Residual-Based Diagnostics for Conditional Heteroscedasticity Models

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Abstract: We examine the residual-based diagnostics for univariate and multivariate conditional heteroscedasticity models. The tests involve the parameter estimates of an autoregression with the squared standardised residuals and the cross products of the standardised residuals as dependent variables. Noting that the regression involves estimated regressors and the standard distribution theories of the ordinary least squares estimates do not apply, we derive the asymptotic variance of the regression estimates. Diagnostic statistics, which are asymptotically distributed as \( \chi^2 \); are constructed. We conduct a Monte Carlo experiment to investigate the finite sample properties of the residual-based tests for both univariate and multivariate models. The results are favourable to the residual-based diagnostics compared to the portmanteau statistics used in the literature.

Key Words: conditional heteroscedasticity, Lagrange multiplier test, Monte Carlo experiment, portmanteau statistic, residual-based diagnostic

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1 Introduction

Since the seminal paper by Engle (1982) on the autoregressive conditional heteroscedasticity (ARCH) models, many alternative models have been suggested to capture the time-varying variance of time series. Bollerslev's (1986) generalized ARCH (GARCH) model is a natural extension, with lagged conditional variances introduced as explanatory variables in the conditional-variance equation. Further extensions were suggested by Nelson (1991) (exponential GARCH (EGARCH) model), Higgins and Bera (1992) (nonlinear ARCH (NARCH) model), Glosten, Jagannathan and Runkle (1993) (an asymmetric model commonly called the GJR model), Ding, Granger and Engle (1993) (asymmetric power ARCH (APARCH) model) and Zakoian (1994) (threshold ARCH (TARCH) model). Hentschel (1995) proposed a model that encompasses many of the alternative existing models. For surveys of the developments and applications of these models, see Bollerslev, Chou and Kroner (1992), Bera and Higgins (1993) and Bollerslev, Engle and Nelson (1994).

The success of the conditional heteroscedasticity models in modelling univariate time series has motivated many researchers to extend these models to the multivariate case. Formulations of multivariate conditional heteroscedasticity models include the vech-representation form due to Bollerslev, Engle and Wooldridge (1988), the constant-correlation multivariate GARCH (CC-MGARCH) model due to Bollerslev (1990) and the BEKK (named after Baba, Engle, Kraft and Kroner) model due to Engle and Kroner (1995). Within the vech-representation family, the diagonal form, which we shall denote as the VD model, has been applied in many empirical works. Recently, a new multivariate conditional heteroscedasticity model has been suggested by Tse and Tsui (1998).

As empirical researchers are equipped with various conditional heteroscedasticity models, the checking of the adequacy of a fitted model becomes important for model...
selection. The diagnostics applied in the literature can be divided into three categories: portmanteau tests of the Box-Pierce-Ljung type, Lagrange multiplier (LM) tests and residual-based diagnostics. The Box-Pierce-Ljung portmanteau statistic is perhaps the most widely used diagnostic. It is readily computable from the standardised residuals and has been used in many empirical works for model diagnostics (see, for example, the papers by Bollerslev (1990), Ballie and Myers (1991) and Karolyi (1995)). Although it has been noted that the portmanteau statistics do not have an asymptotic $\chi^2$ distribution, many authors, nonetheless, apply the $\chi^2$ distribution as an approximation. Li and Mak (1994) pointed out the lack of rigour in this approach and proceeded to derive the asymptotic distribution of the portmanteau statistics in the univariate case. Ling and Li (1997) further developed this work and derived the asymptotic distribution of the portmanteau statistics in the multivariate case. The Ling-Li statistic is based on the serial correlation coefficients of the transformed vector of residuals. The recent test suggested by Hong and Shehadeh (1999) is a portmanteau test based on the spectral density function rather than the serial correlation coefficients. Tse and Zuo (1998) and Tse and Tsui (1999) reported some Monte Carlo results on the finite-sample distributions of the Li-Mak test and the Ling-Li test, respectively.

The LM test has an advantage over the portmanteau test due to its efficiency when the alternative hypothesis is correct. The test, however, requires the specification of an alternative, and it may not have good power against other alternatives. Also, the calculation of the test statistic depends on the alternative and thus makes this approach less attractive as a general diagnostic. For the applications of LM tests to conditional heteroscedasticity models, see Bollerslev, Engle and Wooldridge (1988), Engle and Ng (1993) and Engle and Kroner (1995). Bera and Higgins (1992) suggested a diagnostic of the ARCH models against the NARCH alternatives based on the LM principle. Tse (1999) proposed a test for constant correlations in a multivariate GARCH model using the LM approach. Lundbergh and Terasvirta (1998) provided a unified framework
for testing univariate conditional heteroscedasticity models based on the LM principle. Some results on the equivalence between the LM and the portmanteau tests in certain cases can be found in Ling and Li (1997) and Lundbergh and Terasvirta (1998).

