Credit Market Development and Human Capital Accumulation*

Wai-Hong Ho†

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Abstract

In a two period overlapping generations economy with asymmetric information, we investigate the interaction between credit market development and human capital accumulation. As is typical, young borrowers supply their endowed unit of labor time to earn wage income which is used as internal funds. In contrast to conventional setups, young lenders distribute theirs between acquiring education and working for earnings. Through identifying the risk types of borrowers by a costly screening technology, a self selection equilibrium is achieved. We find that, at steady state, lenders will allocate more time to acquire education if the cost of screening borrowers falls. Furthermore, a longer duration of lenders’ schooling time suppresses borrowers’ incentive to cheat thereby enabling lenders to screen less frequently. These results suggest the possibility of a mutual beneficial interplay between credit market development and human capital accumulation. At last, our comparative static analysis show that improvements on borrowers’ investment technology promote human capital accumulation but that on lenders’ does the opposite.

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†Assistant Professor, Department of Economics, Faculty of Social Sciences and Humanities, University of Macau, Macao, China Tel: 853-3978469; Fax: 853-838312. Email Correspondence: whho@umac.mo.
I. Introduction

After decades of research, human capital now is widely viewed as one of the crucial elements in a nation’s take-off process. While human capital may accumulate through an array of important channels like public funding and private funding (see Glomm and Ravikumar (1992)), education that requires time to build has also drawn much attention from economists, see for example Lucas (1988) and Azariadis and Drazen (1990). In a standard model of this sort, individual agent allocates his endowed unit of labor time between skill augmentation and current production. The following trade-off pins down the optimal duration of schooling: while allocating more time to schooling can increase effective labor supply and hence higher wage income in the future, the foregone current labor income as the opportunity cost must go up. Essentially, the longer is the time for education, the higher is the economic growth.

As is well known, the problem of asymmetric information is a common characteristic in credit markets of many underdeveloped countries. The last decade has witnessed a rising interest in the relationship between asymmetric information and economic growth. Though there is a wide variety of ways capturing the essence of information asymmetry in a growth model, a more or less common thesis is reached: different forms of informational imperfection erect hurdles blocking the flow of productive resources from lenders to borrowers. For instance, Azariadis and Smith (1993) and Bencivenga and Smith (1993) find that the credit rationing problem can prohibit the first best loan contract. In a model allowing for both rationing and screening contracts, Bose and Cothren (1996, 1997) show similar adverse effects of asymmetric information on growth. Ho and Wang (2005) find that the presence of asymmetric information is growth retarding in an endogenous growth model with tax-financed public capital. The basic essence behind these models is captured by the equilibrium loan contracts and their impact on capital accumulation. As is typical in these models, credit market is composed of borrowers and lenders looking for pairwise investment opportunities. Since borrowers’ investment risk levels cannot be observed by lenders\(^1\), the revelation principle is called for to establish a type/identity

\(^1\)Lenders cannot observe the actions taken by borrowers in models with moral hazard. For contributions in this regard see Tsiddon (1992) and Fu (1996).
revealed equilibrium. As such, the optimal loan contracts in models with credit rationing (see Bencivenga and Smith (1993), Bose and Cothren (1996, 1997)) exhibit the feature that some borrowers are excluded from the credit market whereas in models with screening (see Bose and Cothren (1996, 1997), Ho and Wang (2005)), a fraction of borrowers are screened by lenders along with a smaller amount of loan size. Since sustaining such a self-selection mechanism wastes productive resources, capital accumulation is impaired.

By combining these two strands of literature, this paper examines the interplay between credit market development and human capital accumulation in a two period overlapping generations model. In particular, young lenders allocate a fraction of their endowed time unit to work for wage income which will be invested in a credit market with the presence of asymmetric information between lenders and borrowers. The rest of their time is used to accumulate human capital which allows them to enjoy higher wage income from supplying more effective labor next period. In contrast with lenders, borrowers supply their endowed labor time inelastically for wage earnings which is used as internal investment funds. In this environment, the main question we want to address is: how would the lenders’ duration of schooling time interact with credit market development?\(^2\) We find that, at steady state, reducing the screening cost will induce lenders to devote more time to acquiring education. This result seems consistent with the causal observation that duration of schooling time is generally longer in countries with more developed credit market. In addition, a longer duration of lenders’ schooling time suppresses borrowers’ incentive to cheat and hence enables lenders to screen with lower frequency. If we interpret the screening probability as indicating the quality of institutional arrangement of contractual enforcement, being high screening probability associated with low quality of institutional arrangement, this result suggests that education may improve the financial institution quality.\(^3\) While the beneficial effect of credit market development on human capital accumulation is well known in the literature (see

\(^2\)It is useful to point out that, as the lenders invest in education, they are not subject to any kinds of borrowing constraints that play critical roles in the literature on education and growth, like those in Gregorio (1996) and Fender and Wang (2003). Our focus is on the interaction between credit market frictions and lenders’ allocation of time but not their allocation of funds.

\(^3\)The proposition that education or high income may cause institutional improvement is not new. There is a small but growing empirical literature on this topic, including Barro (1999) and Glaeser et al (2004).
Gregorio (1996)), the current paper proposes a channel through which education may exert a positive impact on credit market development\(^4\).

More precisely, we consider an overlapping generations economy populated with heterogeneous agents who live for two periods. Lenders who are endowed with one unit of time in both periods are responsible for human capital accumulation and credit provision. When lenders are young, they first have to decide the fractions of time allocated to schooling and working. Subsequently, they supply their wage income as the source of external investment funds in the credit market. When they are old, they consume the sum of the wage income they earn when old and the interest payment from the loan they made when young. Human capital accumulation involves decreasing returns with respect to the fraction of time allocated to education by young lenders. Borrowers who are endowed with investment projects work when they are young. Their own wage incomes are used as internal funds to implement their projects. Since project outputs are strictly increasing with the amount of inputs, borrowers approach lenders for extra amount of investment funds. However, the risk types of the investment projects are known only to their owners not to the lenders. It is this feature which gives rise to the problem of asymmetric information. By adopting a costly screening technology, lenders can design loan contracts to reveal borrowers’ true types. The final output in the economy is produced according to a constant returns technology with respect to the physical capital and the efficient units of labor.

