Abstract

We analyse incentive problems in team and partnership structures where the only available information to condition a contract on is some noisy ranking of the partners’ efforts. This enables us to ensure both first best efficient effort levels for all partners and the redistribution of output only among partners. Our efficiency result is obtained for a wide range of cost and production functions. (JEL C7, D7, D8, L2. Keywords: Moral hazard, Teams and Partnerships, Tournaments.)

1 Introduction

We study teams and partnerships in which risk neutral partners jointly produce output which they share among themselves. It is generally accepted that such partnerships are inefficient if the partners’ actions are not verifiable. The argument is that partners shirk because they must share their marginal benefit of effort with others but bear the cost alone. The question is, then, why are there well publicized examples of extremely profitable partnerships which seem to have very little trouble incentivating partners?

We provide an intuitive answer to this question by focusing on team compensation schemes which reward partners on the basis of the relative ranking of their efforts. We find that full efficiency can be obtained under the assumption that some noisy ranking of the partners’ efforts is verifiable—which should be less costly to acquire than cardinal information on efforts. Our result thus helps explaining a phenomenon where economic theory seems to have previously been at odds with reality. In the motivating examples below, there are two main elements

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present on which we build our analysis: team output determines the pool from which prizes are taken and a relative performance ranking is used as the means of allocating these prizes.

Partnerships are the dominant organizational form in several fields of the professional services industry, especially in law, accounting and, until recently, investment banking. The perceived advantages of partnerships are highly valued by dominant firms in these sectors. For instance, when Goldman Sachs was converted into a public company in 1999 it retained important elements of the partnership that it maintained for 130 years previously. Through the first nine months of 2006, the $13.9 billion that Goldman Sachs set aside for salaries and bonuses was roughly 50% of its net revenue. This amounts to bonuses averaging $542,000 for its 26,000 staff. New partners are elected every two years. As their share of this compensation pool is disproportionately larger than the associates’ share, there is a fierce promotion tournament going on among the lower ranks.

Similarly, consider a partnership of lawyers. Rebitzer and Taylor (2007) argue that “these firms are typically structured as partnerships. Attorneys become partners via up-or-out promotion contests.” Promotions are indeed lucrative, as “at Sullivan & Cromwell, for example, according to the American Lawyer, the average partner earned $2.35 million last year” while young lawyers at the same firm have to make do with a meagre $145,000.

In these partnership examples, it is crucial that the tournament prizes are determined by the joint output. This distinguishes our setup from the fixed prizes which are usually studied in the tournaments literature. We capture this feature by using final output as the total sum of prizes awarded in a tournament. In our model, the sharing rule which specifies the percentage of output allocated to the winner, second, etc, is proposed by an arbitrary player and the partnership is only formed if all players agree. A (subgame perfect) equilibrium consists of two elements: a sharing rule which specifies the prizes in the compensation tournament and a set of efforts which determines the output and the probabilities of winning these prizes. There are two main incentive effects of a player increasing effort. On the one hand, additional effort increases the total output of the team of which the agent only receives a share. On the other hand, however, increased effort also raises the agent’s chance of winning the tournament. Relative to the socially optimal level of effort, the first effect leads to under-investment while the second gives the leverage to counter this adverse effect. For the offered sharing rule, these two effects exactly cancel out in symmetric equilibrium and we obtain full efficiency. Moreover, for a sufficiently precise ranking technology, the players who are not ranked first in the tournament also get a positive prize in symmetric equilibrium.

If we allow for limited liability of partners, there is the additional caveat that the (marginal)

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1 Greenwood and Empson (2003) list the percentage of partnerships as form of governance for the top 100 firms per industry as follows: Law 100%, Accounting 56%, Management consulting 17%, Architecture 18%.
2 Reuters, 25-Oct-06.
3 The Wall Street Journal, 1-April-06. We discuss further applications in the concluding section.
chance of winning the tournament must be reasonably responsive to changes in effort.\textsuperscript{4} Thus relative performance compensation schemes under limited liability are only useful if the ranking technology is not too inaccurate. This is arguably easier to achieve in partnerships where professionals share a certain specialization than in general corporations. Our model can therefore explain why partnerships emerge rather between similarly specialized professionals than between professionals with complementary skills.

The remainder of the paper is organized as follows. In section 2, we introduce the model and our team game. Section 3 illustrates the main result through example and in section 4, we prove the efficiency result. We provide an extension to teams and partnerships with more than two members in the appendix which also contains all proofs and technical details.