Like the portmanteau tests, residual-based diagnostics have no specific alternative. General model adequacy is investigated using the residuals. The diagnostics involve running artificial regressions and testing for the statistical significance of the regression parameters. To a certain extent, the form of the regression depends on a particular type of model inadequacy the researcher wants to investigate. Extensive discussions of this approach can be found in Pagan and Hall (1983). For testing against adequacy of the conditional variance structure, lagged squared standardised residuals may be used, as suggested in Bollerslev (1990). The asymptotic distributions of the estimated regression parameters, however, have not been established. As the regressors are estimated, the usual ordinary least squares (OLS) result does not apply. Empirical research, however, typically adopts the usual OLS procedure as an approximation. In the multivariate case, the Monte Carlo results reported by Tse and Tsui (1999) showed that the use of the OLS inference procedure grossly under-rejects the null hypothesis of model adequacy.

In this paper we derive the asymptotic distributions of the residual-based diagnostics for the conditional heteroscedasticity models. Both univariate and multivariate models are considered. In the multivariate case we propose to examine the squared standardised residuals as well as the cross products of the standardised residuals. Diagnostic statistics based on the correct asymptotic variance of the OLS regression parameter estimates are constructed. We examine the finite-sample properties of the residual-based diagnostics using Monte Carlo methods. It is found that in the univariate case the residual-based diagnostics have favourable power against the Li-Mak portmanteau test. In the multivariate case, our results also show that the residual-based diagnostics provide a useful check for model adequacy.

The plan of the rest of the paper is as follows. In Section 2 we present the results for
the asymptotic distributions of the residual-based diagnostic tests for the univariate as well as multivariate conditional heteroscedasticity models. Section 3 reports the Monte Carlo results of the finite-sample distributions of the residual-based diagnostics. We consider a variety of univariate and multivariate conditional heteroscedasticity models. Both the size and the power of the diagnostics are studied. Finally, we give some concluding remarks in Section 4.

2 Residual-Based Diagnostic Tests

Residual-based diagnostics are tests constructed to search for particular residual patterns implied by the deviation of the fitted model from its underlying assumptions. Pagan and Hall (1983) provides a wide-ranging and comprehensive coverage of residual-based tests. These tests may be designed to diagnose particular types of model misspecifications, including serial correlation, heteroscedasticity, constancy of coefficients, nonnormality, simultaneity and so on. In this paper, our concern is conditional heteroscedasticity in time series models.

2.1 The Univariate Case

Consider a univariate time series \(X_t\), for \(t = 1; \ldots; T\), with conditional heteroscedasticity generated by the following equations:

\[
X_t | \tau_t = \epsilon_t; \quad (1)
\]

\[
\epsilon_t = \frac{\gamma}{\sqrt{t}} \tau_t; \quad (2)
\]

where \(\gamma_{t}\) are independently and identically distributed with mean zero and variance 1, and \(\tau_t\) and \(\gamma_t\) are, respectively, the conditional mean and variance of \(X_t\) based on the information set \(\mathcal{F}_{t-1}\) at time \(t = 1\). For the exposition in this paper, \(\gamma_{t}\) is assumed to be Gaussian. This framework incorporates many ARCH and GARCH type of conditional heteroscedasticity models applied in the literature. Furthermore, \(\tau_t\) may be nonlinear
functions of past observations and/or dependent on weakly exogenous variables.

Let $\mu$ be the parameter vector of the model and $\hat{\mu}$ be the maximum likelihood estimator (MLE) of $\mu$. Here we assume that $\mu$ is with $N$ elements and contains all the parameters appearing in $\xi_t$ and $\eta_t$: We define

$$\gamma_t(\mu) = \frac{1}{2} \left( \log \frac{\eta_t^2}{\xi_t^2} + \frac{\eta_t^2}{\xi_t^2} \right);$$

(3)

so that the log-likelihood function $\gamma(\mu)$, ignoring the constant, is given by $\gamma(\mu) = P \gamma_t(\mu)$. We assume that the usual regularity conditions hold so that

$$P_T (\hat{\mu} - \mu) \xrightarrow{D} N(0; G);$$

(4)

where $P$ denotes convergence in distribution and $G$ is the asymptotic variance of $P_T (\hat{\mu})$. In addition, $G$ is the limit of $\left[ T^{-1} \mathbb{E} \left( \partial^2 \gamma(\hat{\mu}) \right) \right]^{-1}$ and can be consistently estimated by $\hat{G} = T^{-1} \mathbb{E} \left( \partial^2 \gamma(\hat{\mu}) \right)^{-1}$.\(^3\)

Denoting $\eta_t^2$ as $\eta_t^2$ evaluated at $\hat{\mu}$ and $\gamma_t$ as the estimated residual, we define the standardised residual as $e_t = \gamma_t - \eta_t$. Following Pagan and Hall (1983) and Bollerslev (1990), model diagnostics can be conducted using the standardised residuals. Noting that the squared standardised residuals tend to 1 in probability, we run a regression of $e_t^2$ on some information variables and examine the statistical significance of the regression parameters. The lagged standardised residuals are natural regressors to use. Thus, denoting $d_t = (e_{t-1}^2; \ldots; e_{t-M}^2)^0$, we consider the regression

$$e_t^2 = d_t + \xi_t;$$

(5)

where $\pm$ is an $M$-vector of regression parameters. We denote the OLS estimator of $\pm$ by $\hat{\pm}$

\(^1\)For simplicity we drop the summation indexes, unless there is a possibility of confusion. Also, we assume that presample observations, if required, are fixed and known. This assumption, of course, has no effect on the asymptotic theory.

