E_{x_1}} \right)^{-\varepsilon}}{n_1 + (n_2 - 1) \left( \frac{E_{x_2}}{\tau w E_{x_1}} \right)^{-\varepsilon}}.
\]

These formulas depend upon the domestic spending share \( \lambda \) and the trade elasticity \( \varepsilon \) used to calculate the ACR statistics. They also are a function of the firm counts \( n_1 \) and \( n_2 \) at each location for each differentiated good that in practice might be difficult to estimate. (And they depend upon the value \( Ex_i \) of a productivity draw at location \( i \), the friction \( \tau \) and relative wage \( w_i \).) But notice that \( \pi_{1,2} \geq \lambda (1 - \lambda) \) and \( \pi_{2,1} \geq \lambda (1 - \lambda) \), so we have a bound on \( \pi_{1,2} \) and \( \pi_{2,1} \) that is easy to calculate. We can substitute the bound into formula (11) to obtain the following bound,

\[
\eta^A \equiv -\lambda (1 - \lambda) \left[ \left( \frac{Ec_2 - Ec_1}{Ec} \right) \right] \left( 1 + \frac{d \ln w}{d \ln \tau} \right),
\]

where \( Ec_i \) here indicates the expectation conditional on source \( i \). If condition (12) holds so that \( \eta^A \), is negative, and if the productivity distribution is Fréchet, then the bound \( \underline{\eta^A} \) is a lower bound for the (absolute value of) \( \eta^A \), i.e. \( |\eta^A| > |\underline{\eta^A}| \). The statistic \( \eta^A \) requires data on cost shares by source and the import share (and a wage term that may be negligible).

In the last column of Table 1, we calculate \( \underline{\eta^A} \) and compare it to \( \eta^A \). For the Fréchet and \( n = 2 \), the bound \( \underline{\eta^A} \) is -.025, which is half the value of \( \eta^A \) in this case. The bound is not very tight, but it is still relatively informative about the magnitude. For \( n \geq 4 \), where \( \eta^A \) becomes small, the bounds become tight. Next consider other distributions besides the Fréchet. While we have no formal analytic comparison of \( |\eta^A| \) and \( |\underline{\eta^A}| \), we can see how they compare in simulations. Table 1 illustrates for the other distributions both that \( |\eta^A| > |\underline{\eta^A}| \) and that the bound is relatively informative about the general magnitude of \( \eta^A \).
5 Conclusion

In this paper, we have decomposed welfare change into a productive efficiency component and an allocative efficiency component. Under the assumption that preferences are Cobb-Douglas, we have obtained the following results. First, the ACR statistic in elasticity form exactly tracts the elasticity of the productive efficiency component of welfare change. Second, we derive a formula for the allocative efficiency component of welfare change that depends upon differences in the cost shares of price between domestic firms and foreign firms. Third, if domestic firms have an overall cost advantage to serve the local market, then a decrease in the trade friction in a sector increases allocative efficiency within the sector. Fourth, allocative efficiency increases across sectors if the sector experiencing a decline in frictions has mark-ups that are at least as high as the economy-wide average. Fifth, if firms in both countries draw from the same productivity distribution, small trade frictions have a negligible effect on allocative efficiency, so the effects of small trade frictions operate entirely through the productive efficiency, to a first-order approximation. Sixth, large trade frictions can have significant effects on allocative efficiency. In an illustrative numerical calculation using example numbers from ACR, the effects on allocative efficiency can be similar in magnitude to the effects on productive efficiency, depending upon the shape of the productivity distribution, and the number of head-to-head competitors in each country.