1.1 Related literature

Alchian and Demsetz (1972) and Holmström (1982) pose the original problem of unattainability of first best efforts in neoclassical partnerships when output is ex ante non-contractible and shared among partners. Legros and Matthews (1993) show that full efficiency can be obtained in some cases, for example, for partnerships with finite action spaces or with Leontief technologies. Nevertheless, they confirm and generalize Holmström (1982)’s result that full efficiency is unattainable for neoclassical partnerships, ie. the case which we study where the production and utility functions are smooth. They show that approximate efficiency can be achieved by mixed-strategy equilibria, where one partner takes an inefficient action with small probability. However, sustaining such equilibria depends crucially on the partners bearing full liability. If the partners are subject to limited liability, the mechanism does not work since it is impossible to impose the large fine on a partner which is necessary to prevent deviation. In our result, full efficiency is attainable even with limited liability, provided that the ranking technology gives a sufficiently high marginal probability of winning for symmetric efforts. Battaglini (2006) discusses the joint production of heterogeneous goods. For multi-dimensional output he finds that implementing the efficient allocation is possible whenever the average dimensionality of the agents’ strategy spaces is lower than the number of different goods produced. He confirms, however, that efficiency is unattainable in the standard case.

The classic reference on efficiency in tournaments is Lazear and Rosen (1981).\textsuperscript{5} They compare rank order wage schemes to wages based on individual output and find that, for risk-neutral agents, both allocate resources efficiently. In their setup, the fixed prizes—and thus their efficiency result—arise from perfectly competitive and centralized markets. However, influential

\textsuperscript{4} Under limited liability, a partner cannot lose more than the amount invested, that is, his share of output is non-negative. Allowing for limited liability is important, because “since the introduction of legal forms such as the limited liability partnership and the limited liability company, unlimited liability partnerships are rarely seen in the professional services.” Levin and Tadelis (2005, p162)

\textsuperscript{5} Kurshid and Sahai (1993) survey the measurements literature which lends support to the tournaments approach by arguing that ordinal statistics are inherently cheaper to produce than cardinal statistics.
studies such as Mortensen and Pissarides (1994) argue convincingly that, for example, the labour market can be viewed as neither centralized nor competitive. Yet any market imperfection leaving positive profits to a firm will render the efficiency result in Lazear and Rosen (1981) inapplicable. In contrast, by endogenizing contests, we do not require any market at all, and therefore, our model credibly lends itself to the analysis of incentive problems in a single partnership regardless of the industry market structure.

Moldovanu and Sela (2001) characterize the optimal prize structures in tournaments. They analyze an exogenously given, fixed budget for prizes and show that for convex cost functions, it is optimal to give positive prizes not only to the winner. Our analysis shows that convexity per se does not lead to the multiple prizes in partnerships. In order to get multiple prizes, the ranking technology has a crucial role as well. Cohen, Kaplan, and Sela (2004) also characterize optimal prize structures. In contrast to Moldovanu and Sela (2001), they allow an agent’s reward to depend on his effort. They do not, however, address the problem of efficiency in partnerships or teams. Galanter and Palay (1991) discuss compensation in elite law firms and argue that promotion-to-partner schemes indeed constitute tournaments. The contests literature has flourished recently and is surveyed by Konrad (2004).

Lazear and Kandel (1992) show that the existence of peer pressure can weaken the free rider problem in teams and partnerships. Their concept of peer pressure among partners captures factors such as guilt, norms, and mutual monitoring which all serve as disciplinary devices. The difference to our approach is that their compensation scheme is not a tournament but consists of constant shares of output. Miller (1997) shows that whenever a single partner can observe and report on at least one other’s actions, efficient efforts can be implemented. Strausz (1999) shows that when agents choose their efforts sequentially and observe the actions taken by their predecessors, there exists a sharing rule which implements efficient efforts. This sharing rule induces players to reveal a shirking partner by influencing final output in a particular way.

## 2 The model

There are two identical, risk neutral agents exerting unobservable individual efforts \( e_i \in [\delta, \infty) \), \( i \in \{1, 2\} \), for some positive \( \delta \) arbitrarily close to zero.\(^6\) However, some noisy ranking of efforts of the partners is assumed to be observable and verifiable. The technology that translates a partner’s effort into his place in the ranking is described, for \( x := e_i/e_j \), by

\[
\Gamma(e_i, e_j) = [f_i(x), f_j(x)]
\]  

\(^6\) The natural effort choice set would be \([0, \infty)\) but we avoid zero effort for technical reasons. We generalize our results to more than two players in the appendix but the full intuition can be understood from the two players case.

