Abstract

Coase (1972) first pointed out that a monopoly selling a durable good would behave differently from the monopoly selling a perishable good. Given a choice between renting and selling, a monopoly firm would like to rent the durable good, rather than selling it, in order to maximize its profit. Given this, a two-fold question comes to mind. Does this conjecture always remain true in any durable good monopoly market, and what happens when we move from a monopoly market to an oligopoly market? In this paper, first, we show that Coase conjecture does not necessarily remain valid in all monopoly situations. In particular, we show that when a foreign firm operating in another country faces exchange rate fluctuations Coase conjecture fails to hold. Then, when we move from a monopoly situation to an oligopoly situation, we also come across some interesting findings. For instance, when two symmetric (domestic) firms compete with each other in one market, selling turns out to be the unique dominant behaviour of the firms. On the other hand, allowing the same competition between a foreign and a domestic firm, where the foreign firm is exposed to exchange rate fluctuations, we get equilibrium outcomes which are very different from the previous one.

Keywords: Durable good, Renting, Selling, Market structure, Exchange rate.

1. Introduction

Coase (1972) first pointed out that a monopoly selling a durable good would behave differently from the monopoly selling a perishable good. Given a choice between renting and selling, a monopoly firm would like to rent the durable good, rather than selling it, in order to maximize its profit. The intuition behind this is that rational consumers are able to calculate that a monopolist selling the durable good would lower future prices due to the future fall in the demand resulting from having some consumers purchasing the durable product in earlier periods. This calculation reduces the willingness of consumers to pay high prices in the first period when the monopoly offers the product for sale. Hence, the current demand facing the monopoly falls, implying that the monopoly will charge a lower price than what a monopoly selling a perishable good would charge.\(^1\) This is also what is known as Coase conjecture in the literature.

Given this, a two-fold question comes to mind. The first question is: Does this conjecture always remain true in any durable good monopoly market? And the second question is: What happens when we move from a monopoly market to an oligopoly market? In this paper, we tried to find answers to these questions. First, we show that Coase’s conjecture does not necessarily remain valid in all monopoly situations. For example, consider the situation, where a foreign firm wants to operate in the domestic market of some other country. Assume that it’s the only firm operating in that domestic market. The foreign firm has the option to rent or sell the durable good. Question is, what would be the optimal choice of that firm regarding renting and selling, when it wants to repatriate all the profit to its own country, and at the same time, faces future exchange rate fluctuations in currencies between the two countries? In such a situation, we find that Coase’s conjecture under monopoly market structure does not always remain valid. We find that in a wide variety of situations, the foreign monopoly firm would actually find it profitable to adhere to selling rather than renting.

Next, when we move from a monopoly situation to an oligopoly situation, we also come across some interesting findings. For instance, we find that without any effect of exchange rate fluctuations or uncertainty when two firms compete with each other in one

market, selling turns out to be the unique dominant behaviour of the competing firms. Now, when we allow the same competition between a foreign and a domestic firm, where the foreign firm only faces future exchange rate fluctuations while repatriating its profits, we get a mix of equilibrium outcomes (symmetric as well as asymmetric) where in some situations one firm prefers renting while the other prefers selling and in other situations both prefer selling. In any case, pursuing a renting policy by both the firms is never optimal in any situation. On the other hand, in some cases, we get prisoners’ dilemma like situations, where selling by both firms turns out to be a dominant strategy, yet renting by both firms yield higher profits.

Thus, in this study, we find that not only the market structure plays a crucial role in choosing between a renting or selling policy by the durable goods firms, but also the impact of the future exchange rate fluctuations (uncertainty) plays an important role too.

The rest of the paper is organized as follows. Section 2 introduces the model. Section 3 considers the monopoly case. Section 4 deals with the duopoly case. Section 5 characterizes the equilibrium outcomes in the duopoly situation, and explains the impact of exchange rate fluctuations on the competing firms behaviour. Finally, section 6 concludes.

2. The Model

Consider a continuum of consumers having different valuations for a certain durable good, and who live for two periods denoted by \( t, t = 1,2 \). The different valuations for the good are summarized by the familiar downward sloping demand curve. More precisely, at period \( t = 1 \), the aggregate inverse demand for one period of service is given by 

\[ p(Q) = a - Q \]

where \( p \) is the price and \( Q \) is the aggregate supply. The good is perfectly durable. Renting and selling are essentially distinguished in the first period only in the following way. If a consumer rents a good in time period one, she has to give it back to the renter at the end of period one, whereas if a consumer buys a good in time period one, she gets to keep it till the end of time period two. Since period two is the last period (end of time) renting or buying a product in period two is equivalent. For simplicity, we
assume no time discounting in the model for the firms and consumers.\(^2\) There is no secondhand market.

Let the prevailing period 1 exchange rate between the currencies of country 1 and 2 be \(e\).

Let’s define \(e = \left( \frac{\text{country 1's currency}}{\text{country 2's currency}} \right)\). In period 2 the exchange rate could be \(e_H\) or \(e_L\) \((e_L < e_H)\) with probabilities \(\theta\) and \((1-\theta)\) respectively.

### 3. The Monopoly Case

Consider a foreign monopoly firm in country 1 who would like to provide a durable good to country 2 and repatriate back the profits. In period 1, this firm knows the prevailing exchange rate between the currencies of the two countries, but only knows the probability distribution regarding the uncertainty of the exchange rate in period 2. It has to decide between renting or selling the good in period 1. Recall that in this framework, the act of renting and selling can only be distinguished in time period 1. For simplicity, we assume no operational or transaction cost to the firm.\(^3\)

In the forthcoming analysis, first we consider the benchmark case with no exchange rate uncertainty and establish the Coase conjecture. Then we move on to the situation with exchange rate uncertainty and show that the conjecture is not true in a wide range of parametric configuration.

#### 3.1 No Exchange Rate Uncertainty

Since the exchange rate does not fluctuate, and thus, has no differential effect on firm’s profit on renting and selling, without loss of generality we drop the exchange rate for simplicity in the following analysis.