This paper has the following major findings. First, lenders’ screening probability is decreasing with their duration of schooling. This is mainly due to the following property of borrowers’ payoffs: they are decreasing with lenders’ duration of schooling time. It can be shown that the payoffs to high risk type borrowers who self-claim to be a low risk type will be hurt more relative to those high risk type borrowers who truthfully reveal their type. As a result, high risk type borrowers have lower incentive to pretend as a low risk counterpart after lenders’ schooling time goes up. When borrowers’ incentive to cheat is lowered, lenders can screen less frequently. Secondly, when credit markets

\[^4\text{The term “credit market development” is referred to a process in which a credit market experiences either decreasing cost of screening or declining probability of screening in the paper.}\]
become more efficient in the sense that less resources are needed to screen out borrowers, physical capital stock piles up, driving down the equilibrium rental rate of capital and hence the returns on loan business, which makes lending less attractive. As a response, lenders would choose to devote a longer period of time to acquiring education. After obtaining these results, we perform other comparative static analysis for some model parameters, including the levels of lenders’ and borrowers’ production technologies, to study their impacts on the duration of schooling. We find that an improvement on the technological level of borrowers’ production technology has a positive impact on lenders’ incentive to go schooling. However, an improvement on the technological level of lenders’ home (a default) production technology tilts the balance between working and schooling in favor of the former.

For sure, this is not the first paper which studies the relationship between financial market development and human capital accumulation. Some recent papers on economic development have also addressed this issue. For example, Galor and Zeira (1993) show that in the presence of credit market frictions and indivisibilities in investment in human capital, the initial distribution of wealth affects aggregate output and investment both in the short run and in the long run. Gregorio and Kim (2000) are closest to our work. They study an endogenous growth model in which credit markets affect time allocation of individuals with different educational abilities. In particular, by comparing two disjoint models in which one has credit market frictions and one does not, they show that in the latter case more able people would specialize in studying and the less talented would specialize in working. In contrast, we show that the avenue along which the interaction between credit market development and education takes place can be two way - when credit market development facilitates human capital accumulation, it can also benefit from the latter.

The rest of this paper is organized as follows. Section 2 lays out the structure of the overlapping generations framework and the human capital accumulation process. Section 3 is about the equilibrium loan contracts. In Section 4, we derive the steady state. Section 5 contains some comparative statics analyses. We conclude and discuss some possible extensions in section 6. All technical proofs are relegated to the appendix.
II. The Model

Our model is similar to Bencivenga and Smith (1993) and Bose and Cothren (1996, 1997) except that lenders now play dual roles - channelling credits and accumulating human capital. In the economy, there is an infinite sequence of two-period lived overlapping generations. All generations are identical in size and composition. The population size of each generation is normalized to one. Young agents in each period are equally divided into lenders and borrowers. All agents have one unit of labor to supply. When a lender is young, he uses a fraction of his time for works to earn wage income which becomes the source of external investment funds in the credit market. The rest of his time is used for education. When he becomes old, he works and consumes. Let $e_{t+1}$ be the human capital produced in period $t$ and to be used in period $t+1$. $e_{t+1}$ is measured by units of effective labor and is owned by old lenders born in period $t$. Human capital during the lifetime of an lender evolves according to:

$$e_{t+1} = (1-\alpha)e_t + \prod(n_t); 0 < \alpha < 1$$  \hspace{1cm} (1)

where $\alpha$ is the depreciation rate of human capital and $n_t$ is the fraction of time allocated to accumulate human capital. $e_t$ is the average level of the parental generation’s human capital. For simplicity, we assume that $\prod(n_t) = \gamma n_t^\theta$ where $0 < \theta < 1$ and $1 \leq \gamma$. We will take $e_0$ and $n_0$ to be given initial conditions. Notice that, since population size has been normalized to 1, the total amount of human capital produced at period $t$ is also equal to $e_t$.

Young borrowers earn the real wage rate, $w_t$, by supplying their labor to firm. When they are old, they implement their investment projects which require exactly one unit of labor time to convert inputs of consumption good into capital. However, the risk levels of borrowers’ investment projects are not identical. Specifically, $\lambda$ fraction of borrowers have type $H$ projects with a lower probability of success and $1-\lambda$ fraction of borrowers have type $L$ projects. A type $i \in \{L, H\}$ investment projects can success with probability $P_i$ to convert one unit of time $t$ output into $Q$ units of capital goods at time $t+1$. The investment projects may fail with probability $1-P_i$ and produce zero capital goods. We assume that $0 \leq P_h < P_l \leq 1$. The owner of a successful investment project will become
a firm operator at his old age.

A young lender can lend his wage to a borrower in exchange for consumption goods in the next period. Another option he has is to convert his time $t$ wage into $Q\varepsilon$ units of time $t$ capital by using a risk free technology where $\varepsilon$ is assumed to be sufficiently smaller than $P_h$ to guarantee that loan business will take place between lenders and borrowers. An old lender works for firm to earn wage income. Since this is the end period of his life, he simply consumes all his income. To obtain maximum simplicity, both lenders and borrowers are assumed risk neutral and consume only when they are old.

In this model, the total effective labor supply at period $t$, $L_t$, comes form the young borrower, the young lenders who supply the fraction of labor not used for human capital accumulation and the old lenders who supply all human capital measured by effective labor. Its value is $L_t = 0.5 + 0.5(1 - n_t) + 0.5e_t = 0.5(2 - n_t + e_t)$.