We have assumed Cobb-Douglas preferences for tractability. We make some remarks about the more general CES case with elasticity of substitution parameter \( \sigma \). The basic formula (7) for the allocative efficiency also applies to this case. For \( \sigma > 1 \), mark-ups are bounded above by the simple monopoly mark-up \( \sigma / (\sigma - 1) \). Thus, as \( \sigma \) gets large the distribution of mark-ups will be compressed, and this will tend to attenuate the relevance of allocative efficiency.\(^8\) We expect low values of \( \sigma < 1 \) (in which all prices are limit prices same as with \( \sigma = 1 \)) to tend to accentuate the relevance of allocative efficiency. Note that when trade economists work with quantitative models of monopolistic competition, they tend to work with values of \( \sigma \) well above one, so that mark-ups are not implausibly high. With monopolistic competition, as \( \sigma \) goes to one, the mark-up goes to infinity. Nonetheless, in a model of head-to-head competition like the one considered here, it is possible both for \( \sigma \) to be low and to have reasonable mark-ups at the same time. For example, Table 1 illustrates cases where the mark-up is less than 20% of price, while \( \sigma = 1 \) throughout. On the consumer side, substitutability across goods is relatively low, but mark-ups are relatively low because

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\(^8\)Consider for example the case where firms draw from the Fréchet. In this case, as in BEJK, \( \sigma < 1 + \theta \) in order for the price function to be defined. In the limit as \( \sigma \) goes to its upper bound of \( 1 + \theta \), the outcome converges to the BEJK outcome where allocative efficiency is invariant to the level of \( \tau \).
on the firm side, head-to-head competitors have relatively similar costs.

There are a number of fruitful directions for future research. In the analysis here, we have held fixed labor resources at each location and the number of firms. It would be interesting to relax these assumptions, allowing labor mobility across locations and entry and exit of firms. Of particular interest in future research would be quantitative assessment of allocative efficiency’s contribution to the welfare gains from trade.

Appendix

Proof of Lemma 1

For expositional simplicity we set the relative wage \( \omega = 1 \). The more general case is obtained by substituting in \( \tau w \) for \( \tau \) in the expressions below and by adding a \( d(\tau w)/d\tau \) multiplicative term. Differentiating \( Ec \) given by equation (10) yields

\[
\frac{dEc}{d\tau} = (1) - \frac{1}{\tau} [\pi_{1,2}Ec_{1,2} - \pi_{2,1}Ec_{2,1}]
\]

\[
(2) + \int_{0}^{\infty} \int_{0}^{x_1^*} \int_{0}^{x_2^*} h(x_1^*, x_1^{**}, \tau x_1^*, x_2^{**}) dx_2^{**} dx_1^{**} dx_1^*
\]

\[
(3) - \int_{0}^{\infty} \int_{0}^{\tau x_1^*} \int_{0}^{x_2^*} \frac{x_2^*}{\tau x_1^*} h(x_1^*, \frac{x_2^*}{\tau}, x_2^{**}) dx_2^{**} dx_2^{*} dx_1^*
\]

\[
(4) - \int_{0}^{\infty} \int_{0}^{x_1^*} \int_{0}^{x_2^*} h(x_1^*, x_1^{**}, \tau x_1^*, x_2^{**}) dx_2^{**} dx_1^{**} dx_1^*
\]

\[
(5) + \int_{0}^{\infty} \int_{0}^{\infty} \int_{0}^{x_1^*} \frac{\tau x_1^*}{x_2^*} h(x_1^*, x_1^{**}, x_2^{**}, \tau x_1^*) dx_2^{**} dx_1^{**} dx_1^*
\]

\[
(6) + \int_{0}^{\infty} \int_{0}^{x_1^*} \int_{0}^{x_2^*} \frac{x_2^*}{\tau x_1^*} h(x_1^*, x_1^{**}, \tau x_1^*, x_2^{**}) dx_2^{**} dx_1^{**} dx_1^*
\]

\[
(7) - \int_{0}^{\infty} \int_{0}^{\tau x_1^*} \int_{0}^{x_2^*} \frac{x_1^*}{x_2^*} h(x_1^*, \frac{x_2^*}{\tau}, x_2^{**}) dx_2^{**} dx_2^{*} dx_1^*
\]

\[
(8) - \int_{0}^{\infty} \int_{0}^{x_1^*} \int_{0}^{x_2^*} \frac{x_2^*}{\tau x_1^*} h(x_1^*, x_1^{**}, \tau x_1^*, x_2^{**}) dx_2^{**} dx_1^{**} dx_1^*
\]

\[
(9) - \int_{0}^{\infty} \int_{0}^{\tau x_1^*} \int_{0}^{x_1^*} \frac{\tau x_1^*}{x_2^*} h(x_1^*, x_1^{**}, x_2^{**}, \tau x_1^*) dx_1^{**} dx_2^{**} dx_1^*
\]