##### 3.1.1 A Renting Monopoly

Assume that each period the monopoly rents a durable product for one period only. Suppose that in each of the two periods the monopoly faces the demand \(p(Q) = a - Q\).

\(^2\) It can be verified that the qualitative results obtained in this paper remain unaffected if we allow time discounting for the firms and consumers.

\(^3\) It can also be shown that assumption of any transaction type of cost can only reinforce our main results in this analysis.
Naturally, in each period the monopoly behaves like a usual profit maximizer and maximizes the sum of two period profits.

In each period the monopoly charges a price \( p_i^R = \frac{a}{2} \) and earns a profit \( \pi_i^R = \frac{a^2}{4} \), \( t = 1,2 \).

Hence in two period the monopoly earns a total profit of \( \pi^R = \frac{a^2}{2} \).

### 3.1.2 A Selling Monopoly

A seller monopoly knows that those consumers who purchase the durable good in \( t = 1 \) will not repurchase in period \( t = 2 \). That is, in \( t = 2 \) the monopoly will face a demand for its product that is lower than the period 1 demand by exactly the amount its sold in \( t = 1 \). Therefore, in period 2 the monopoly will have to sell at a lower price resulting from a lower demand, caused by its own earlier sales.

Formally, we define this two period game as follows. The payoff to the monopoly is the total revenue generated by period 1 and period 2 sales. The strategies of the seller are the price set in period 1, \( p_1^S \); and the price set in period 2 as a function of the amount purchased in period 1, \( p_2^S(q_1) \); where \( q_1 \) is period one sales. The strategies of the buyer are to buy or not to buy (in the first period) depending on the net utility she gets in consuming the good for two periods as opposed to one period (second period) consumption only. We look for a subgame perfect equilibrium and work backward to solve the game.

If \( q_1 \) is period 1 sales of the firm, then the effective demand in period 2 is, \( p = [(a - q_1) - q] \).

The profit maximizing levels of quantity, price and profits in period 2 are:

\[
q_2 = \frac{(a - \bar{q}_1)}{2}, \quad p_2 = \frac{(a - \bar{q}_1)}{2}, \quad \pi_2^S = \left[ \frac{(a - \bar{q}_1)}{2} \right]^2
\]

We derive the effective demand curve for the firm in period 1 from the indifference condition of the marginal consumer between her purchase in period 1 and period 2. This condition implies: \( 2(a - \bar{q}_1) - p_1 = (a - \bar{q}_1) - p_2 \).
Substituting $p_2$ above we get the effective demand curve for the firm in period 1, viz, $p_1 = \frac{3(a - \bar{q}_1)}{2}$. 

Hence, the total profit of the firm is,

$$\pi^s_1 = \left[\left(\frac{3}{2}\right)a - \bar{q}_1\right] + \left[\left(a - \bar{q}_1\right)^2\right]$$

Maximizing the above with respect to $\bar{q}_1$, we get the following:

The equilibrium price, quantity and profit of the selling monopoly in period one are:

$$p^s_1 = \left[\frac{9a}{10}\right]; \quad q^s_1 = \left[\frac{2a}{5}\right]; \quad \pi^s_1 = \left[\left(\frac{9}{25}\right)a^2\right];$$

and in period 2 are:

$$p^s_2 = \left[\frac{3a}{10}\right]; \quad q^s_2 = \left[\frac{3a}{10}\right]; \quad \pi^s_2 = \left[\left(\frac{9}{100}\right)a^2\right].$$

Hence total profit of a seller monopoly is: $\pi^s = \left[\left(\frac{9}{20}\right)a^2\right]$. 

It is straight forward to compare and see that renting monopoly earns a higher profit than a seller monopoly confirming Coase’s conjecture. Hence, we have the following result.

**Proposition 1**

*Given a choice between renting and selling, a durable good monopolist will always choose renting.*

### 3.2 With Exchange Rate Uncertainty

Now let us introduce exchange rate uncertainty for the future period 2.

#### 3.2.1 A Renting Monopoly

As before, assume that each period the firm rents the durable good for one period only. Hence, in each of the two periods the firm faces the demand $p(Q) = a - Q$.

Naturally, in each period the firm would like to behave like a usual monopolist and maximize the sum of its two period profits. Given the exchange rate uncertainty in period 2, its total expected profit in terms of its home currency would be:
\[ E(\pi^R) = e \left( \frac{a^2}{4} \right) + \theta e_H + (1 - \theta) e_L \left( \frac{a^2}{4} \right) \]

3.2.2 A Selling Monopoly

We need the following assumption to guarantee the existence of selling equilibrium.

Assumption: \[ e > \frac{1}{3} \theta e_H + (1 - \theta) e_L \].

We proceed as before and incorporating exchange rate uncertainty we get the following. The total expected profit of the selling firm is,

\[ E(\pi^S) = e \left[ \frac{3}{2} (a - \bar{q}_i) \bar{q}_i \right] + \theta e_H + (1 - \theta) e_L \left[ \frac{(a - \bar{q}_i)}{2} \right]^2 \]

Maximizing the above with respect to \( \bar{q}_i \), we get the following:

\[ \bar{q}_i = \frac{a \left[ 3e - \theta e_H + (1 - \theta) e_L \right]}{6e - \theta e_H + (1 - \theta) e_L} \] (assumption ensures \( \bar{q}_i > 0 \), \( p_1 = \frac{3ae}{6e - \theta e_H + (1 - \theta) e_L} \))

\[ E(\pi^S) = \frac{9a^2 e^2}{4(6e - \theta e_H + (1 - \theta) e_L)^2} \]

The other expressions like \( p_2, q_2 \) follow immediately.

3.3 Renting Versus Selling

Now we would like to compare the total expected profit of the firm between two regimes of renting and selling. The firm decides between the two options in period 1 depending on the expected profits.

**Proposition 2**

The firm will choose selling over renting whenever \( e > 1.43 \theta e_H + (1 - \theta) e_L \).