\(^2\)See, for example, Bollerslev and Wooldridge (1992) and White (1994) for the details.

\(^3\)Note that $@^2(\hat{\mu}) = @^2(\hat{\mu})$ denotes $@^2(\hat{\mu}) = @^2(\hat{\mu})$ evaluated at $\mu = \hat{\mu}$. Likewise, $@^2(\hat{\mu}) = @^2(\hat{\mu})$ denotes $@^2(\hat{\mu}) = @^2(\hat{\mu})$ evaluated at $\mu = \hat{\mu}$. This convention will be extended to other derivatives such as $@e_t^2 = @e_t^2$, which is $@^2 e_t^2 = @^2 e_t^2$ evaluated at $\mu = \hat{\mu}$.
As \( \hat{d}_t \) consists of estimated regressors, the inference procedure based on the usual OLS results is invalid. This point was stressed by Pagan and Hall (1983) in a broader context. The appropriate procedure is to correct for the asymptotic variance of the OLS estimate. The proposition below provides the necessary correction (see Appendix I for the proof).

**Proposition 1:** If equations (1) and (2) specify the correct model for the univariate time series \( fX_tg \), then \( P \mathbf{T} \hat{d} \mathbf{P} \ N(0; L^{-1} L^{-1}) \), where

\[
L = \text{plim}\left( \frac{1}{T} X_t^\prime d_t d_t^\prime \right); \quad (6)
\]

\[
- \hat{\omega}_L = 2L^\prime QGQ \hat{\omega}^{\prime}; \quad (7)
\]

with

\[
Q = \text{plim}\left( \frac{1}{T} X_t^\prime d_t \frac{\partial^2}{\partial \mu^\prime} \right) \quad (8)
\]

and \( d_t = (\hat{\mu}_2^2; \ldots; \hat{\mu}_M^2)^\prime \).

In empirical applications, \( L \) and \( Q \) may be estimated by \( \hat{L} = \left( \frac{1}{T} \hat{d}_t^\prime d_t \right) = T \) and \( \hat{Q} = \left( \frac{1}{T} \hat{d}_t \frac{\partial^2}{\partial \mu^\prime} \right) = T \), respectively. Under Gaussian assumption, \( \hat{L} \) may be replaced by the \( M \times M \) matrix with 3 in the diagonal and 1 elsewhere. Thus, when there is no misspecification in the model, \( T \hat{L} \hat{\omega}_L \) is asymptotically distributed as a \( \chi^2_M \), where

\[
\hat{\omega}_L = 2\hat{L}^\prime \hat{Q}\hat{G}\hat{Q}^{\prime}; \quad (9)
\]

The derivatives in \( \hat{Q} \) may be computed using numerical methods. Alternatively, for GARCH type of models recursive formulae as given by Fiorentini, Calzolari and Panattoni (1996) and Tse (1999) may be used.

Following the arguments of Ling and Li (1997) and Lundbergh and Terasvirta (1998) it can be shown that the residual-based diagnostic is asymptotically equivalent to the portmanteau statistic as well as the LM statistic of no ARCH in the standardised errors against ARCH(M). This result is particularly clear when we use the sample moment of \( \hat{d}_t \) as an estimate of \( L \) so that the residual-based statistic becomes \( P \hat{d}_t^\prime \hat{V}_t \hat{d}_t^\prime \mathbf{T} \),
where \( v_t = e_t^2 \). As the vector of autoregressive coefficients are asymptotically equivalent to \( (P \hat{\alpha}_t)^{(2T)} \) under the null, the residual-based statistic is asymptotically equivalent to the portmanteau statistic. However, two points should be made here. First, although the tests are asymptotically equivalent under model adequacy, they may differ under model misspecification. That is, the power of the tests may differ. Second, the performance of the tests may differ in finite samples.

### 2.2 The Multivariate Case

In this subsection notations are redefined to cater for multivariate observations. Thus, \( X_t = (X_{t1}; \ldots; X_{tK})^0 \) denote a \( K \) -vector of observations generated by the following equations

\[
X_{ti} = \nu_t + \epsilon_t
\]

\[
\nu_t = \nu_t^{1 \Rightarrow 2},
\]

where \( \nu_t \) and \( \epsilon_t \) are \( K \)-vectors of residual and conditional mean, respectively, \( \nu_t = f \gamma_{ti} g \) is the conditional variance matrix of \( \nu_t = (\nu_{t1}; \ldots, \nu_{tK})^0 \), and \( \epsilon_t = (\epsilon_{t1}; \ldots, \epsilon_{tK})^0 \) are independently and identically distributed normal variates with mean zero and variance \( \Gamma K \) (the \( K \times K \) identity matrix). Again we let \( \mu \) be the \( N \) -vector of parameters of the model and \( \hat{\mu} \) be the MLE of \( \mu \). Thus, defining

\[
\hat{\nu}(\mu) = \frac{1}{2} \log |V_t| + \nu_t^{0 \Rightarrow 1 \Rightarrow 2},
\]

the log-likelihood function \( \nu(\mu) \), ignoring the constant term, is given by \( \nu(\mu) = P \nu(\mu) \). Under regularity conditions, \( P T (\hat{\mu}, \mu) \) \( P \) \( N (0, G) \), where \( G \) is the limit of \( \nu_1 E (T^{1 \Rightarrow 2}) \nu(\mu) = \mu (\mu) \) and can be consistently estimated by \( \hat{G} = \nu_1 (T^{1 \Rightarrow 2}) \nu(\mu) = \mu (\mu) \).