\(^7\) This class includes the Tullock success function \( \frac{e_i}{e_i + e_j} \) where \( f_i(x) = \frac{1}{1+x} \).
where \( f_k(\cdot) \) is the probability that partner \( k \in \{i, j\} \) gets the first place in the ranking given his effort \( e_i \) and the rival partner’s effort \( e_j \), and \( f_i + f_j = 1 \). We make the following assumptions on \( f(\cdot) \):

**A1** Symmetry: \( f_i(x) = f_j(1/x) \), for \( x \in [\delta, \infty) \);

**A2** Responsiveness: \( \frac{df_i(x)}{dx} > 0, \frac{df_j(x)}{dx} < 0 \); \( \lim_{x \to \delta} f_i(x) = 0 \) and \( \lim_{x \to \infty} f_i(x) = 1 \);

**A3** \( f(\cdot) \) is twice continuously differentiable.

Assumption **A1** captures the symmetry of the two partners. Assumption **A2** captures the idea that the probability that one partner is ranked first in effort is dependent upon the relative performance of the two partners, measured by \( x = e_i/e_j \). In particular, partner \( i \)'s winning probability is increasing in \( x \), but partner \( j \)'s winning probability is decreasing in \( x \).

If a partnership is formed, the output of the partnership is a function of the total efforts of the partners. Denote the production function as \( y := y(\sum_i e_i) \). The production function is smooth and twice continuously differentiable, with \( y(2\delta) = 0, y'(\cdot) > 0 \) and \( y''(\cdot) \leq 0 \). A partner who receives a share \( s \) of the final output, given his own effort \( e_i \) and the other partner’s effort \( e_j \), gets utility

\[
u_i(e_i, e_j) = sy(e_i + e_j) - C(e_i)
\]

where \( C : [\delta, \infty) \to \mathbb{R} \) is a cost function with \( C(\delta) = 0, C'(\cdot) > 0 \) and \( C''(\cdot) > 0 \).

The objective of a partner is to maximize his own expected utility. Our goal is to analyze whether it is possible to induce the players to exert the efficient level of efforts using a rank order compensation scheme.

### 2.1 Timing

At the first stage, an arbitrary partner initiates the partnership formation by making a proposal to the other partner, offering a sharing rule \((s, 1 - s)\). Without loss of generality let partner 1 be the proposer. Partner 2 then decides whether to accept the proposal or not. If he accepts, the partnership is set up, and the game proceeds to the next stage. If he rejects, the game ends and each player obtains his reservation utility which we normalize to zero. At the second stage, conditional on the formation of the partnership, the partners choose their efforts simultaneously to maximize their own expected utility. Some noisy ranking of efforts is observed and the final output is realized. The final output is distributed between the two partners. The partner who ranks first in efforts obtains the share \( s \) of the final output, and the other partner gets \( 1 - s \).
2.2 (In-)Efficiency benchmark

Efficient actions are those which maximize the total welfare of the two partners absent of any incentive aspects

\[
\max_{(e_i, e_j)} w(e_i, e_j) := y(e_i + e_j) - C(e_i) - C(e_j).
\]

The first best effort level is determined by

\[
y'(2e^*) = C'(e^*)
\]

where \(e^*_i = e^*_j = e^*\). Suppose the two partners fix the shares \((s_i, s_j)\) ex ante, with \(s_i + s_j = 1\). As shown by Holmström (1982), there is no sharing rule that achieves full efficiency and satisfies a balanced budget at the same time. Given the sharing rule \((s_i, s_j)\), the partners choose their efforts to maximize

\[
u_i(e_i, e_j) = s_iy(e_i + e_j) - C(e_i).
\]

Conditional on the formation of the partnership, partner \(i\)'s best response is given by

\[
s_iy'(e_i + e_j) = C'(e_i),
\]

where equilibrium efforts are dependent upon the share \(s_i\) received. The bigger the share received, the higher the effort. However, since \(s_i + s_j = 1\), at least one of the partner always chooses suboptimal effort.

3 Example of efficient team production

In this section we use a specific example to illustrate that the proposed partnership tournament game achieves full efficiency. Let the production function be linear in total efforts

\[
y = \alpha(e_i + e_j), \ \alpha > 0
\]

and assume cost functions to be quadratic

\[
C(e_i) = \frac{1}{2}e_i^2, \ i \in \{1, 2\}.
\]