**Proof:** See appendix.

This shows whenever the firm anticipates that the expected exchange rate will be much lower than the prevailing exchange rate, i.e when it anticipates depreciation in the exchange rate in future periods, it chooses selling over renting. This is because with a durable good, selling mechanism is an arrangement in which a firm can extract more revenue in the initial period by charging a higher price than renting. And also since
demand for the good in future periods gradually decrease (because some consumers already bought it), it is unable to keep up the same high price in future periods, and hence wants to sell more in the favourable exchange rate regime. The second reason is, selling by its inherent nature acts as a hedge against expected exchange rate depreciation in the durable goods market. This behaviour of the foreign durable goods firm facing exchange rate uncertainty sharply contrasts with the behaviour of a usual durable goods monopolist operating in a similar market facing no such uncertainty. In such a market, we saw that renting always dominates selling whereas here we see that is not the case anymore. Hence, Coase’s conjecture is not necessarily valid under the situation when a foreign firm operates in the domestic market of another country and faces a future exchange rate uncertainty, even though the firm operates under a monopoly market structure.

Equilibrium in other ranges of $e$, $e_H$, $e_L$ can be characterized as follows.

**Proposition 3**

(i) If $0.33\left[\theta e_H + (1 - \theta) e_L\right] \leq e \leq 1.43\left[\theta e_H + (1 - \theta) e_L\right]$ then the firm will choose renting over selling as the former is more profitable than later.

(ii) If $e < 0.33\left[\theta e_H + (1 - \theta) e_L\right]$ then selling equilibrium does not exist, hence renting remains the only option to the firm.

*Proof: See appendix A.*

Following figure describes all the equilibrium regions under renting and selling regimes.
4. The Duopoly Case

Now suppose there are two firms, one foreign and one domestic, produce the same durable good and operate in the domestic firm’s market. The foreign firm (due to repatriation of its profit to its home country) is exposed to future exchange rate uncertainty. The domestic firm values its profits in its own currency, naturally remains unexposed to any such exchange rate uncertainty. Like before, first we will consider the case, where the exchange rate uncertainty is absent, and hence do the exercise without involving any exchange rate in the calculation. Then we will move on to the case with exchange rate fluctuations. In this way, we will be able to separate out the strategic effect from the impact of exchange rate fluctuations. The game between the competing firms is as follows. Before choosing the quantities for the respective periods, they decide simultaneously whether to rent or sell the good. This can be thought of as a pre-play stage. Once they make their decision on renting and selling they commit to it and choose appropriate quantities in period 1 and 2 in order to maximize total individual profits.\(^4\) We look for a subgame perfect equilibrium of

\(^4\) Here since our focus is on comparing between renting and selling, we assume that firms cannot mix renting with selling. In other words, either they opt for renting or selling.
this two period game. As before, we start solving the game by the usual method of backward induction. In this game, in the first period four scenarios may arise where (i) both the firms rent (ii) foreign firm rents and the domestic firm sells (iii) foreign firm sells and the domestic firm rents and (iv) both the firms sell. We will take up each of these cases separately while solving the game.

4.1 No Exchange Rate Uncertainty

4.1.1 Both Firms Rent
In this case both the firms are faced with the same demand curve in each period. Hence, we get the standard duopoly outcomes in each period. The price, quantity and profits (in domestic currency) of each firm in each period are:

\[ p_r = \frac{a}{3}, \quad q_r = \frac{a}{3}, \quad \pi_r = \left( \frac{a}{3} \right)^2 \]

where \( R \) denotes renting; \( t=1,2 \).

Hence, sum of the two period profits for the firm is \( \pi_{rr} = 2 \left( \frac{a}{3} \right)^2, \quad i = 1, 2 \) \(^5\)

4.1.2 One Firm Rents and the Other Sells

The second period
Since the two firms are symmetric when we calculate price, quantity and profits in terms of domestic currency, we assume just the presence of two firms 1, 2 instead of distinguishing them as domestic or foreign. Without loss of generality, suppose firm 1 sells and firm 2 rents in the first period. That means in the second period a group of consumers (those who already bought in the first period) will not buy in the second period. Let firm 1 sold \( q_1^s \) units of the good in the first period. Thus, the residual demand faced by the firms in period two is \( p(Q) = a - q_1^s - Q \), where \( Q \) is the aggregate supply in period two. Hence each firm will maximize its profit given this demand in period two.

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\(^5\) We used the following convention in the notation. In the superscript of all the profit expressions \( \pi \), the first component denotes foreign firm’s action, while the second component denotes domestic firm’s action. The subscript is self explanatory.
This gives quantity, price and profits, respectively in the second period as
\[ q_2^s = q_2^r = \left( \frac{a - q_1^s}{3} \right); \quad p_2 = \left( \frac{a - q_1^s}{3} \right); \quad \text{and} \quad \pi_2^s = \pi_2^r = \frac{1}{9} \left[ a - q_1^s \right]^2; \]
where \( S \) and \( R \) refers to seller and renter respectively.

**The first period**

Here we have to find the effective demand faced by the seller and the renter in period one. First notice that for both the seller and the renter to co-exist in the market, we must have: \( p_1^s = p_1^r + p_2 \). Otherwise, if \( p_1^s > p_1^r + p_2 \), then any consumer who plans to enjoy the good for two periods would only rent it in the first period and buy it in the second period instead buying the good in first period from the seller. Similarly, if \( p_1^s < p_1^r + p_2 \), then the reverse will happen.

Now, to find the effective demand of the seller we have to identify the marginal consumer who is indifferent between enjoying the service of the good in both periods as oppose to second period only. More precisely, suppose the seller sells \( q_1^s \) units of the good and the renter rents \( q_1^r \) units of the good in the first period to the consumers with the highest reservation prices. Then, the marginal consumer (who is buying from the seller) with a reservation price \( [a - q_1^s - q_1^r] \) will be indifferent between buying in the first period (gaining utility of \( 2[[a - q_1^s - q_1^r] - p_1^s]] \)) and buying in the second period (gaining utility of \( ([a - q_1^s - q_1^r] - p_2) \)) if and only if \( 2(a - q_1^s - q_1^r) - p_1^s = (a - q_1^s - q_1^r) - p_2 \).