Now we will turn to the description of the credit market. In each period, a lender offers a set of loan contracts designed for different type of borrowers. If these contracts are not dominated by others, a borrower will approach this lender and select a contract. Following Bencivenga and Smith (1993) and Bose and Cothren (1996, 1997), each borrower can apply to one lender only to impose a upper bound on the loan size. Furthermore, the credit market is assumed to be perfectly competitive and hence the lenders' economic profit is zero. Since lenders cannot observe the risk types of borrowers, the problem of asymmetric information arises. However, by squandering a $\delta$ fraction of the amount lent, a lender can determine a borrower’s type. Therefore, the maximum amount of loan a lender can make is equal to $(1 - \delta)$ fraction of the lender’s wage when screened. If a borrower is caught mimicking the other type of borrowers, he will be expelled from the credit market. The contracts offered at time $t$ to a type $i \in \{H, L\}$ take the following form, $C^i_t = [(\phi^i_t, R^i_{st}, q^i_{st}), (1 - \phi^i_t, R^i_{nt}, q^i_{nt})]$, where $\phi^i_t$ is the probability that a type $i$ borrower is screened, $R^i_{st}$ and $q^i_{st}$ are the gross loan rate and the loan size for a type $i$ borrowers in the event of screening respectively. $R^i_{nt}$ and $q^i_{nt}$ are the gross loan rate and the loan size respectively when screening does not take place.
A firm produces the final output according to the following production function:

\[ y_t = k_t^{\beta} l_t^{1-\beta} \]  

(2)

where \( y_t \) is the output per firm, \( k_t \) is the capital input per firm and \( l_t \) is the units of effective labor per firm. Firms rent capital and labor competitively from the markets to maximize

\[ k_t^{\beta} l_t^{1-\beta} - w_t l_t - \rho_t k_t \]

where \( w_t \) is the wage rate and \( \rho_t \) is the rental rate of capital. Profit maximization by firms yield:

\[ \rho_t = \beta k_t^{\beta-1} l_t^{1-\beta}, \]  

(3)

\[ w_t = (1 - \beta) k_t^{\beta} l_t^{1-\beta}. \]  

(4)

We assume that physical capital is fully depreciated after one period of use for simplicity. As usual, output can be used as consumption goods or capital goods.

### III. Credit Markets

A type \( i \in \{H, L\} \) borrower of generation \( t \) has an expected payoff function of the following form:

\[
\phi_t^i P_t[Q\rho_{t+1}(w_t + q_{st}^i) - R_{st}^i q_{st}^i] + (1 - \phi_t^i) P_t[Q\rho_{t+1}(w_t + q_{nt}^i) - R_{nt}^i q_{nt}^i] \\
= P_t Q\rho_{t+1} w_t + \phi_t^i P_t[Q\rho_{t+1} - R_{st}^i] q_{st}^i + (1 - \phi_t^i) P_t[Q\rho_{t+1} - R_{nt}^i] q_{nt}^i. \]  

(5)

In equilibrium, borrowers will self select by choosing the contracts that match with their own risk type. In other words, the following incentive compatibility constraint must be satisfied:

\[
\phi_t^h P_h[Q\rho_{t+1}(w_t + q_{st}^h) - R_{st}^h q_{st}^h] + (1 - \phi_t^h) P_h[Q\rho_{t+1}(w_t + q_{nt}^h) - R_{nt}^h q_{nt}^h] \\
\geq (1 - \phi_t^i) P_h[Q\rho_{t+1}(w_t + q_{nt}^i) - R_{nt}^i q_{nt}^i], \]  

(6)

\[
\phi_t^l P_l[Q\rho_{t+1}(w_t + q_{st}^l) - R_{st}^l q_{st}^l] + (1 - \phi_t^l) P_l[Q\rho_{t+1}(w_t + q_{nt}^l) - R_{nt}^l q_{nt}^l] \\
\geq (1 - \phi_t^i) P_l[Q\rho_{t+1}(w_t + q_{nt}^i) - R_{nt}^i q_{nt}^i]. \]  

(7)
Because the credit market is assumed to be perfectly competitive, lenders always earn zero expected economic profit in equilibrium. This zero profit condition can be expressed as

\[ \phi_i^t P_i R_{st}^i q_{st}^i + (1 - \phi_i^t) P_i R_{nt}^i q_{nt}^i = [\phi_i^t Q \varepsilon - \frac{q_{st}^i}{1 - \delta}] + (1 - \phi_i^t) Q \varepsilon q_{nt}^i \rho_{t+1} \]  

(8)

for \( i \in \{H, L\} \). The left hand side of this equation is the expected income from making loans and the right hand side is the forgone income of the loan. The equilibrium contracts must satisfy the following two feasibility conditions:

\[ q_{st}^i \leq (1 - \delta)(1 - n_t) w_t, \]  

(9)

\[ q_{nt}^i \leq (1 - n_t) w_t, \]  

(10)

for \( i \in \{H, L\} \). Now, we define an equilibrium in the credit market as follows:

**Definition 1.** An equilibrium in the credit market is represented by a sequence of \( \{C_L^t, C_H^t\} \) where the contract \( C_i^t = [(\phi_i^t, R_{st}^i, q_{st}^i), (1 - \phi_i^t, R_{nt}^i, q_{nt}^i)] \), for \( i \in \{H, L\} \) maximizes (5) subject to (6) - (10), taking the sequences of \( \{\rho_t\}, \{w_t\} \) and \( \{n_t\} \) as given.

Since \( \varepsilon < P_h < P_l, Q \rho_{t+1} - R_{st}^i \) and \( Q \rho_{t+1} - R_{nt}^i \) must be positive in equilibrium. It follows that (9) and (10) must hold with equality signs in equilibrium, which determine the loan sizes in different states for both types of borrowers. After substituting the equilibrium loan sizes into (8), the zero profit condition of lenders can be rewritten as

\[ (1 - \delta) \phi_i^t P_i R_{st}^i + (1 - \phi_i^t) P_i R_{nt}^i = Q \varepsilon \rho_{t+1}. \]  

(11)

In what follows, we will proceed by assuming that in equilibrium, only the incentive compatibility constraint (6) is binding but not (7). In appendix, we will provide a proof for this assumption after the complete equilibrium contracts are derived.