Let the technology which transforms partners’ efforts into a ranking of efforts be described by the simplified Tullock success function. Partner \(i \in \{1, 2\}\) is ranked first with probability

\[
f_i(e_i) = \frac{e_i}{e_i + e_j},
\]

if he exerts effort \(e_i\) and the other partner exerts effort \(e_j\). The partner who is ranked first receives share \(s\) of the final outcome, and the partner who is ranked second receives share \(1 - s\).
The efficient effort levels are given by \((e_1^*, e_2^*) = (\alpha, \alpha)\). In our tournament game, given the shares agreed on at the first stage, the partners choose their efforts non-cooperatively at the second stage. Thus partner \(i \in \{1, 2\}\) chooses effort \(e_i\) to maximize

\[
u_i(e_i, e_j) = \frac{e_i}{e_i + e_j} s\alpha(e_i + e_j) + \frac{e_j}{e_i + e_j} (1 - s)\alpha(e_i + e_j) - \frac{1}{2}e_i^2.
\] (2)

The equilibrium efforts depend on \(s\) and are symmetric: \(e_i(s) = e_j(s) = s\alpha\). We point out that equilibrium efforts are increasing in the share \(s\). In the extreme case of \(s = 1\), both partners exert the efficient level of efforts. The intuition is straightforward. As one partner increases effort, given the other partner’s effort level, he increases the final output, and at the same time increases his probability of being ranked first. This implies that he has a higher probability of receiving the winning share of a bigger final outcome. The larger the winner’s share \(s\), the higher the incentive for a partner to exert high effort. This incentive reaches its maximum when \(s\) takes its maximum value. This result is similar to—but in our case stronger than—the standard tournaments result with fixed prizes where incentives increase with the spread between the prizes.

Comparing a partner’s objective function (2) with that of a social welfare maximizer

\[
w(e_1, e_2) = \alpha(e_1 + e_2) - \frac{1}{2}e_1^2 - \frac{1}{2}e_2^2
\]

we see that in (2), each partner’s incentive to exert effort consists of two parts. The first one is that exerted effort increases total output \(\alpha(e_i + e_j)\), which increases a partner’s payoff no matter whether he is ranked first or second. This motive also exists in the social welfare maximization problem. In a partnership, an agent expects to receive only part of the output and thus does not internalize the positive externality of higher effort on the other partner. This is the usual incentive to free-ride leading to under-investment of effort in partnerships. In our game, however, a tournament is used to allocate the shares. Therefore, partners have an additional motive to exert effort, because higher effort increases the probability of getting a bigger share of the output and decreases the probability of getting the smaller share. With a Tullock success function and \(s = 1\), this extra incentive brought about by the tournament exactly offsets the disincentive from profit sharing.

To illustrate that full efficiency is achieved, we still need to show that it is optimal for partner 1 to propose the share \(s = 1\) at the first stage and for partner 2 to accept. Given the equilibrium efforts \(e(s)\), partner 1 chooses share \(s\) at the first stage to maximize

\[
u_1(s) := u_1(e_1(s), e_2(s)) = \frac{1}{2}(2 - s)s\alpha^2
\] (3)
subject to participation of the second player. Since choosing a minimal effort of $\delta$ generates nonnegative utility, this participation is ensured.\(^8\) As $s = 1$ maximizes (3), the shares are chosen appropriately and the efficient equilibrium effort levels are implemented.

In this example, when a tournament is used as the share allocation mechanism, the optimal allocation rule is to give the entire outcome to the partner who ranks first in efforts. This is not a feature of the efficient mechanism in general. As we show in the next section, for a sufficiently precise ranking technology, the efficient mechanism shares output between the players such that each agent receives a positive prize. To illustrate this result, change the above used simplified Tullock ranking technology to the more general

$$f_i(x) = \frac{1}{1 + x^{-r}} \iff f'_i(1) = \frac{r}{4}.$$  

Now any $r > 1$ will result in sharing rules giving a positive share also to the partner coming second in the contest.\(^9\)

### 4 Results

We now show that in the general setup, full efficiency is attainable for linear and concave production functions and a large class of ranking technologies. Recall the production technology

$$y = y(e_i + e_j), \quad \text{with} \quad y(2\delta) = 0, \quad y'(\cdot) > 0, \quad y''(\cdot) \leq 0.$$  

Given the sharing rule $s$ and partner $j$’s effort of $e_j$, partner $i$’s expected utility from exerting effort $e_i$ is

$$u_i(e_i, e_j) = f_i \left( \frac{e_i}{e_j} \right) sy(e_i + e_j) + \left( 1 - f_i \left( \frac{e_i}{e_j} \right) \right) (1 - s)y(e_i + e_j) - C(e_i).$$  

Assuming the existence of interior solutions, this implies for $i = 1, 2,$

$$f'_i \left( \frac{e_i}{e_j} \right) \frac{1}{e_j} (2s - 1)y(e_i + e_j) +$$

$$\left( f_i \left( \frac{e_i}{e_j} \right) s + \left( 1 - f_i \left( \frac{e_i}{e_j} \right) \right) (1 - s) \right) y'(e_i + e_j) - C'(e_i) = 0. \tag{4}$$  

Given $j$’s effort $e_j$, (4) implies that marginally increasing effort $e_i$ has three effects: 1) a marginal increase of final output, 2) a marginal increase of partner $i$’s winning probability, and 3) a marginal increase of effort cost.