Substituting the value of \( p_2 \) in the above equation, we get effective demand curve for the seller in period one, viz: \( p_1^s = \left[ \frac{4}{3} (a - q_1^s) - q_1^r \right] \)

Substituting the value of \( p_1^s \) and \( p_2 \) in \( p_1^s = p_1^r + p_2 \), we get the effective demand of the renter in period one, viz: \( p_1^r = \left( a - q_1^s - q_1^r \right) \). Now the first order condition of maximizing the sum of the profits in period 1 and in period 2 each for the renter and the seller gives us two equations with the first stage variables \( q_1^s \) and \( q_1^r \). Once we solve for
\(q^s_i\) and \(q^R_i\) we can solve for the price, quantity and profits of both the firms in the two periods.

Solving these two equations we get, \(q^s_i = \left[\frac{11a}{35}\right]\), \(q^R_i = \left[\frac{12a}{35}\right]\); \(p^s_i = \left[\frac{4a}{7}\right]\), \(p^R_i = \left[\frac{12a}{35}\right]\);

Hence, \(p_2 = \left[\frac{8a}{35}\right]\) and \(q^{s,R}_2 = \left[\frac{8a}{35}\right]\).

We find that the price of the seller in the first period exceeds that of the renter which can be easily explained by the fact that the seller is selling the good for two period of consumption while the renter is allowing for one period of consumption only.

The total profit of the seller and the renter respectively are:

\[
\pi^{SR}_1 = \left[\frac{284}{1225}\right]a^2, \quad \pi^{SR}_2 = \left[\frac{208}{1225}\right]a^2
\]

Hence, we observe the following.

**Lemma 1**

*In a durable good market with a seller and a renter, the seller earns a higher profit than the renter.*

The intuition behind this is the following. In the second period, both the seller and the renter share the market equally and earn equal profits. So the only difference in the profits comes in the first period. In the first period, the seller charges a significantly higher price than the renter since it sells the good for two-period of consumption as oppose to the renter, who rents the good only for one-period of consumption. The high price of the good enables the seller to earn higher revenue (even if it sells a lesser unit of goods compared to the renter) than the renter in the first period.

**4.1.3 Both Firms Sell**

When both the firms opt for selling in period one, then in period two the demand they face is restricted by their previous period sales. If \(q^1_i\) and \(q^2_i\) are the sales of each of the firms in period one, then the effective demand in period two is, \(p = \left[a - q^1_i - q^2_i\right] - q^2_1 - q^2_2\).
The profit maximising levels of quantity, price and profits for each firm in period 2 are:

\[ q_{i2}^{*} = \left[ a - \bar{q}_1^i - \bar{q}_2^i \right] \]
\[ p_2 = \frac{1}{3} \left[ a - \bar{q}_1^i - \bar{q}_2^i \right] \]
\[ \pi_2^{i*} = \frac{1}{9} \left[ a - \bar{q}_1^i - \bar{q}_2^i \right] \]

We derive the effective demand curve for the sellers in period 1 from the indifference condition of the marginal consumer between her purchase in period 1 and period 2. This condition implies:

\[ 2 \left( a - \bar{q}_1^i - \bar{q}_2^i \right) - p_1^S = \left( a - \bar{q}_1^i - \bar{q}_2^i \right) - p_2. \]

Substituting \( p_2 \) above we get the effective demand curve for each seller in period one,

\[ p_1^S = \left( \frac{4 \left( a - \bar{q}_1^i - \bar{q}_2^i \right)}{3} \right). \]

As previously, from the first order condition of maximising the sum of period 1 and period 2 profits for each firm with respect to their respective quantity in period 1, we get two equations with the variables \( \bar{q}_1^i \) and \( \bar{q}_2^i \). Solving them simultaneously we get,

\[ \bar{q}_1^i = \bar{q}_2^i = \left[ \frac{5a}{16} \right] ; \quad p_1^S = \left[ \frac{a}{2} \right] ; \quad \pi_1^i = \pi_2^i = \left[ \frac{5}{32} \right] \]

and for the second period

\[ \bar{q}_2^i = \bar{q}_2^i = \left[ \frac{a}{8} \right] ; \quad p_2^S = \left[ \frac{a}{8} \right] ; \quad \pi_2^i = \pi_2^i = \left[ \frac{a^2}{64} \right]. \]

Hence the sum of the two periods profit for each firm is:

\[ \pi_i^{ss} = \left[ \left( \frac{11}{64} \right) a^2 \right], \quad i=1, 2 \]

4.2 The Extended Game and the Equilibrium Outcomes

The following table summarizes the two period game.
Table 1

<table>
<thead>
<tr>
<th>FIRM 1</th>
<th>RENTING</th>
<th>SELLING</th>
</tr>
</thead>
<tbody>
<tr>
<td>RENTING</td>
<td>$\pi^R_1, \pi^R_2$</td>
<td>$\pi^R_1, \pi^R_2$</td>
</tr>
<tr>
<td>SELLING</td>
<td>$\pi^S_1, \pi^S_2$</td>
<td>$\pi^S_1, \pi^S_2$</td>
</tr>
</tbody>
</table>

Now, it is easy to verify that $\pi^S_1 > \pi^R_1$ and $\pi^S_2 > \pi^R_2$, thus making selling a dominant strategy for firm 1. The same holds for firm 2 as well. Hence, (selling, selling) turns out to be the unique dominant strategy equilibrium of this two period game. Interestingly, it is also easy to verify that $\pi^S_i > \pi^R_i; i=1,2$. Thus, we arrive at a situation of prisoners dilemma. It can also been shown that the total welfare (consumer surplus plus profits of the firms) is lower in (selling, selling) equilibrium as opposed to (renting, renting) equilibrium.

**Proposition 4**

In a duopoly durable good market where firms are allowed to rent or sell; (selling, selling) turns out to be the unique dominant strategy equilibrium. Moreover, since (renting, renting) payoff dominates (selling, selling), we arrive at a situation of prisoner’s dilemma.