Terms (6) and (8) are zero. Terms (3) and (7) cancel (2) and (4) cancel, and (9) and (5) cancel. This leaves term (1), as claimed. The derivation of \( dE \ln c/d\tau \) is similar.
Probability Formulas for the Fréchet Case

Following BEJK, under the Fréchet assumption, the probabilities the lowest cost location is at 1 or 2 are given by

\[
\lambda = \frac{n_1 \xi_1}{n_1 \xi_1 + n_2 \xi_2 (\tau w)^{-\theta}}
\]

\[
1 - \lambda = \frac{n_2 \xi_2 (\tau w)^{-\theta}}{n_1 \xi_1 + n_2 \xi_2 (\tau w)^{-\theta}}
\]

In the separate appendix, we calculate

\[
\pi_{1,2} = \lambda \left( \frac{n_2 \xi_2 (\tau w)^{-\theta}}{(n_1 - 1) \xi_1 + n_2 \xi_2 (\tau w)^{-\theta}} \right) = \lambda \left( 1 - \lambda \right) \frac{n_1 + n_2 \xi_2 (\tau w)^{-\theta}}{(n_1 - 1) + n_2 \xi_2 (\tau w)^{-\theta}}
\]

\[
\pi_{2,1} = (1 - \lambda) \left( \frac{n_1 \xi_1}{(n_2 - 1) \xi_2 (\tau w)^{-\theta}} + n_1 \xi_1 \right) = (1 - \lambda) \lambda \left( \frac{n_1 + n_2 \xi_2 (\tau w)^{-\theta}}{n_1 + (n_2 - 1) \xi_2 (\tau w)^{-\theta}} \right).
\]

From the above expression of \( \lambda \) and the definition of trade elasticity \( \varepsilon \), it is straightforward to verify that \( \varepsilon = -\theta \). Also note that for the Fréchet, \( E_{x_2}/E_{x_1} = (\xi_2/\xi_1)^{1/\theta} \). Plugging this into the above expression for \( \pi_{1,2} \) and \( \pi_{2,1} \) results in the formula reported in (20).

References


Robinson, Joan (1934), *The Economics of Imperfect Competition*, Macmillan.


Table 1
Welfare Elasticities from Increasing the Friction $\tau$
Two Symmetric Countries with Half of the Firms in Each Country
Parameters $\tau$ and $\theta$ are Selected to Match $\lambda = .93$ and $\epsilon = -5$

Case A: Productivities Drawn from Frechet

<table>
<thead>
<tr>
<th>Total Number of Firms</th>
<th>$\tau$</th>
<th>$\theta$</th>
<th>Ec</th>
<th>$\eta_{Total}$</th>
<th>$\eta^{Prod}$</th>
<th>$\eta^{A}$</th>
<th>$\eta^{A}$ Frechet Bound</th>
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Case B: Productivities Drawn from Power Function

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<th>Ec</th>
<th>$\eta_{Total}$</th>
<th>$\eta^{Prod}$</th>
<th>$\eta^{A}$</th>
<th>$\eta^{A}$ Frechet Bound</th>
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Case C: Productivities Drawn from Log Normal

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<th>$\theta$</th>
<th>Ec</th>
<th>$\eta_{Total}$</th>
<th>$\eta^{Prod}$</th>
<th>$\eta^{A}$</th>
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<td>2.59</td>
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Case D: Productivities Drawn from Pareto

<table>
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<th>$\tau$</th>
<th>$\theta$</th>
<th>Ec</th>
<th>$\eta_{Total}$</th>
<th>$\eta^{Prod}$</th>
<th>$\eta^{A}$</th>
<th>$\eta^{A}$ Frechet Bound</th>
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