Since the incentive compatibility constraint for type \( H \) borrowers are never binding, it can be shown that the expected payoff to a high risk borrower is strictly decreasing with the screening probability. Therefore, in equilibrium, it will be optimal to set \( \phi_i^h = 0 \) implying that lenders never screen borrowers who claim themselves to be high risk type. As a result, from (11), the equilibrium loan rate for high risk borrowers is

\[ P_{nt}^h = \frac{Q \varepsilon \rho_{t+1}}{P_h}. \]  

(12)
Hence, the equilibrium loan contract for a high risk borrower can be summarized by
\( C^H_t = (R^H_{nt}, q^H_{nt}) \) where \( R^H_{nt} \) is given by (12), \( q^H_{nt} \) by (10) with equality for \( i = H \) and \( \phi^H_t = 0 \).

Since \( q^i_{nt} = q^h_{nt} = q^l_{nt} \) from (10) with equality sign and \( \phi^h_t = 0 \), the binding incentive compatibility constraint (6) yields us
\[
\phi^l_t = \frac{(1 - n_t)(R^H_{nt} - R^l_{nt})}{(2 - n_t)Q \rho_{t+1} - (1 - n_t)R^l_{nt}} .
\]

(13)

For \( i = L \), we substitute (9), (10) with equality sign, (11) and (13) into (5), it can be shown that the expected payoff to a low risk borrower is strictly increasing in \( R^l_{nt} \).

This result has the following intuitive explanation. A type \( H \) borrower has the incentive to mimic a type \( L \) counterpart whenever no screening happens. In order to suppress this incentive to cheat, setting the loan rate in the event of no screening, \( R^l_{nt} \) as high as possible would be desirable. From the lenders’ zero profit condition, it is easy to see that setting \( R^h_{nt} \) as high as possible is indeed equivalent to setting \( R^l_{nt} \) as low as possible. With \( R^h_{nt} = 0 \), from (11), the equilibrium loan for low risk borrowers in the event without screening is:
\[
R^l_{nt} = \frac{Q \rho_{t+1}}{P_t(1 - \phi^l_t)} .
\]

(14)

If we substitute (12), (14) into (13), we will derive the equilibrium screening probability for low risk borrowers:
\[
\phi^l_t = \phi_t = 1 - n_t \left( \frac{\varepsilon}{P_h} - \frac{\varepsilon}{P_l} \right). 
\]

(15)

Since \( \varepsilon < P_h < P_l \), the equilibrium screening probability is between 0 and 1 for any plausible values of \( n_t \). It is worth noting that the equilibrium screening probability \( \phi_t \) is inversely related to the lenders’ duration of schooling at period \( t \), \( n_t \). In view of the importance of this linkage between \( \phi_t \) and \( n_t \) in the subsequent discussion, we state it in a corollary.

**Corollary 1:** A longer duration of lenders’ schooling time leads to a lower equilibrium screening probability for low risk borrowers.

We explain this result in the following way. When the duration of schooling \( n_t \) goes up, it decreases the gross return to a high-risk borrower if his investment project is
successful regardless of being truthful or not, $Q\rho_{t+1}(w_t + q^h_{nt}) = Q\rho_{t+1}(2 - n_t)w_t$ with $i = \{H, L\}$. Since $R^h_{nt} > R^l_{nt}$ holds in equilibrium, increasing $n_t$ implies a lower percentage decrease in the total loan payment by a high risk borrowers from being honest, $R^h_{nt}q^h_{nt}$, than that from misrepresenting his type, $R^l_{nt}q^l_{nt}$. Therefore, with a longer duration of schooling time, the potential net return to a high-risk borrower from being truthful, $Q\rho_{t+1}(w_t + q^h_{nt}) - R^h_{nt}q^h_{nt}$, will be hurt less relative to that from self-claiming to be the other type, $Q\rho_{t+1}(w_t + q^l_{nt}) - R^l_{nt}q^l_{nt}$. Consequently, a high-risk borrower has smaller incentives to masquerade as a low-risk one after an increase in the duration of schooling time. As a result, the equilibrium probability of screening has to decrease in order to keep the incentive compatibility constraint binding. Since increasing $n_t$ induces lenders to screen less frequently, a longer duration of lenders’ schooling time improves the problem of asymmetric information. If we interpret the screening probability as measuring the quality of institutional arrangements that enforce financial contracts in that economy, the above analysis will suggest that the educational level may potentially explain the varying degree of financial institution quality across countries. Specifically, an economy with high (low) education level should associate with a credit market with high (low) quality of institutional arrangements to enforce financial contracts. Now we summarize the above results in the following proposition.

**Proposition 1.** In each period $t$, the equilibrium contract for type $H$ borrowers is given by $C^h_t = (R^h_{nt}, q^h_{nt})$ with $R^h_{nt} = \frac{Q\epsilon\rho_{t+1}}{P^h_h}$, $q^h_{nt} = (1 - n_t)w_t$, and no screening. The equilibrium contract for type $L$ borrowers is given by $C^l_t = [(\phi_t, R^l_{nt}, q^l_{nt}), (1 - \phi_t, R^l_{nt}, q^l_{nt})]$ with $\phi_t = \frac{1 - n_t}{2 - n_t}(\frac{\epsilon}{P^h_h} - \frac{\epsilon}{P^l_l})$, $R^l_{nt} = 0$, $q^l_{nt} = (1 - \delta)(1 - n_t)w_t$, $R^l_{nt} = \frac{Q\epsilon\rho_{t+1}}{P^l_l(1 - \phi_t)}$ and $q^l_{nt} = (1 - n_t)w_t$.