Let \( \hat{\nu}_t = f \gamma_{ti} g \) be the estimated conditional variance matrix, \( \nu_t = (\nu_{t1}; \ldots; \nu_{tK})^0 \) be the estimated residual and \( \epsilon_t = (\epsilon_{t1}; \ldots; \epsilon_{tK})^0 \) be the standardised residual with \( \epsilon_{ti} = \nu_{ti} \nu_{ti}^{1 \Rightarrow 2} : \text{We also denote } \gamma_{tij} = \gamma_{tij} = (\gamma_{tij} \gamma_{tij})^{1 \Rightarrow 2} \text{ as the conditional correlation and } \gamma_{tij} = \)
\(\gamma_{ti} = (\gamma_{ti}, \gamma_{tj})^{\frac{1}{2}}\) as its estimated value. Residual-based diagnostics can be conducted on the squared standardised residuals and the cross products of the standardised residuals. Extending the results on the univariate case, we run regressions of \(e_{ti}^2\) 1, for \(i = 1, \ldots, K\); and \(e_i e_j\) \(\gamma_{tij}\), for \(1 \leq i < j \leq K\), on some information variables. Again, the lagged squared standardised residuals and the lagged cross products of the standardised residuals are natural candidates. Thus, denoting \(\hat{d}_{ti} = (e_{ti}^2, e_{tj}^2; e_{ti} e_{tj}; e_{ti}^2 e_{tj}; e_{ti} e_{tj} e_{ti} e_{tj})^0\) and \(\hat{d}_{tij} = (e_{ti} e_{tj}; e_{ti} e_{tj}; e_{ti}^2 e_{tj}; e_{ti} e_{tj}; e_{ti} e_{tj} e_{ti} e_{tj})^0\), we consider the following regressions\(^4\)

\[
e_{ti}^2 = \hat{d}_{ti}^0 \pm_i + \nu_{ti}; \quad i = 1, \ldots, K; \tag{12}
\]

\[
e_i e_j \gamma_{tij} = \hat{d}_{tij}^0 \pm_{ij} + \nu_{tij}; \quad 1 \leq i < j \leq K; \tag{13}
\]

where \(\pm\) and \(\pm_i\) are \(M\)-vectors of regression parameters. We further define \(\hat{s}_{ti} = \frac{e_{ti}}{\gamma_{ti}}\), \(\hat{d}_{ti} = (\hat{s}_{ti}^2; \ldots; \hat{s}_{ti}^2)^0\) and \(\hat{d}_{tij} = (\hat{s}_{ti} \hat{s}_{tj}; \ldots; \hat{s}_{ti} \hat{s}_{tj} \hat{s}_{ti} \hat{s}_{tj})^0\). The following propositions provide the asymptotic distributions of the OLS estimators \(\hat{\pm}\) and \(\hat{\pm}_i\) of \(\pm\) and \(\pm_i\); respectively (see Appendix II for the proof).

**Proposition 2:** If the equations (9) and (10) specify the correct model for the multivariate time series \(fX_t\), then \(P \mathcal{T} \hat{f}_X \mathcal{T} P \mathcal{N}(0; L \mathcal{I}^{-1} L \mathcal{I}^{-1})\), where

\[
L = \text{plim} \left( \frac{1}{T} X \hat{d}_t \hat{d}_t^0 \right); \tag{14}
\]

\[
\mathcal{I}^{-1} = 2L + Q_i G Q_i^0 + P_i G Q_i^0 + Q_i G P_i^0; \tag{15}
\]

with

\[
Q_i = \text{plim} \left( \frac{1}{T} X \hat{d}_t \frac{\hat{\mu}_i^2}{\hat{\mu}_i^0} \right); \tag{16}
\]

\[
P_i = \text{plim} \left( \frac{1}{T} (X \hat{d}_t (\hat{s}_{ti}^2; 1)^0) \frac{\hat{\mu}_i (\hat{\mu})}{\hat{\mu}_i^0} \right); \tag{17}
\]

Note that for \(K = 1\), \(P_i = i Q_i\) and equation (15) can be reduced to equation (7). For \(K > 1\), such a simplification is generally not obtainable. To compute a diagnostic we...
replace the matrices $L_i$, $Q_i$, $P_i$ and $-i$ by their sample analogues and estimates, denoted by hats, so that

\[
\hat{L}_i = \frac{1}{T} \sum d_{ti} d_{tij}^0; \quad (18)
\]
\[
\hat{Q}_i = \frac{1}{T} \sum d_{ti} \frac{\hat{e}_{tij}}{\hat{e}_{tij}^0}; \quad (19)
\]
\[
\hat{P}_i = \frac{1}{T} \left( \sum d_{tij} (\hat{e}_{tij}^2 - 1) \right) \frac{\hat{(\mu)/\hat{\sigma}_t^2}}{\hat{\sigma}_t^0}; \quad (20)
\]

and

\[
\hat{-i} = 2\hat{L}_i + \hat{Q}_i \hat{G} \hat{Q}_0 + \hat{P}_i \hat{G} \hat{Q}_0 \hat{P}_0; \quad (21)
\]