4.3 With Exchange Rate Uncertainty

Now let’s introduce the fact that the foreign firm faces an exchange rate uncertainty in time period 2. The foreign firm knows the prevailing exchange rate $e$ in time period 1 between two countries, but only knows the underlying probability distribution (namely,
or $e_L$ with probabilities $\theta$ and $(1-\theta)$ respectively where $(e_L < e_H)$ of the exchange rate for time period 2.

### 4.3.1 Both Firms Rent

Now, the sum of the two period profits for the foreign firm becomes

$$\pi^{RR}_F = e \left( \frac{a^2}{9} \right) + \theta e_H + (1-\theta) e_L \left( \frac{a^2}{9} \right)$$

and for the domestic firm it remains as

$$\pi^{RR}_D = 2 \left( \frac{a}{3} \right)^2 .$$

### 4.3.2 Foreign Firm Sells and the Domestic Firm Rents

When the foreign firm sells and the domestic firm rents we have their profits as follows:

Profit of the foreign firm: $\pi^{SR}_F = \left( \frac{4}{3} (a - q_{F1}^S) - q_{R1}^R \right) q_{F1}^S + \left( \frac{\theta e_H + (1-\theta) e_L}{9} \right) [a - q_{F1}^S]$.  

Profit of the domestic firm: $\pi^{SR}_D = (a - q_{F1}^S - q_{D1}^R) q_{D1}^R + \frac{1}{9} [a - q_{F1}^S]^2$.  

Where $q_{i,j}^K, i=F,D; j=1,2; K=S,R.$ respectively. $F$ & $D$ implying foreign and domestic firm, 1, 2 implying the first and second time periods and $S, R$ denotes selling and renting respectively.  

Maximizing these two profits we get two equations in terms of two variables, $q_{F1}^S$ and $q_{D1}^R$. Solving these two equations gives us the equilibrium values of these two variables as:

$$q_{F1}^S = \frac{2(12e - 2\theta e_H + (1-\theta)e_L)}{2(24 e - 2\theta e_H + (1-\theta)e_L)} - 9e$$

$$q_{D1}^R = \frac{12ae}{2(24 e - 2\theta e_H + (1-\theta)e_L)} - 9e$$

For both $q_{F1}^S$ and $q_{D1}^R$ to be positive i.e., to coexist simultaneously the condition $2(12e - 2\theta e_H + (1-\theta)e_L) - 9e > 0$ should hold. This implies $\left( \frac{e}{\theta e_H + (1-\theta)e_L} \right) > 4 \frac{15}{15}$
i.e., over any range of any expected exchange rate appreciation of less than 275% of the domestic currency against foreign currency.\(^6\)

The profits of the foreign firm who sells (when the domestic firm rents) is:

\[
\pi_{FD}^S = \left[ \frac{2ae}{2(24e - 2\theta e_H - (1-\theta)e_L)} - 9e \right]^2 [75e - 4\theta e_H + \{1-\theta\}e_L].
\]

Profits of the domestic firm who rents (when the foreign firm sells) is:

\[
\pi_{DS}^R = \left[ \frac{208a^2e^2}{2(24e - 2\theta e_H + (1-\theta)e_L)} - 9e \right]^2 .
\]

### 4.3.3 Foreign Firm Rents and the Domestic Firm Sells

When the foreign firm rents and the domestic firm sells the total profit of the domestic firm is

\[
\pi_{RD}^S = e\left[ a - q_{D1}^S - q_{F1}^S \right] q_{D1}^S + \left[ \frac{\theta e_H + (1-\theta)e_L}{9} \right] a - q_{D1}^S.
\]

Similarly maximizing these two profits we get two equations in terms of two variables, \(q_{D1}^S\) and \(q_{F1}^S\). Simultaneous solution of these two equations gives us the equilibrium values of these two variables as well that of \(p_{F1}^S\), \(p_{D1}^S\), \(q_{F2}^R\), \(p_{D2}^R\) and \(p_2\). They being

\[
q_{D1}^S = \left( \frac{11}{35} \right) a; \quad q_{F1}^S = \left( \frac{12}{35} \right) a; \quad p_{D1}^S = \left( \frac{20}{35} \right) a, \quad p_{F1}^S = \left( \frac{12}{35} \right) a \quad \text{and} \quad p_2 = q_{D2}^S = q_{F2}^R = \left( \frac{8}{35} \right) a.
\]

\(^6\) The reason behind this being that at an expected appreciation of 275%, the foreign firm (when selling) will stop its operations since expected exchange rate appreciation always favours it renting over selling through a larger market in the second period.

The existence of renting by the domestic firm needs the fulfillment of the condition,

\[
2\left( 24e - 2\theta e_H + (1-\theta)e_L \right) - 9e > 0 \quad \text{i.e., over any range of exchange rate depreciation to an appreciation of less than 875%}. \quad \text{This is because at an expected exchange rate appreciation of 875% the domestic firm's (who is renting) supply in the first period turns out to be undefined. Though this is the necessary condition for the simultaneous existence of selling by domestic firm and renting by the foreign firm, the sufficient condition remains to be} \left( \frac{e}{\theta e_H + (1-\theta)e_L} \right) > \frac{4}{15} .
\]
This leads the total profit of the domestic and foreign firms to be $\overline{\pi}_D = \left[ \frac{284}{35^2} \right] a^2$ and $\overline{\pi}_F = a^2 \left[ \frac{144e + 64\theta e_H + (1-\theta)e_L}{35^2} \right]$ respectively.

### 4.3.4 Both Firms Sell

When both the foreign and domestic firm adheres to selling in both the periods the inverse demand curve faced by the firms in period 2 is

$$p_2 = (a - q_{F1}^S - q_{D1}^S) - (q_{F2} + q_{D2})$$

Therefore the profits of both the domestic and foreign firms in domestic currency turns out to be $\pi_{i2}^S = \left[ a - \sum_i q_{iA}^S \right] - \sum_i q_{i2}^S q_{i2}$ for $i = F, D$. Maximizing these second period profits we solve for the second period output, price and profits in terms of the first period sales of the two firms.