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\(^8\) For the case of unlimited liability, the second player’s participation constraint has to be examined separately.  
\(^9\) The sharing optimally proposed for the case of $r = 3$ is $(2/3, 1/3)$. As $r$ gets large and the ranking technology gets very responsive to marginal deviations from equal effort, we approximate equal sharing between partners.
When the symmetric Nash solution exists, \( e_i = e_j = e \) and \( f_i(1) = \frac{1}{2} \). Substituting these, we obtain from (4) that
\[
\frac{f_i'(1)}{e}(2s - 1)y(2e) + \frac{1}{2}y'(2e) = C'(e).
\] (5)

As equilibrium effort \( e \) is a function of \( s \) we write effort as \( e(s) \) and the associated output as \( y = y(2e(s)) \). Intuitively, \( f_i'(1) \) relates to the precision of the tournament’s ranking technology. A high value of \( f_i'(1) \) corresponds to a highly precise ranking technology. A high-precision technology involves a drastic change of the winning probability as the ratio \( e_i/e_j \) approaches 1.\(^{10}\) In the following lemma we begin the analysis of equilibrium effort choice.

**Lemma 1.** *Equilibrium effort \( e \) at the second stage is increasing in \( s \).*

This corresponds to the standard tournament literature result that a partner’s incentive is increasing in the spread between prizes. Here we have replaced the fixed prizes with a fixed sharing of the final output. It is natural that effort is increasing in the share \( s \) since a larger share means a bigger prize for the winner.

**Lemma 2.** *Denoting the first best, efficient efforts by \( e^* \), the sharing rule \( s \) which satisfies
\[
f_i'(1) \frac{1}{e^*}(2s - 1)y(2e^*) = \frac{1}{2}y'(2e^*)
\] (6)
elicits the efficient effort choice at the second stage.*

Limited liability restricts the partners’ possible shares to \( s \in [0, 1] \). The next lemma establishes a threshold precision for the ranking technology for efficiency to obtain under limited liability.

**Lemma 3.** *Under unlimited liability, there always exists a share \( s^* \) such that (6) is satisfied. Under limited liability with \( s \in [0, 1] \), there exists an \( s^* \) that satisfies (6) if \( f_i'(1) \geq \frac{1}{4} \).*

We now show that with unlimited liability, first best can always be implemented.

**Proposition 1.** *Under unlimited liability, full efficiency is obtained. At the first stage, partner 1 proposes a sharing rule \((s^*, 1 - s^*)\) and at the second stage, each partner exerts first best efforts.*

Notice that when \( f_i'(1) \) is sufficiently low, the equilibrium share which induces efficient efforts may exceed 1. Denote by \( \bar{s} \) the solution to (6).

\(^{10}\) The role of output variance in ensuring pure strategy equilibrium existence in Lazear and Rosen (1981) or Nalebuff and Stiglitz (1983) is in our case taken by the assumed differentiability of the ranking technology.
Proposition 2. Under limited liability, if \( s \in [0, 1] \), then full efficiency can always be obtained. If \( s \notin [0, 1] \), then player 1 proposes shares \((s^*, 1 - s^*) = (1, 0)\) and the agents choose suboptimal efforts.

Since for a sufficiently precise ranking technology it is always the case that \( s \in [0, 1] \), there is a threshold precision above which efficiency is guaranteed.

Corollary 1. If the ranking technology is sufficiently precise, that is if \( f'_i(1) \geq \frac{1}{4} \), then full efficiency can always be obtained.

The above propositions show that for the class of production functions studied, as long as the ranking technology is such that the marginal winning probability for symmetric efforts is sufficiently large, full efficiency can always be achieved, even under limited liability. There is no necessity for a budget breaker. The only requirement is the observability of some noisy ranking of efforts. This result does not depend on whether or not one can deduce the other partner’s effort after output is observed. The efficiency result is robust to production functions of other forms, as long as the concept of symmetric equilibrium can be applied.

The condition on the marginal winning probability for symmetric efforts is critical for limited liability. In the symmetric equilibrium we consider here, a partner is only willing to increase his efforts if doing so significantly increases his probability of winning a bigger share of the final output. We emphasize that \( f'(1) < \frac{1}{4} \) is a necessary but not sufficient condition for inefficiency under limited liability. When inefficiency occurs, it depends on the curvature of the production function and the tournament ranking technology. In the example of section 3, if we replace the Tullock success function with the more general function (1) and leave the linear production function unchanged, \( f'(1) = \frac{1}{4} \) is exactly the critical value between full efficiency and inefficiency. If the production function is strictly concave, the difference between \( y(e) \), where \( e = e_1 + e_2 \), and its linear approximation \( y'(e)e \) is positive and the required threshold on the marginal probability of winning decreases.