The equilibrium in the credit market derived in above takes the marginal product of labor, the marginal product of capital, the fraction of time devoted to education and the flow of human capital as given. In the following section, we will establish the general equilibrium of the model.
IV. The Steady State

We first study the lender’s utility maximization problem to look for the optimal fraction of time for education. A representative lender maximizes his utility,

\[ U_t = d_{t+1} \]

subject to the budget constraint

\[ d_{t+1} = Q \varepsilon \rho_{t+1} (1 - n_t) w_t + w_{t+1} e_{t+1} \]

where \( d_{t+1} \) is consumption in period \( t + 1 \) of a lender born in period \( t \). After substituting for \( e_{t+1} \) with (1) and solving for lender’s maximizing problem, the first order condition yields:

\[ n_t = \left( \frac{w_{t+1}}{w_t} \frac{\gamma}{Q \varepsilon \rho_{t+1}} \right)^{1/\sigma} \]

Equation (16) determines the time spent on human capital accumulation, \( n_t \), with given \( w_t, w_{t+1} \) and \( \rho_{t+1} \). Obviously, the rental rate of capital at period \( t + 1 \) is inversely related to \( n_t \), implying that, when the rate of return to physical capital goes up, lenders prefer to spend less time on education. This equation exhibits the well known equilibrium trade-off between returns from receiving education and the forgone current income. Substituting (3) and (4) for \( w_t, w_{t+1} \) and \( \rho_{t+1} \) into (16) gives:

\[ n_t = \left( \frac{\gamma}{Q \varepsilon} \frac{k_{t+1}}{k_t} \frac{l_{t+1}^2}{l_t} \right)^{1/\sigma} \]  (17)

Because none of the borrowers cheats in equilibrium, all projects of both types will be financed. The total number of firms in each period is 0.5\( [\lambda P_h + (1 - \lambda) P_l] \). Therefore, the effective labor supply per firm is equal to:

\[ l_t = \frac{2 - n_t + e_t}{\lambda P_h + (1 - \lambda) P_l} \]  (18)

The total capital stock in period \( t + 1 \) is given by the successful investment projects. Recalling that a fraction of low risk borrowers is screened with probability \( \phi_t \) and physical
capital is fully depreciated after one period of usage, we can determine the total capital stock at $t + 1$:

$$K_{t+1} = 0.5Q\{[\lambda P_h + (1 - \lambda)P_l](2 - n_t) - (1 - n_t)\delta \phi_t(1 - \lambda)P_l\}w_t. \quad (19)$$

It follows immediately that the capital stock per firm at $t + 1$ is given by:

$$k_{t+1} = \frac{Q\{[\lambda P_h + (1 - \lambda)P_l](2 - n_t) - (1 - n_t)\delta \phi_t(1 - \lambda)P_l\}w_t}{\lambda P_h + (1 - \lambda)P_l}. \quad (20)$$

Combining (4) and (18) and substituting for $w_t$ in (20) yields:

$$k_{t+1} = \frac{Q(1 - \beta)\{[\lambda P_h + (1 - \lambda)P_l](2 - n_t) - (1 - n_t)\delta \phi_t(1 - \lambda)P_l\}k_t^\beta}{[\lambda P_h + (1 - \lambda)P_l]^{1-\beta}(2 - n_t + e_t)^\beta}. \quad (21)$$

A competitive equilibrium for the economy is defined as a set of quantities $\{e_{t+1}, k_{t+1}, l_{t+1}, n_t, w_t, \rho_t, l_t, \phi_t, e_t, k_t\}$ satisfying equations (1), (3), (4), (15), (17), (18) and (21). In the steady state equilibrium, all endogenous variables are constant over time, i.e. $w_{t+1} = w_t, \rho_{t+1} = \rho_t, k_{t+1} = k_t, e_{t+1} = e_t, l_{t+1} = l_t, \phi_{t+1} = \phi_t = \phi$ and $n_{t+1} = n_t = n$ and all markets are clear. The steady state equilibrium can be characterized by the following equations:

$$e = \frac{\gamma n^\beta}{\alpha}, \quad (22)$$

$$w = (1 - \beta)k^\beta\left[\frac{2 - n + e}{\lambda P_h + (1 - \lambda)P_l}\right]^{-\beta}, \quad (23)$$

$$\rho = \beta k^{\beta-1}\left[\frac{2 - n + e}{\lambda P_h + (1 - \lambda)P_l}\right]^{1-\beta}, \quad (24)$$

$$\phi = \frac{1 - n_t}{2 - n}(\frac{\lambda}{P_h} - \frac{\epsilon}{P_l}), \quad (25)$$

$$k = \frac{(Q\epsilon \beta \gamma \theta)^{1-\gamma}}{\gamma \theta}(2 - n + e)n^{\frac{1-\gamma}{1-\beta}}\left[\frac{\lambda P_h + (1 - \lambda)P_l}{(2 - n + e)}\right]^\frac{1-\gamma}{1-\beta}. \quad (26)$$

Equations (22), (23), (24), (25) and (27) are simply steady state versions of (1), (4), (3), (15) and (21) respectively. Substituting (18) into (17) and assuming steady state give (26). Furthermore, combining (22), (26) and (27) gives us the following equation:

$$L(n) := \frac{[\lambda P_h + (1 - \lambda)P_l](2 - n) - (1 - n)\delta \phi(1 - \lambda)P_l}{n^{1-\theta}(2 - n + \gamma n^\beta / \alpha)} = \frac{\epsilon \beta}{\gamma \theta(1 - \beta)} \quad (28)$$
where \( \phi \) is given by (25). Some properties of the function \( L(n) \) are summarized in the following lemma. The technical aspects of which can be found in appendix.

**Lemma 1.** (a) \( \lim_{n \to 0} L(n) = \infty \), (b) \( L(1) = \frac{\lambda P_h + (1-\lambda)P_l}{1+\frac{\gamma}{\alpha}} \), and (c) the function \( L(n) \) is strictly decreasing in \( n \).