Under Gaussian assumption, $\hat{L}_i$ can be replaced by the $M \times M$ matrix with 3 in the diagonal and 1 elsewhere. The test statistic can then be calculated as $T \hat{\hat{\hat{\hat{\hat{\hat{L}}}}}}_{-i} - 1 \hat{L}_i \hat{Q}_i$, which is asymptotically distributed as a $\chi^2_M$.

**Proposition 3:** If equations (9) and (10) specify the correct model for the multivariate time series $fX_t g$, then $P \sum \hat{\hat{\hat{\hat{\hat{\hat{L}}}}}}_{-i} - 1 \hat{L}_i \hat{Q}_i \sim \chi^2_M$, where

\[
L_{ij} = \text{plim} \left( \frac{1}{T} \sum d_{tij} d_{tij}^0 \right) \quad (22)
\]
\[
-_{ij} = C_{ij} + Q_{ij} \hat{G} \hat{Q}_0 + P_{ij} \hat{G} \hat{Q}_0 \hat{P}_0 \quad (23)
\]

with

\[
C_{ij} = \text{plim} \left( \frac{1}{T} \sum (1 + \frac{1}{\hat{e}_{ij}^0}) d_{tij} d_{tij}^0 \right); \quad (24)
\]
\[
Q_{ij} = \text{plim} \left( \frac{1}{T} \sum d_{tij} \frac{\hat{e}_{tij}^3}{\hat{e}_{tij}^0} \right); \quad (25)
\]
\[
P_{ij} = \text{plim} \left( \frac{1}{T} \sum d_{tij} (\hat{e}_{tij}^3 - 1) \right) \frac{\hat{(\mu)/\hat{\sigma}_t^2}}{\hat{\sigma}_t^0}; \quad (26)
\]

The matrices $L_{ij}; C_{ij}; Q_{ij}; P_{ij}$ and $-_{ij}$ can be consistently estimated by their sample analogues given by

\[
\hat{L}_{ij} = \frac{1}{T} \sum d_{tij} d_{tij}^0; \quad (27)
\]
\[
\hat{C}_{ij} = \frac{1}{T} \sum_{t=1}^{T} \left( 1 + \frac{\gamma_{ij}^2}{2} \right) \hat{d}_{ij} \hat{d}_{ij}^0 ;
\]

(28)

\[
\hat{Q}_{ij} = \frac{1}{T} \sum_{t=1}^{T} \left( \frac{\hat{d}_{ij}^T \hat{e}_{it} \hat{e}_{jt} \gamma_{ij}^2}{\hat{e}_{it}^T \hat{e}_{jt}^0} \right) ;
\]

(29)

\[
\hat{P}_{ij} = \frac{1}{T} \sum_{t=1}^{T} \left( \frac{\hat{d}_{ij}^T (\hat{e}_{it} \hat{e}_{jt} \gamma_{ij}^2) \hat{e}_{it}^T \hat{e}_{jt}^0}{\hat{e}_{it}^T \hat{e}_{jt}^0} \right) ;
\]

(30)

and

\[
\hat{\gamma}_{ij} = \hat{C}_{ij} + \hat{Q}_{ij} G \hat{Q}_{ij}^0 + \hat{P}_{ij} G \hat{P}_{ij}^0 + \hat{Q}_{ij} G \hat{P}_{ij}^0 ;
\]

(31)

Under Gaussian assumption \( \hat{C}_{ij} \) may be replaced by the \( M \times M \) matrix with \( 1 + 2(\frac{\gamma_{ij}^2}{2} = T) \) in the diagonal and \( P(\frac{\gamma_{ij}^2}{2} = T) \) elsewhere. Likewise, \( \hat{C}_{ij} \) may be replaced by the \( M \times M \) matrix with \( P(1 + \gamma_{ij}^2)(1 + 2\gamma_{ij}^2) = T \) in the diagonal and \( P(1 + \gamma_{ij}^2) \gamma_{ij}^2 = T \) elsewhere. A diagnostic test statistic may be calculated as \( T \hat{\gamma}_{ij}^0 \hat{\gamma}_{ij} \), which is asymptotically distributed as a \( \chi^2_M \).

We conclude this section by making the caveat that we have proposed a battery of tests which are obviously dependent. Thus, there is the issue of controlling the size of multiple tests. Unlike the Ling-Li test, which is a general diagnostic for multivariate conditional heteroscedasticity, the diagnostics we propose for the multivariate case examine separate aspects of possible model misspecification. Notwithstanding the problem of multiple test, examining a battery of diagnostics provides additional information about the likely source of misspecification and thus may provide clues as to how the model may be reformulated if a diagnostic is found to be significant.