They are given by $q_{F2}^S = q_{F2} = \frac{1}{3} \left[ a - \left( q_{D1}^S + q_{F2}^S \right) \right]$, $p_2 = \frac{1}{3} [a - (q_{D1}^S + q_{F1}^S)]$ and the profits $\pi_{F2}^S = \pi_{D2}^S = \frac{1}{9} [a - (q_{D1}^S + q_{F1}^S)]^2$. Here we get symmetric solution since there exist no difference between these two firms when both their profits are valued in domestic currency.

The total profits of the foreign firm (valued in foreign currency) and that of the domestic firm are $\pi_F^S = \frac{4e}{3} \left[ a - q_{F1}^S - q_{D1}^S \right] g_{F1}^S + \frac{1}{9} \left[ a - q_{F1}^S - q_{D1}^S \right] \left[ \theta e_H + (1-\theta) e_L \right]$ and $\pi_D^S = \left[ a - q_{F1}^S - q_{D1}^S \right] H_{F1}^S + \frac{1}{9} \left[ a - q_{F1}^S - q_{D1}^S \right]^2$ respectively.

Maximizing the above profits with respect to $q_{F1}^S$ and $q_{D1}^S$ and simultaneously solving the first order conditions we obtain the equilibrium values of these two variables as

$$\overline{q}_{D1}^S = \left[ \frac{5ae}{17e - \theta e_H + (1-\theta)e_L} \right]$$

and $\overline{q}_{F1}^S = \left[ \frac{6e - \theta e_H + (1-\theta)e_L}{17e - \theta e_H + (1-\theta)e_L} \right]$ respectively.
Here, $6e - (\theta e_H + \{1 - \theta\} e_L) > 0$ is the sufficient condition that ensures positive values of $\bar{q}_{D_1}^S$ and $\bar{q}_{F_1}^S$. This imposes a limit that the expected appreciation should be less than 500%.\(^7\)

The profits of the foreign and domestic firms are:

$$\bar{\pi}_F^{SS} = \frac{4d^2 e^2}{\left[17e - (e_H + (1 - \theta) e_L)\right]^2} \left[12e - (\theta e_H + (1 - \theta) e_L)\right]$$

$$\bar{\pi}_D^{SS} = \frac{44d^2 e^2}{\left[17e - (\theta e_H + (1 - \theta) e_L)\right]^3}.$$

### 5. The Extended Game and the Characterization of Equilibrium Outcomes

The following table summarizes the two period game. For all the relevant proofs in the forthcoming subsections, see appendix B.

<table>
<thead>
<tr>
<th>FOREIGN FIRM (F)</th>
<th>DOMESTIC FIRM (D)</th>
<th>RENTING</th>
<th>SELLING</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>RENTING</td>
<td>$\pi_F^{RR} ; \pi_D^{RR}$</td>
<td>$\pi_F^{RS} ; \pi_D^{RS}$</td>
</tr>
<tr>
<td></td>
<td>SELLING</td>
<td>$\pi_F^{SR} ; \pi_D^{SR}$</td>
<td>$\pi_F^{SS} ; \pi_D^{SS}$</td>
</tr>
</tbody>
</table>

In our quest for the equilibrium outcomes in this strategic set up under exchange rate uncertainty, we compare individual firm’s profits between renting and selling to look out for the optimal strategies of each firm. We find that the optimal strategy for each firm depends on the degree of expected exchange rate fluctuations in a significant way.

### 5.1 Optimal Strategies of Firms

\(^7\)However any appreciation in the expected exchange rate less than 275% continues to be binding condition.
5.1.1 Strategy of the foreign firm

The profit difference of the foreign firm between renting and selling when the domestic firm adheres to renting shows the following pattern:

\[ \pi_F^{RR} - \pi_F^{SR} : \]

(a) +ve for \( 0.26666666 < \frac{e}{\text{Exp}(e)} < 0.879736 \) implying "renting is better than selling" over the expected exchange rate appreciation greater than 13.6704% but less than 27.5%.

(b) -ve for \( 0.879736 < \frac{e}{\text{Exp}(e)} \) implying "selling is better than renting" for any expected exchange rate appreciation less than 13.6704%.

The profit difference of the foreign firm between renting and selling when the domestic firm adheres to selling shows the following pattern:

\[ \pi_F^{RS} - \pi_F^{SS} : \]

(a) +ve for \( 0.26666666 < \frac{e}{\text{Exp}(e)} < 0.957099 \) implying "renting better than selling" in between the range of expected exchange rate appreciation greater than 4.482399% but less than 27.5%.

(b) -ve for \( 0.957099 < \frac{e}{\text{Exp}(e)} \) implying "selling is better than renting" over the range of expected exchange rate appreciation less than 4.482399%.

5.1.2 Strategy of the domestic firm

The profit difference of the domestic firm between renting and selling when the foreign firm adheres to renting shows the following pattern:

\[ \pi_D^{RR} - \pi_D^{RS} = -\left( \frac{106}{11025} \right) \alpha^2 \] i.e. always –ve implying selling is always better than renting.
Lemma 2

The domestic firm will always choose to sell whenever the foreign firm adheres to renting.

The profit difference of the domestic firm between renting and selling when the foreign firm adheres to selling shows the following pattern:

\[ \pi_D^{SR} - \pi_D^{SS} : \]

(a) For any value of \( \left( \frac{e}{E_{xp}(e)} \right) > 0.8958186 \) or the level of expected exchange rate appreciation less than 11.62974\% “selling is better than renting”.

(b) For any value of \( \left( \frac{e}{E_{xp}(e)} \right) < 0.8958186 \) or expected exchange rate appreciation greater than 11.62974\% “renting is better than selling”.