Lemma 1 indicates that the function \( L(n) \) is a downward-sloping curve. Since the right hand side of (28) is a constant, an unique steady state value of \( n \) between 0 and 1 must exist if the following condition is met:

\[
\frac{\gamma(1-\beta)[\lambda P_h + (1-\lambda)P_l]}{\beta(1+\frac{\gamma}{\alpha})} < \epsilon.
\]

This condition requires that the value of parameter \( \epsilon \) cannot be too small in order to keep lenders from spending all the time on education, according to (16). We state this result in the following proposition.

**Proposition 2.** If \( \frac{\gamma(1-\beta)[\lambda P_h + (1-\lambda)P_l]}{\beta(1+\frac{\gamma}{\alpha})} < \epsilon \), there exists a unique steady state value of lenders’ schooling time, \( n \), called \( n^* \), lies in the internal \((0, 1)\).

After \( n^* \) is found, we can substitute the result into (22) to derive the steady state level of human capital \( e^* \). The steady state capital stock per firm \( k^* \) follows immediately from either (26) or (27). (23) and (24) then will give us the equilibrium wage rate and the equilibrium rental rate of capital respectively. Through (26), it is easy to verify that \( \frac{\partial k^*}{\partial n^*} > 0 \) satisfies. Therefore, when lenders increase their schooling time, the economy converges to a steady state with higher capital stock per firm. We state this result in the following lemma and the proof of it can be found in appendix.

**Lemma 2.** At steady state, the capital stock per firm, \( k^* \), is increasing with lenders’ duration of schooling time, \( n^* \).

Since the borrowers’ expected payoffs are strictly increasing with the loan size, they tend to borrow as much as possible from lenders. Equations (9) and (10) indeed impose an upper limit on the loan sizes and help clear the loan market. Equations (18) and (19) guarantee the clearing of labor market and capital market respectively. At last, it is easy
to check that the following condition is satisfied so that the market of the final output is also cleared at the steady state.

$$0.5[\lambda P_h + (1 - \lambda)P_l]y = 0.5(2 - n + \epsilon)w + 0.5[\lambda P_h + (1 - \lambda)P_l]\rho k.$$  (29)

The left hand side of the above equation represents the total output and the right hand side is the use of the output. Hence, we have proved the following proposition,

**Proposition 3.** There exists a unique competitive equilibrium at the steady state.

### V. Comparative Statics

In this section, we are going to examine how the duration of lenders’ schooling time at the steady state, $n^*$, will be affected by changing some of the parameters, including $\delta$, $P_h$, $P_l$ and $\epsilon$. Of special interest is the change in $\delta$ because varying this parameter corresponds to changing the degree of sophistication of credit markets. We find that lowering $\delta$ always encourages lenders to devote a longer period of time to human capital accumulation. Increasing the values of $P_h$ and $P_l$ represents technological progress. As we will see, just similar to the situation of lowering $\delta$, increasing the value of these two parameters also induces lenders to allocate more time to acquire education. The last comparative static study concerns the improvement on lenders’ default (home production) technology, which is represented by $\epsilon$. We find that increasing $\epsilon$ is harmful for human capital accumulation.

#### V.1 Changing $\delta$

The consequence of decreasing $\delta$ is depicted in figure 1. It is easy to see from (28) that $L$ is negatively associated with $\delta$:

$$\frac{\partial L}{\partial \delta} = -\frac{(1 - n)(1 - \lambda)\phi P_l}{n^{1-\theta}(2 - n + \frac{\lambda P_h}{\alpha})} < 0.$$  

Therefore, as $\delta$ becomes smaller, the entire $L$ curve shifts upward, implying a longer duration of schooling (The economy moves from the steady state E to F in figure 1.). The
intuition behind this result is very straightforward. Lowering screening cost enables young lenders to increase the supply of external investment funds. When more capital goods are produced, the equilibrium rental rate of capital must go down. Relative to physical capital accumulation, human capital accumulation then becomes more attractive, which will encourage lenders to allocate more time to acquire education.

[Figure 1 about here]

V.2 Changing $P_h$ and $P_l$

An increase in the value of $P_i$, $i = \{H, L\}$, shifts the $L$ curve upward since the value of $L$ is increasing with $P_i$. From (28), this is given by

$$\frac{\partial L}{\partial P_h} = \frac{[\lambda (2-n) + \delta (1-\lambda) P_l (1-n)^2 \varepsilon]}{n^{1-\theta}(2-n + \frac{2n^\theta}{\alpha})} > 0,$$

$$\frac{\partial L}{\partial P_l} = \frac{(2-n)(1-\lambda)}{n^{1-\theta}(2-n + \frac{2n^\theta}{\alpha})} [1 - \delta \frac{(1-n)^2 \varepsilon}{(2-n)^2 P_l}] > 0.$$

The consequence of this change can also be illustrated by figure 1. Increasing $P_h$ benefits human capital accumulation since, when $P_h$ becomes bigger, the equilibrium loan rate offered to high risk borrowers, $R^h_{nt}$, declines and so does the informational rent from mimicking a low risk borrowers. As a result, high risk borrowers have lower incentive to cheat, leading to a lower screening probability and hence more funds for financing investment projects. In addition, a higher $P_h$ also implies a higher expected level of physical capital production coming from high risk borrowers’ investment projects. Since both effects promotes physical capital production, the rental rate of physical capital falls. Therefore, lenders choose to spend more time on education.

Increasing $P_l$, however, generates two countervailing forces on human capital accumulation. On the one hand, a higher $P_l$ induces lenders to screen more frequently (i.e. a higher $\phi_t$). This is because the loan rate offered to low risk borrowers, $R^l_{nt}$, is decreasing with $P_l$. As $P_l$ becomes bigger, the benefit from cheating rises. In order to keep the incentive compatibility constraint binding, lenders need to step up their screening effort. As a result, the amount of funds for physical capital production goes down. But a higher
$P_l$ increases the expected amount of physical capital produced by high risk borrowers’ investment projects. Since the latter effect always dominates the former one in this model, increasing $P_l$ unambiguously promotes physical capital production. When more physical capital goods are produced, the capital rental rate declines. Consequently, lenders choose to allocate more time to receive education.