Finally it is worth pointing out that whenever the ranking technology is precise enough, the player who comes out second also receives a positive share. The exact precision threshold depends on the used production function.

Corollary 2. If the ranking technology is sufficiently precise, that is if \( f'_i(1) > \frac{1}{4} \), then \( s^* < 1 \), i.e. both players receive a positive share. Moreover, the winner’s share \( s^* \) exceeds \( \frac{1}{2} \) and decreases with \( f'_i(1) \).

Although the previous discussion assumes homogenous agents, the existence of sharing rules leading to efficient efforts does not depend on this symmetry. In order to achieve efficiency, however, a mechanism will then generically need to resort to identity-dependent sharing rules specifying different rewards for different players. In the appendix we show that our main findings extend to the case of more than two partners.
Concluding remarks

The model’s most direct application is to partnerships in the professional services. However, since our setup is applicable to any partnership or team structure as long as there exist performance related bonuses paid from the joint product, there is a much wider area of application in virtually any form of cooperation. Instances which match our model precisely are, for example, political contests where the partners in a coalition government work jointly on what may be viewed as maximising the countries’ tax base. A follow-up election is a rank order tournament which may not confirm all coalition parties in office. Joint research among tenure track researchers at a university which may grant tenure based on the perceived quality of individual research is a further example. Apart from this promotion aspect, publishing itself can be viewed as a tournament: in the sciences and engineering, several researchers usually work together on one project. Although they share joint output, the most important contributor is typically made the first author of a resulting publication.

Warfare history and the vassal system are rich sources for anecdotes. In the Thirty Years’ War, for instance, Albrecht von Wallenstein, a Bohemian nobleman, was rewarded for his services to the Catholic emperor Ferdinand II against the Danish King Christian: in 1628, Wallenstein received the duchy of Mecklenburg where combined forces of Wallenstein and Count Tilly had previously defeated the Danes.11 Similarly, during the Napoleonic wars, a Royal Navy man-of-war capturing an enemy prize split the proceeds according to a fixed rule specified by the Cruizers and Convoys Act (1708). It granted three eighths to the ship’s captain, one eighth each to the (increasingly numerous sets of) wardroom, principal warrant and petty officers, and the final two eighths to the crew.12 Promotion, thus, was more than a source of pride.

Finally, a team or partnership in which information quality is the key in the selection of projects is a natural application. Partners spending more effort in collecting information increase the quality of information, hence the quality of project selection, and thus expected output. Therefore, a partner with good information plays a more important role in decision making than the ones with bad information who would rather rubber stamp the suggestion of the former. The partners, as a matter of fact, engage in a contest in the collection of information about projects. In order to provide incentives to exert effort, a larger share of output should be granted to better informed partners.13

For a sufficiently precise ranking technology, the players who are not ranked first in the tournament also get a positive prize in symmetric equilibrium. This is a property which, together with limited liability, corresponds to actual compensation schemes. If we were to compare two similar limited liability partnerships, one with a very precise ranking technology and one where

11 F. Schiller gives a literary but historically accurate account in his 1792 History of the Thirty Years’ War.
12 Kert (1997) details related incentive systems existing in other navies and the private sector.
13 Incidentally, many TV game shows—for example the CBS reality show Survivor—have this format. Players start out in teams but the final prizes are awarded to individuals based on their earlier team performance.
rankings are rather imprecise, we could obtain two very different sharing rules which nevertheless both implement first best efforts. In the case of the precise ranking technology, all partners could get nearly the same share of output while for the imprecise effort ranking a sharing rule giving all output to the winner could be optimal. Thus, in our setting, decidedly egalitarian looking partnerships may actually arise from pure efficiency considerations.

Appendix

More than two partners

In this subsection we show that our full efficiency result is not an artifact of two member partnerships, where one can deduce the effort level of the other partner from observing the output. We prove that given the formation of a partnership of $n$ members, the mechanism which allocates the entire final outcome among the partners achieves first best efforts. Moreover, we show that this efficient sharing rule will be proposed at the first stage of the game.