5.2 Equilibrium Outcomes

Given the calculations in the previous section, we can characterise the whole spectrum of equilibria for different ranges of expected exchange rate appreciation or depreciation of the domestic currency against the foreign currency. We find two predominant equilibria, one symmetric and another asymmetric in whole range of expected exchange rate fluctuations. The symmetric equilibrium corresponds to the dominant strategy equilibrium (as before with the case of no exchange rate uncertainty) where both the firms engage in selling in the range from an appreciation of less than 4.82\% to any possible depreciation of the expected exchange rate. The asymmetric equilibrium corresponds to a (subgame perfect) equilibrium where the foreign firm rents and the domestic firm sells in the range of expected exchange rate appreciation of greater than 4.82\% but less than 275\%. If the expected exchange rate appreciates further more (i.e. beyond 275\%) the selling strategy does not remain feasible for the foreign firm, hence if it wants to serve the market, it has to do so only by renting. There also exists another asymmetric (subgame perfect) equilibrium where the foreign firm sells and the domestic
firm rents within the short range of expected exchange rate appreciation of 11.62% and 13.67%. This means, within this small range of expected exchange rate appreciation we have multiple asymmetric equilibria of renting and selling.

The above is complete characterization of all possible renting and selling equilibrium with the foreign and domestic firm under different ranges of the expected exchange rate. The important lesson, we can learn from this is that there is indeed a significant impact of exchange rate uncertainty on the two commercial transactions, namely renting and selling by the competing firms. Before, we have seen that in an environment without any exchange rate uncertainty (selling, selling) is the unique dominant strategy equilibrium (see proposition 4); here interestingly, this is not anymore true. Thus, we establish that the strategic effect as well as the effect of exchange rate uncertainty plays a very crucial role in determining the outcome in a durable goods market operated by foreign and domestic firms.

**Proposition 5**

*Under the exchange rate uncertainty, the whole spectrum of equilibrium outcomes between a foreign and a domestic firm in the durable goods market can be characterized as follows:*

(i)

(a) For \( \frac{e}{\text{Exp}(e)} > 0.957 \) i.e. from an appreciation of less than 4.82% to any possible depreciation of the expected exchange rate, (selling, selling) is the unique dominant strategy equilibrium.

(b) For \( 0.266 < \frac{e}{\text{Exp}(e)} < 0.957 \) i.e. in the range of expected exchange rate appreciation of greater than 4.82% but less 275%, the equilibrium is given by (renting, selling) i.e. foreign firm rents and domestic firm sells.

(c) For \( 0.879 < \frac{e}{\text{Exp}(e)} < 0.895 \) i.e. within the expected exchange rate appreciation of 11.62% and 13.67%, there also exits the equilibrium of
(selling, renting), i.e. foreign firm sells and domestic firm rents, along with the previous equilibrium; hence we observe multiple equilibria in this range.

(ii) These equilibrium outcomes are in sharp contrast with unique dominant strategy equilibrium that we obtain in the environment without any exchange rate uncertainty (see proposition 4). Thus, the impact of exchange rate uncertainty plays a significant role in determining the optimal behaviour of firms.

5.3 Prisoners’ Dilemma

We find an interesting prisoners’ dilemma situation associated with the dominant strategy equilibrium with or without exchange rate uncertainty. (See proposition 4 and proposition 5(i)(a)). In both these equilibria, both the firms end up earning lower profit than they would in the (renting, renting) configuration. It can also been shown that the total welfare (consumer surplus in the domestic country plus profit of the domestic firm) in the domestic country is lower in (selling, selling) equilibrium as opposed to (renting, renting) equilibrium. Hence, a natural policy of the domestic country’s government would be to give the competing firms a proper incentive to rent their products in order to move towards the preferred equilibrium from the social point of view.

Proposition 6

(i) Under the exchange rate uncertainty, in the durable good markets with a foreign and domestic firm, the dominant strategy equilibrium of (selling, selling) gives the firms a lower profit than (renting, renting) configuration and hence firms end up in a situation of prisoners’ dilemma.

(ii) Social welfare of the domestic country is also lower in (selling, selling) equilibrium as opposed to the (renting, renting) configuration.

Proof: See appendix B.
6. Conclusion

In this paper, we analyzed what would be the optimal policy regarding renting and selling by durable goods firm(s). First we considered the case of a monopoly market, and then we moved on to a strategic situation. Our first finding is that a foreign durable goods monopolist facing future exchange rate fluctuations, would behave differently from a durable goods monopolist facing no such fluctuations. We showed that a foreign durable goods monopolist would rather stick to selling when it anticipates an unfavourable expected exchange rate in the future periods (i.e. when the expected exchange rate will be much lower than the present exchange rate). This implies Coase’s conjecture regarding the behaviour of durable goods monopolist is not necessarily valid in all situations even if the market structure is of monopoly.

Secondly, we also focused on the impact of the exchange rate uncertainty in a strategic framework in a durable goods market. While in an environment without any exchange rate uncertainty, we find that selling is the unique dominant strategy of the competing firms; with the introduction of exchange rate uncertainty the results change dramatically. There we find a mix of dominant strategy as well as other interesting asymmetric (subgame perfect) Nash equilibria under various ranges of expected exchange rate appreciation or depreciation of the domestic currency against foreign currency.

Thus, this study establishes the fact that both market structure as well as exchange rate uncertainty plays an important role in determining the optimal behaviour of firms operating in the durable goods market.

References


**Appendix A**

*Proof of Proposition 2 and 3.*

\[ E(\pi^s) > E(\pi^a) \]

\[ \iff (\theta e_H + (1-\theta)e_L)^2 - 3 \left( e \left( e / (\theta e_H + (1-\theta)e_L) \right) \right)^2 > 0 \]

\[ \iff (\theta e_H + (1-\theta)e_L)^2 - 1.43 \left( e / (\theta e_H + (1-\theta)e_L) - 0.23 \right) > 0 \]

Above expression is positive if \( \frac{e}{(\theta e_H + (1-\theta)e_L)} > 1.43 \) or \( \frac{e}{(\theta e_H + (1-\theta)e_L)} < 0.23 \)

We know selling equilibrium exists only when \( \frac{e}{(\theta e_H + (1-\theta)e_L)} > 0.33 \) (see the expression of \( \bar{q}_1 \)); thus a comparison between renting and selling is valid only in that parameter region.