V3. Changing $\varepsilon$

Varying the value of $\varepsilon$ can affect both sides of (28). To see this, let $V = \frac{\varepsilon \beta}{\gamma \theta (1 - \beta)}$. Note that when $\varepsilon$ goes up, $L$ must decrease and $V$ must increase. Algebraically,

$$\frac{\partial L}{\partial \varepsilon} = -\frac{(1 - n)^2(1 - \lambda)\delta P_l}{n^{1 - \theta}(2 - n)(2 - n + \frac{\gamma \phi}{a})} \left( \frac{1}{P_h} - \frac{1}{P_l} \right) < 0,$$

$$\frac{\partial V}{\partial \varepsilon} = \frac{\beta}{\gamma \theta (1 - \beta)} > 0.$$

From (28), increasing $\varepsilon$ shifts the $L$ curve downward and $V$ curve upward (see figure 2), leading to a lower steady state $n^*$. The economy moves from the initial steady state $E$ to $F$ in figure 2). The intuition behind this finding is as follows. Increasing $\varepsilon$ generates the following two reinforcing effects on reducing $n^*$. First, a higher $\varepsilon$ implies higher expected returns from investment in credit markets, which, according to (16), would encourage lenders to allocate more time to working. Second, from (15), $\varepsilon$ is positively related to screening probability. Since screening activities squanders consumption goods, a higher $\varepsilon$ and hence a higher $\phi$ lowers the amount of funds for physical capital production in credit markets, resulting a higher rental rate of physical capital. Subsequently, lenders will respond to work more. Combining the above two effects yields a lower $n^*$.

VI. Conclusion

This paper argues that the interaction between credit market development and human capital accumulation can be mutually reinforcing. Our starting point is a standard neo-

\footnote{Even though increasing $P_l$ will induce lenders to screen more frequently and thereby adversely affecting the physical capital production, this effect is secondary and indirect. Note that when $\delta = 0$, such effect vanishes.}

[Figure 2 about here]
We suggest two possible ways to extend the model. The first extension is to look at the transitional dynamic property of the current model. Throughout the paper, our analysis only focus on the steady state situation. It will be of interest to see how this transitional path will be affected by changing the degree of sophistication of credit markets. The second extension is to include government spending to be an input on human capital production. This kind of government spending can be financed by taxes on lender’s wage income and borrowers’ capital income. This framework would allow us to study how the degree of capital market imperfection may affect the structure of the fiscal policies.
Appendix

Claim 1: In equilibrium, the incentive compatibility constraint (7) holds with strict inequality while (6) holds with equality.

Proof: It is easy to show that (6) holds with equality after substituting in \( \phi_t^h = 0 \), \( R_{nt}^h = \frac{Q \rho_{t+1}}{P_n^h} \), \( \phi_t^l = \phi_t = \frac{1 - n_t}{2 - n_t} \left( \frac{1}{P_h^t} - \frac{1}{P_l^t} \right) \), \( R_{nt}^l = \frac{Q \rho_{t+1}}{P_l(1 - \phi_t)} \), and \( q_{nt}^h = q_{nt}^l = (1 - n_t) w_t \). Under the equilibrium contracts, we obtain

\[
(1 - \phi_t^l) P_l(Q \rho_{t+1}(w_t + q_{nt}^l) - R_{nt}^l q_{nt}^l) + \phi_t^l P_l(Q \rho_{t+1}(w_t + q_{nt}^l) - R_{nt}^l q_{nt}^l)
= P_l Q \rho_{t+1} w_t + (1 - \phi_t^l) P_l(Q \rho_{t+1} - R_{nt}^l) q_{nt}^l
+ \phi_t^l P_l(Q \rho_{t+1} - R_{nt}^l) q_{nt}^l
= P_l Q \rho_{t+1} w_t + P_l \left[ Q \rho_{t+1} \left( 1 - \frac{1 - n_t}{2 - n_t} \left( \frac{\varepsilon}{P_h^t} - \frac{\varepsilon}{P_l^t} \right) \right) + \phi_t Q \rho_{t+1}(1 - \delta) - \frac{Q \varepsilon \rho_{t+1}}{P_l} \right] (1 - n_t) w_t
\]

Thus we prove the claim. QED

Claim 2: Since the expected payoff to a high-risk borrower is strictly decreasing with the screening probability \( \phi_t^h \), it is optimal to set \( \phi_t^h = 0 \).

Proof: With \( q_{nt}^h = (1 - \delta)(1 - n_t) w_t \), \( q_{nt}^l = (1 - n_t) w_t \) and making use of the zero-profit condition (10) for \( i = H \), the expected payoff to a high-risk borrower is

\[
\phi_t^h P_h[Q \rho_{t+1} + (Q \rho_{t+1} - R_{st}^h)(1 - \delta)(1 - n_t) w_t] + (1 - \phi_t^h) P_h[Q \rho_{t+1} + (Q \rho_{t+1} - R_{nt}^h)(1 - n_t) w_t]
= P_h Q \rho_{t+1} w_t + \left[ P_h Q \rho_{t+1} \left( \phi_t^h(1 - \delta) + (1 - \phi_t^h) \right) \right] - \left[ (1 - \delta) \phi_t^h P_h R_{st}^h + (1 - \phi_t^h) P_h R_{nt}^h \right] (1 - n_t) w_t
= P_h Q \rho_{t+1} w_t + P_h Q \rho_{t+1} (1 - \delta \phi_t^h)(1 - n_t) w_t - Q \varepsilon \rho_{t+1}(1 - n_t) w_t.
\]

which is decreasing in \( \phi_t^h \). Free from the self-selection constraints, the optimal solution is to set \( \phi_t^h \) as low as possible, i.e., at zero. QED