At the first stage, partner 1 proposes a sharing rule of $(s_1, s_2, \cdots, s_n)$. Suppose that the production function takes the form

$$y(e) = \alpha \sum_i e_i$$

and the winning probability technology is described by the Tullock success function which specifies the probability of partner $i$ coming out first in the ranking of effort with probability

$$p_i^1(e) = \frac{e_i}{e_1 + e_2 + \cdots + e_n}, \quad i \in \{1, \ldots, n\},$$

which is also the probability that partner $i$ wins the share $s_1$ of the final outcome. Denote $e_{-i} := (e_1, \ldots, e_{i-1}, e_{i+1}, \ldots, e_n)$. Then, partner $i$ wins share $s_2$ with probability

$$p_i^2(e) = p_1^2(e) \cdot \frac{e_1}{e_2} + \cdots + p_n^1(e) \cdot \frac{e_1}{e_n}.$$ 

Probabilities $p_i^3, \ldots, p_i^n$ are given similarly. Then, given that a partnership of $n$ partners is set up, an allocation rule that assigns the entire output to the partner who is ranked first in effort elicits first best efforts.

**Proposition 3.** Under the sharing rule $(1, 0, \ldots, 0)$, agents choose efforts efficiently.

In the next proposition, we show that for production functions (7), if some sharing rules elicit first best effort levels at the second stage, they are among partner 1’s choice set of sharing rules.

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14 Assume that, if any of the $n$ partners fails to participate in the mechanism, the partnership is not formed and the game ends.

15 Important are not so much the exact probabilities of coming out second, third etc, but the symmetry of the ranking probabilities between agents and the existence of a symmetric equilibrium for any sharing rule.
rules in the first stage. This implies that the sharing rule \((1, 0, \cdots, 0)\) stipulated in proposition 3 is part of a subgame perfect equilibrium. That this sharing rule is indeed chosen is shown in the following proposition.\(^{16}\)

**Proposition 4.** Suppose the ranking technology is such that, in symmetric equilibrium, each partner is ranked at each place with equal probability \(1/n\). If there exist shares \(\hat{s} = (s_1, s_2, \cdots, s_n)\) which elicit first best efforts at the second stage, then such \(\hat{s}\) also maximizes partner 1’s expected payoff at the first stage.

As in the case of \(n = 2\), if the production function is strictly concave, full efficiency is easier to obtain than in the above linear case. A full characterization of the case of arbitrary ranking technologies is difficult for partnerships with more than two partners since the generalization of the ranking technology poses conceptual and technical problems.

**Proofs**

**Proof of lemma 1.** From the implicit function theorem, it follows that

\[
\frac{de(s)}{ds} = \frac{2f_i'(1)}{e(s)} y(2e(s)) \left( C'' - y''(2e(s)) + f_i'(1) \frac{1}{e(s)} (2s - 1) \left( \frac{y(2e(s))}{e(s)} - 2y'(2e) \right) \right).
\]

If (4) is the foc leading to an equilibrium, then an additional derivative wrt \(e_i\) must be negative. This derivative equals

\[
\frac{d^2 u_i(e_i, e_j)}{de_i^2} = f_i'' \left( \frac{e_i}{e_j} \right) \frac{1}{e_i^2} (2s - 1) y(e_i + e_j) + 2f_i' \left( \frac{e_i}{e_j} \right) \frac{1}{e_j} (2s - 1) y'(e_i + e_j) + \left( f_i \left( \frac{e_i}{e_j} \right) s + \left( 1 - f_i \left( \frac{e_i}{e_j} \right) \right) (1 - s) \right) y''(e_i + e_j) - C''(e_i).
\]

At the point of symmetric efforts \(e = e_i = e_j\), we have

\[
f_i''(1) \frac{1}{e^2} (2s - 1) y(2e) + 2f_i'(1) \frac{1}{e} (2s - 1) y'(2e) + \frac{1}{2} y''(2e) - C''(e) < 0. \tag{8}
\]

We will now show that

\[
f_i''(1) = -f_i'(1). \tag{9}
\]

We know that \(f_i(x) + f_j(x) = 1\) for any \(x \in (0, \infty)\). Differentiating this expression wrt \(x\) gives

\[
f_i'(x) + f_j'(x) = 0. \tag{10}
\]

\(^{16}\) As shown in the proof, proposition 4 holds for more general production functions than (7).
We know from assumption A1, that for any $x \in (0, \infty)$, $f_i(x) = f_j(1/x)$. Differentiating this expression gives

$$f'_i(x) = -\frac{1}{x^2} f'_j\left(\frac{1}{x}\right). \quad (11)$$

Plugging (11) into (10), we obtain

$$f'_i(x) - \frac{1}{x^2} f'_i\left(\frac{1}{x}\right) = 0.$$

Differentiating this identity wrt $x$, we get

$$f''_i(x) - \left((-2)x^{-3} f'_i\left(\frac{1}{x}\right) - \frac{1}{x^2} f''_i\left(\frac{1}{x}\right)\right) = 0.$$