Now observe that when \( \frac{e}{(\theta e_H + (1-\theta)e_L)} > 1.43 \), selling dominates renting.

On the other hand, when \( 0.33 \leq \frac{e}{(\theta e_H + (1-\theta)e_L)} \leq 1.43 \), renting dominates selling.
Appendix B

B.1.1 Foreign firms strategy

We subtract the profit of the foreign firm when it sells, from the profit when it rent while the domestic firm is adhering to *renting*.

The difference can be judged in terms of the following equation in terms of $\frac{e}{\text{Exp}(e)}$:

$$y_1 = \left[ -73.6875 \left( \frac{e}{\theta e_H + \{1 - \theta\} e_L} \right)^3 + 84.5625 \left( \frac{e}{\theta e_H + \{1 - \theta\} e_L} \right)^2 - 18.5 \left( \frac{e}{\theta e_H + \{1 - \theta\} e_L} \right) + 1 \right]$$

$\pi_{FR}^{RR} - \pi_{FR}^{SR}$ is positive if the value of the above equation is positive and negative when it is negative.

All the three roots of the equation above are real. They are 0.0838292, 0.184017, 0.879736. The first two being ruled out by the binding condition.

Similarly we get a cubic equation in terms of $\frac{e}{\text{Exp}(e)}$ when we subtract the profit of the foreign firm when it sells from the profit when it rents while the domestic firm is adhering to *selling*.

$\pi_{FS}^{RS} - \pi_{FS}^{SS}$ is positive or negative as and when the value of the following equation is positive or negative.

$$y_2 = \left[ - \left( \text{17184} \right) \left( \frac{e}{\theta e_H + \{1 - \theta\} e_L} \right)^3 + \left( \text{18500} \right) \left( \frac{e}{\theta e_H + \{1 - \theta\} e_L} \right)^2 - \left( \text{2032} \right) \left( \frac{e}{\theta e_H + \{1 - \theta\} e_L} \right) + 1 \right]$$

Of the three roots only one is real and the rest two are imaginary. They are 0.9579099, 0.597421-0.0179504i & 0.597421+0.0179504i respectively. We are however concerned only with real roots throughout our analysis.

B.1.2 Domestic firms strategy

Comparing the profits of the domestic firm when the foreign firm sells we get the following.
\( \pi^S_D - \pi^S_S \) is positive or negative as an when the following equation is positive or negative:

\[
y_A = -\left( \frac{6812}{496} + \frac{e}{\theta e_H + \{1-\theta\} e_L} \right)^2 + \left( \frac{6656}{496} + \frac{e}{\theta e_H + \{1-\theta\} e_L} \right) - 1
\]

The quadratic equation has two roots, 0.895816 and 0.0812806 respectively. The second one is however ruled out by the binding condition. \( \pi^S_D - \pi^S_S \) is negative for any \( \left( \frac{e}{\theta e_H + \{1-\theta\} e_L} \right) > 0.89558186 \), implying selling is the optimal strategy for the domestic firm when the foreign firm resorts to selling. However, for any \( \left( \frac{e}{\theta e_H + \{1-\theta\} e_L} \right) < 0.89558186 \), renting would be optimal.

### B.2 Prisoners dilemma comparison

a. \( \pi^R_D - \pi^S_D > 0 \). This holds for \( \frac{e}{\text{Exp}(\theta)} > 0.3414422 \).

b. \( \pi^R_F - \pi^S_F > 0 \). This holds for \( \frac{e}{\text{Exp}(\theta)} < 1.91645 \).

Therefore both "a" and "b" holds simultaneously also over the range \( 0.3414422 < \frac{e}{\text{Exp}(\theta)} < 1.91645 \) i.e., for any any expected exchange rate appreciation less than 192.8753\% to a depreciation of less than 47.8202\%. This result leads to the case of prisoner's dilemma for any expected exchange rate depreciation less than 47.82018837 to an expected appreciation of 4.482399\%.

The equation we arrive at while calculating \( \pi^R_F - \pi^S_F \) to enquire into the possibility of prisoner's dilemma is also cubic in \( \frac{e}{\text{Exp}(\theta)} \). \( \pi^R_F - \pi^S_F \) is positive or negative when the following equation is positive or negative:

\[
y_4 = -143 \left[ \frac{e}{\theta e_H + \{1-\theta\} e_L} \right]^3 + 291 \left[ \frac{e}{\theta e_H + \{1-\theta\} e_L} \right]^2 - 33 \left[ \frac{e}{\theta e_H + \{1-\theta\} e_L} \right] + 1
\]
Two of the three roots of the equation above are imaginary. The roots are 1.91645, 0.0592553-0.011736i, 0.0592553+0.011736i.

Again $\pi^{RR}_D - \pi^{SS}_D$ is positive or negative as an when the following quadratic equation is positive or negative:

$$y_s = 9 \left( \frac{e}{\theta e_H + (1 - \theta) e_L} \right)^2 - 34 \left( \frac{e}{\theta e_H + (1 - \theta) e_L} \right) + 1$$

*This quadratic equation has two roots 0.3414422 and 0.032184 respectively. The second one is however ruled out by the binding condition.*

**B.3 Welfare comparisons**

The difference of welfare when both the firm rents as compared to the case when both the firm sells is:

$$w_{RR} - w_{SS} = 510 \alpha^2 e \left( \frac{e}{\theta e_H + (1 - \theta) e_L} \right) \left[ \frac{e}{9(17e - \theta e_H + (1 - \theta) e_L)^2} - 0.1411764 \right] \left[ \frac{e}{\theta e_H + (1 - \theta) e_L} \right].$$

This implies that $w_{RR} > w_{SS}$ when $\left( \frac{e}{\theta e_H + (1 - \theta) e_L} \right) > 0.1666666$ or any expected exchange rate appreciation less than 500.0024%. This is always satisfied because of the binding condition$^8$.

---

$^8$ The binding condition being that expected exchange rate appreciation should be less than 275%.