Claim 3: The expected payoff to a type L borrower is strictly increasing in \( R_{nt}^l \).
Proof: From (9), (10) (both with equality), and (11) for $i = L$, similarly, the expected payoff to a low-risk borrower is

$$\phi^i P_l [Q \rho_{t+1} + (Q \rho_{t+1} - R^l_{nt}) (1 - \delta) (1 - n_t)] w_t + (1 - \phi^i) P_l [Q \rho_{t+1} + (Q \rho_{t+1} - R^l_{nt}) (1 - n_t)] w_t$$

$$= P_l Q \rho_{t+1} w_t + \{ P_l Q \rho_{t+1} [\phi^i (1 - \delta) + (1 - \phi^i)] - [(1 - \delta) \phi^i R^l_{nt} + (1 - \phi^i) P_l R^l_{nt}] \} (1 - n_t) w_t$$

$$= P_l Q \rho_{t+1} w_t + P_l Q \rho_{t+1} (1 - n_t) w_t (1 - \delta \phi^i) - Q \varepsilon \rho_{t+1} (1 - n_t) w_t.$$

From (13) and $(2 - n_t) Q \rho_{t+1} - (1 - n_t) R^h_{nt} > 0$, we obtain

$$\frac{\partial \phi^i}{\partial R^l_{nt}} = \frac{(1 - n_t) \{(2 - n_t) Q \rho_{t+1} - (1 - n_t) R^h_{nt} \}}{(2 - n_t) Q \rho_{t+1} - (1 - n_t) R^h_{nt}} < 0.$$

It then follows that the expected payoff to a low-risk borrower is strictly increasing in $R^l_{nt}$. QED

**Proof of Lemmas**

**Lemma 1.** (a) $\lim_{n \to 0} L(n) = \infty$, (b) $L(1) = \frac{\lambda P_h + (1 - \lambda) P_l}{1 + \frac{\gamma}{\alpha}}$, and (c) the function $L(n)$ is strictly decreasing in $n$.

Proof: It is easy to check the first two properties. To prove (c), we look at the first order derivative of function $L$ with respect to $n$

$$\frac{\partial L}{\partial n} = \frac{-1}{n^{2 - \theta} \alpha (2 - n + 2n^\alpha)} \{ n (2 - n) + \frac{\gamma n^\alpha}{\alpha} \{ \lambda P_h + (1 - \lambda) P_l [1 - \delta (\frac{\varepsilon}{P_h} - \frac{\varepsilon}{P_l}) (1 - n)(3 - n)] \}$$

$$+ (2 - n)[(1 - \theta)(2 - n) + n^\theta (\frac{\gamma}{\alpha} - n^{1 - \theta})] \{ \lambda P_h + (1 - \lambda) P_l [1 - \delta (\frac{\varepsilon}{P_h} - \frac{\varepsilon}{P_l}) (1 - n)^2] \} \}.$$

To sign $\frac{\partial L}{\partial n}$, we need to determine the sign of the term, $n (2 - n + 2n^\alpha) \{ \lambda P_h + (1 - \lambda) P_l [1 - \delta (\frac{\varepsilon}{P_h} - \frac{\varepsilon}{P_l}) (1 - n)(3 - n)] \}$ and that of the term, $(2 - n)[(1 - \theta)(2 - n) + n^\theta (\frac{\gamma}{\alpha} - n^{1 - \theta})] \{ \lambda P_h + (1 - \lambda) P_l [1 - \delta (\frac{\varepsilon}{P_h} - \frac{\varepsilon}{P_l}) (1 - n)^2] \}.$

One can easily check that $\frac{1 - n(3 - n)}{(2 - n)^2}$ indeed is strictly decreasing with $n$. When $n = 0$, this fraction is equal to 0.75 which is the biggest numerical value that it can take. Hence, for any plausible values of $n$, the first whole term must be positive. Following from $\gamma \geq 1 > \alpha > 0$, the second whole term must also be positive. Therefore, the first order derivative of function $L$ with respect to $n$ is negative. QED
Lemma 2: At steady state, the capital stock per firm, $k^*$, is increasing with lenders’ duration of schooling time, $n^*$.

Proof: According to (26),

$$k^* = \omega (2 - n^* + e^*) (n^*)^{\frac{1-\theta}{1-\beta}}$$

where $\omega = (\frac{Q_\beta}{\eta})^{\frac{1}{1-\beta}} [\lambda P_h + (1 - \lambda) P_l]^{-1}$ and $e^* = \frac{\gamma(n^*)^\theta}{\alpha}$. The partial derivative of $k^*$ with respect to $n^*$ is given by (To economize on notation, we suppress the superscript $*$ in the following derivation):

$$\frac{\partial k^*}{\partial n^*} = \omega \{(1 + \frac{\gamma n^{\theta-1}}{\alpha}) n^{\frac{1-\theta}{1-\beta}} + (2 - n + \frac{\gamma n^\theta}{\alpha}) (1 - \frac{\theta}{1-\beta}) n^{\frac{\beta-\theta}{1-\beta}}\}$$

$$= \omega \{-n^{\frac{1-\theta}{1-\beta}} + \frac{\gamma n^{\beta(1-\theta)}}{\alpha} n^{\frac{1-\theta}{1-\beta}} + \frac{2(1 - \theta)}{1-\beta} n^{\frac{\beta-\theta}{1-\beta}} - \frac{1 - \theta}{1-\beta} n^{\frac{1-\theta}{1-\beta}} + \gamma(1 - \theta) n^{\frac{(1-\theta)}{1-\beta}}\}$$

$$= \omega \{-1 + \frac{1 - \theta}{1-\beta} n^{\frac{1-\theta}{1-\beta}} + \frac{\gamma}{\alpha n^{\frac{1-\theta}{1-\beta}}} n^{\frac{\beta(1-\theta)}{1-\beta}} + \frac{2(1 - \theta)}{1-\beta} n^{\frac{\beta-\theta}{1-\beta}} + \gamma(1 - \theta) n^{\frac{(1-\theta)}{1-\beta}}\}$$

which is positive. Hence we prove the claim. QED
References