3 Some Monte Carlo Results

In this section we report the results of a Monte Carlo experiment on the finite-sample distributions of the diagnostics suggested in Section 2. Subsection 3.1 discusses the results of the univariate case, while Subsection 3.2 discusses the results of the multivariate case.
3.1 The Univariate Case

We consider three data generating processes (DGP), denoted by $M_1$, $M_2$ and $M_3$. These are low-order ARCH and GARCH processes. $M_1$ is an ARCH(1) process, $M_2$ is an ARCH(2) process and $M_3$ is an GARCH(1, 1) process. The parameters of the DGP are given by the following conditional-variance equations:

\[
M_1 : \quad \sigma^2_t = 0.2 + 0.6 \sigma^2_{t-1}; \quad (32)
\]

\[
M_2 : \quad \sigma^2_t = 0.2 + 0.4 \sigma^2_{t-1} + 0.4 \sigma^2_{t-2}; \quad (33)
\]

\[
M_3 : \quad \sigma^2_t = 0.2 + 0.6 \sigma^2_{t-1} + 0.2 \sigma^2_{t-1}; \quad (34)
\]

The conditional mean $\mu_t$ of each DGP is assumed to be zero. Given a DGP we generate samples of $T$ observations and run a conditional heteroscedasticity model to the data. The estimated model (EM) considered are ARCH(1), ARCH(2) and GARCH(1, 1).\textsuperscript{5} Various diagnostics are then calculated. We denote $RB_M$ as the adjusted (with the correct asymptotic variance) residual-based diagnostic with $M$ lagged regressors, $POR_M$ as the Li-Mak portmanteau statistic with $M$ lagged autocorrelation coefficients and $OLS_M$ as the residual-based diagnostic using the unadjusted OLS variance. All test statistics are compared against the $\chi^2$ critical value. We let $T = 200, 500$ and $1000$, and estimate the empirical size and power of the diagnostics based on Monte Carlo sample size of 1000.

It has been noted that the validity of the diagnostics does not depend on $M$ being large. Indeed, the Monte Carlo results of Tse and Zuo (1997) showed that the power of the Li-Mak test drops when $M$ is large. Thus, we consider low values of $M = 1; 2; 3$ and 4. We first examine the empirical size of the test when the correct model is estimated for each of the DGP. We set the nominal size to be 5 percent. The empirical size of the three types of diagnostics are summarized in Table 1.

It can be seen that the OLS test grossly under-rejects the null hypothesis of model adequacy. The $RB$ and $POR$ tests generally have quite reliable size, although there is a

\textsuperscript{5}A constant mean is estimated for all models.
slight tendency for these tests to over-reject rather than under-reject. Rather remarkably, these diagnostics give good empirical size even for relatively small sample size of 200.

Table 2 summarizes the results of the power consideration of the tests. Four combinations of the DGP and EM, in which the DGP is not nested within the EM, are considered. It can be seen that the RB test has higher empirical power than the POR test for most cases. Not surprisingly, the OLS test has the weakest empirical power for most cases. It is clear that the empirical power of all tests is lowest when $M = 1$. Otherwise, there seems to be no clear-cut choice among $M = 2; 3$ or 4. Overall, the results suggest that the properly defined residual-based test provides a useful diagnostic for conditional heteroscedasticity.

3.2 The Multivariate Case

Recently Tse and Tsui (1999) examined the performance of several model diagnostics for multivariate conditional heteroscedasticity models. They compared the Ling-Li portmanteau test with the (uncorrected) equation-by-equation portmanteau test and (uncorrected) residual-based test in a Monte Carlo experiment. They found that the Ling-Li test may have very weak power under certain circumstances. The uncorrected residual-based test grossly under-rejects the null, while the uncorrected portmanteau test based on the cross products of the standardised residuals may provide a useful diagnostic.

The estimation of multivariate conditional heteroscedasticity models is computationally more tricky than the univariate models. The main difficulty lies in controlling the conditional variance matrix to be positive definite in each iteration. There are a number of alternative forms of multivariate conditional heteroscedasticity models in the literature. Of these models, the CC-MGARCH appears to have the best convergence

\footnote{Other combinations in which the DGP is nested within the EM, such as DGP = M1 and EM = ARCH(2) or GARCH(1, 1), are not considered.}

\footnote{Although the Tse-Tsui study showed that the asymptotic $\hat{A}^2$ approximation works well for the portmanteau statistics the correct asymptotic distribution of the test has not been established. It can be seen, however, that the asymptotic distributions of the portmanteau statistics based on the standardised residuals of individual equations can be developed along the lines in Section 2.}
property. While the BEKK model is deemed to provide a positive definite conditional variance matrix regardless of the parameter values, our experience is that the convergence of the BEKK model is extremely slow. Although the experiment started with investigating the CC-MGARCH model and the BEKK model as the EM, we decided to abort the BEKK model as we proceeded. In this paper we report the Monte Carlo results with the CC-MGARCH model as the only EM.