Therefore, for $x = 1$, we obtain the required equality (9). If we plug this identity back into (8), we get

$$C'' - \frac{1}{2} y''(2e(s)) + \frac{f'_i(1)}{e(s)} (2s - 1) \left(\frac{y(2e(s))}{e(s)} - 2y'(2e)\right) > 0.$$

Since $C'' > 0$, $y''(\cdot) \leq 0$ and $y(x) \geq y'(x)x$, we are done. \hfill \Box

**Proof of lemma 2.** Given $s$ that satisfies (6), at stage 2, partner $i$ chooses effort such that (5) is satisfied. Substituting (6) into (5), one obtains

$$y'(2e) = C'(e)$$

which determines the fully efficient effort level. \hfill \Box

**Proof of lemma 3.** Rewrite equation (6) as

$$4f'_i(1)(2s - 1)y(2e^*) = 2y'(2e^*)e^* \quad (12)$$

Since $y(\cdot)$ is concave function, for any $x \in [\delta, \infty)$ holds that $y(x) \geq y'(x)x$. Therefore, whenever $4f'_i(1) \geq 1$, there exists $e^* \in [0, 1]$ that solves (12). \hfill \Box

**Proof of proposition 1.** Expecting the symmetric equilibrium effort levels $e_1(s) = e_2(s) = e(s)$ that are determined by equation (5), in choosing the optimal share $s$, partner 1’s expected utility is

$$u_1(s) = u_1(e(s), e(s)) = f_1(1)sy(2e(s)) + (1 - f_1(1))(1 - s)y(2e(s)) - C(e(s))$$

$$= \frac{1}{2} y(2e(s)) - C(e(s))$$

subject to partner 2’s participation constraint which we will verify later for the derived equilibrium. Solving partner 1’s utility maximization problem gives us the following first order
condition
\[
\frac{d}{ds} u_1(s) = (y'(2e(s)) - C'(e(s))) \frac{de(s)}{ds} = 0.
\]
Partner 1 chooses \( \hat{s} \) such that
\[
y'(2e(\hat{s})) = C'(e(\hat{s})).
\]
Therefore the sharing rule which implements efficient efforts maximizes player 1’s utility. It is now easily verified that partner 2’s participation constraint holds because in symmetric equilibrium both players expect the same utilities and by offering \( s = \frac{1}{2} \), the proposer can ensure non-negative utility.

**Proof of proposition 2.** If \( \tilde{s} \in [0, 1] \), then the proof is exactly as the proof of the previous proposition. If there is no \( \tilde{s} \in [0, 1] \) which solves (6), meaning \( \tilde{s} > 1 \). Since \( de(s)/ds > 0 \) limited liability equilibrium efforts are necessarily lower than the efficient levels \( e^* \). Therefore we have, for the optimal sharing rule \( s^* \),
\[
\frac{d}{ds} u_1(s^*) = (y'(2e(s^*)) - C'(e(s^*)) \frac{de(s^*)}{ds} > 0,
\]
where \( de(s^*)/ds > 0 \) by lemma 1 and \( y'(2e) > C'(e) \) for any \( e < e^* \) from our curvature assumptions on production and cost functions. This implies that the optimal \( s^* = 1 \).

**Proof of proposition 3.** Given the allocation rule, partner \( i \) chooses effort \( e_i \) receives share 1 with probability \( p_i(e_i, e_{-i}) \) and receives a share of 0 otherwise. He chooses his effort \( e_i \) to maximize
\[
U_i(e_i, e_{-i}) = \frac{e_i}{\sum_{j=1}^n e_j} y(\sum_{i=1}^n e_i) - C(e_i) = \alpha e_i - C(e_i).
\]
The optimal choice of \( e_i \) is determined by the first order condition
\[
\alpha = C'(e_i)
\]
which implies that the equilibrium effort level is equal to the first best effort level.

**Proof of proposition 4.** Since in symmetric equilibrium, each player expects a payoff of \( \frac{1}{n}s_1 + \frac{1}{n}s_2 + \ldots + \frac{1}{n}s_n \), partner 1 faces the following maximization problem at the first stage
\[
u_1(s) = u_1(ne(s)) = \frac{1}{n}y(ne(s)) - C(e(s)).
\]
Partner 1 chooses the \( s \) that satisfies the first order condition, which is
\[
(y'(ne(s)) - C'(e(s))) \frac{de(s)}{ds} = 0
\]
The \( s \) solving this first order condition elicits the first best effort level.
References