We consider the following DGP:

1. CC-MGARCH Model:
   \[
   \begin{align*}
   \gamma_{411}^t & = 0.2 + 0.8 \gamma_{411}^{t-1} + 0.1 \gamma_{412}^{t-1} + 0.2 + 0.8 \gamma_{412}^{t-1} + 0.1 \gamma_{412}^{t-1}, \\
   \gamma_{422}^t & = 0.2 + 0.8 \gamma_{422}^{t-1} + 0.1 \gamma_{422}^{t-1} \\
   \gamma_{412}^t & = 0.5 \gamma_{411}^t \gamma_{422}^t
   \end{align*}
   \]

2. BEKK(D) Model:
   \[
   \begin{align*}
   V_t = \begin{bmatrix} \tilde{A} & \tilde{A} & \tilde{A} \\ \tilde{A} & \tilde{A} & \tilde{A} \end{bmatrix} + \begin{bmatrix} 0.2 & 0.1 & 0.8 & 0 & 0.8 \\ 0.1 & 0.2 & 0 & 0.8 & 0 \\ 0.4 & 0 & 0.4 & 0 & 0.4 \\ 0 & 0.4 & 0.4 & 0 & 0.4 \\ 0 & 0.4 & 0.4 & 0.4 & 0.4 \\ 0 & 0.4 & 0.4 & 0.4 & 0.4 \\ 0 & 0.4 & 0.4 & 0.4 & 0.4 \\ 0 & 0.4 & 0.4 & 0.4 & 0.4 \\ 0 & 0.4 & 0.4 & 0.4 & 0.4 \end{bmatrix} V_{t-1} \\
   \end{align*}
   \]

3. BEKK Model:
   \[
   \begin{align*}
   V_t = \begin{bmatrix} \tilde{A} & \tilde{A} & \tilde{A} \\ \tilde{A} & \tilde{A} & \tilde{A} \end{bmatrix} + \begin{bmatrix} 0.2 & 0.1 & 0.8 & 0 & 0.8 \\ 0.1 & 0.2 & 0 & 0.8 & 0 \\ 0.4 & 0 & 0.4 & 0 & 0.4 \\ 0 & 0.4 & 0.4 & 0 & 0.4 \\ 0 & 0.4 & 0.4 & 0.4 & 0.4 \\ 0 & 0.4 & 0.4 & 0.4 & 0.4 \\ 0 & 0.4 & 0.4 & 0.4 & 0.4 \\ 0 & 0.4 & 0.4 & 0.4 & 0.4 \\ 0 & 0.4 & 0.4 & 0.4 & 0.4 \end{bmatrix} V_{t-1} \\
   \end{align*}
   \]

4. VD Model:
   \[
   \begin{align*}
   \begin{bmatrix} \gamma_{411} & \gamma_{412} & \gamma_{422} \\ \gamma_{412} & \gamma_{422} \end{bmatrix} = \begin{bmatrix} \begin{bmatrix} 0 & 0.2 + 0.6 \gamma_{411}^{t-1} + 0.2 \gamma_{412}^{t-1} \\ 0.1 + 0.4 \gamma_{412}^{t-1} + 0.1 \gamma_{412}^{t-1} \\ 0.2 + 0.6 \gamma_{422}^{t-1} + 0.2 \gamma_{422}^{t-1} \\ 0.1 + 0.4 \gamma_{422}^{t-1} + 0.1 \gamma_{422}^{t-1} \end{bmatrix} & \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \end{bmatrix} \\
   \end{align*}
   \]

Table 3 summarises the results of the empirical size and power of the diagnostics. LLM denotes the Ling-Li statistic with lag-M autocorrelation coefficients. R1-M, R2-M and R3-M denote the residual-based statistics with, respectively, $e_{t1}^2$, $1$, $e_{t2}^2$, $1$ and $e_{t3}^2$, $1$.

---

\[8\] This acronym denotes the diagonal form of the BEKK model.
$e_1 e_2 i$ as the dependent variables in the articial regression. We consider $M = 1; 2; 3$ and 4, $T = 500$ and 1000, and use Monte Carlo samples of 1000. Rejection frequencies at nominal size of 5 percent are recorded. It can be seen that the tests have good empirical size for the sample sizes considered. There may be signs of over-rejection for $M = 4$. Otherwise, the nominal size appears to be accurate. Clearly, the R3 test (based on $e_1 e_2 i$) represents the test with the best power against the alternatives considered. For the DGP considered, the Ling-Li test has quite weak power. This reinforces the results of Tse and Tsui (1999). Similarly we can see that the residual-based diagnostic based on $e_2^2 t$ also have rather weak power. Overall, the R3 test provides a useful check for the model adequacy of multivariate conditional heteroscedasticity models.

4 Conclusions

We have derived the asymptotic distributions of the residual-based diagnostics for the conditional heteroscedasticity models. Both univariate and multivariate models are considered. In the univariate case we consider the articial regression of the squared standardised residual on its lagged values. In the multivariate case we propose to examine the squared standardised residuals as well as the cross products of the standardised residuals. Diagnostic statistics based on the correct asymptotic variance of the OLS regression parameter estimate are constructed. We examine the finite-sample properties of the residual-based diagnostics using Monte Carlo methods. In the univariate case we find that the residual-based diagnostics have favourable power against the Li-Mak portmanteau test. In the multivariate case, the residual-based diagnostics based on the cross products of the standardised residuals provide tests with the appropriate empirical size and good power against the alternatives considered.
Appendix I

This appendix provides a proof of Proposition 1. The notations defined in Subsection 2.1 will be used. We denote \( v_t \) and \( e_t \) as \( v_t \) and \( e_t \) in Proposition 1. Thus, \( \hat{\theta} = (\hat{P} \hat{d}_t \hat{d}_t) \). Then, \( \hat{\theta} = (\hat{P} \hat{d}_t \hat{d}_t) \). We write

\[ P \hat{d}_t \hat{d}_t = \frac{1}{T} \sum_{t=1}^{T} X_t \hat{d}_t v_t \]

As \( \hat{d}_t \equiv d_t + (\hat{\sigma}_t = \hat{\sigma}) (\hat{\mu}_i \mu) \), we have

\[ \frac{1}{T} \sum_{t=1}^{T} X_t \hat{d}_t v_t \equiv \frac{1}{T} \sum_{t=1}^{T} d_t v_t \]

Under regularity conditions, \( \text{plim}(\hat{d}_t \hat{d}_t) = \text{plim}(\hat{d}_t \hat{d}_t) = L \). Noting that \( e_t \equiv \hat{\varepsilon}_t + (\hat{\sigma}_t = \hat{\sigma}) (\hat{\mu}_i \mu) \), we have

\[ \frac{1}{T} \sum_{t=1}^{T} X_t \hat{d}_t v_t \equiv \frac{1}{T} \sum_{t=1}^{T} d_t v_t \]

For the first term, we note that \( \hat{\varepsilon}_t \) are independently and identically distributed with mean zero and variance 2, and are contemporaneously uncorrelated with \( d_t \), so that \( A \overset{D}{=} N(0; 2L) \). For the second term, we have \( B \overset{D}{=} N(0; QGQ') \). As \( T(\hat{\mu}_i \mu) \overset{D}{=} T \overset{1}{=} T \overset{1}{=} QG(T \overset{1}{=} QG \overset{1}{=} \hat{\mu}_i \mu) \), we have \( B \overset{D}{=} QG(T \overset{1}{=} QG \overset{1}{=} \hat{\mu}_i \mu) \), so that the asymptotic covariance between \( A \) and \( B \) can be evaluated as follows

\[ E(AB) = \frac{1}{T} \sum_{t=1}^{T} E\left[ X_t \left( \hat{\varepsilon}_t \hat{\varepsilon}_t \right) \right] GQ^0 \]

Taking iterative expectations, we have

\[ E(AB) = \frac{1}{2T} \sum_{t=1}^{T} E\left[ \left( \hat{\varepsilon}_t \hat{\varepsilon}_t \right) \right] GQ^0 \]

We use \( \overset{D}{=} \) to denote equivalence in asymptotic distributions.
\[ \begin{align*}
    & = \frac{1}{2T} \mathbb{E} \left[ X \left( \frac{d_t}{\sqrt{T}} \frac{\left( \hat{\mu} - \mu \right)}{\sqrt{\frac{2}{T}}} \right)^2 \right] GQ^0 \\
    & = \frac{1}{T} \mathbb{E} \left[ X \left( \frac{d_t}{\sqrt{T}} \frac{\left( \hat{\mu} - \mu \right)}{\sqrt{\frac{2}{T}}} \right)^2 \right] GQ^0.
\end{align*} \]

Now
\[ \text{plim} \left( \frac{1}{T} \mathbb{E} \left[ X \left( \frac{d_t}{\sqrt{T}} \frac{\left( \hat{\mu} - \mu \right)}{\sqrt{\frac{2}{T}}} \right)^2 \right] GQ^0 \right) \]
so that \( E(AB) \mid i \in QGQ^0 \). Similarly, it can be shown that \( E(AB) \mid i \in QGQ^0 \). Combining the above results, we have \( E(A + B)(A + B)^0 \mid 2L_i \in QGQ^0 = \), completing the proof of Proposition 1.

**Appendix II**

This Appendix provides proofs of Propositions 2 and 3. We use the notations defined in Subsection 2.2. For Proposition 2, we can see that, following the arguments in Appendix I, \( P \mathbb{E} \left[ X \left( \frac{d_t}{\sqrt{T}} \frac{\left( \hat{\mu} - \mu \right)}{\sqrt{\frac{2}{T}}} \right)^2 \right] GQ^0 \), where \( v_{ti} \equiv e_{ti} \mid i = (3^2 \mid i \mid 1) + \left( e_{ti} \mid 3^2 \right) \). Now \( e_{ti} \equiv e_{ti} + \left( e_{ti} \mid \hat{\mu} \mid \mu \right), \) so that
\[ \begin{align*}
    & \text{plim} \left( \frac{1}{T} \mathbb{E} \left[ X \left( \frac{d_t}{\sqrt{T}} \frac{\left( \hat{\mu} - \mu \right)}{\sqrt{\frac{2}{T}}} \right)^2 \right] GQ^0 \right) \\
    & = \text{plim} \left( \frac{1}{T} \mathbb{E} \left[ X \left( \frac{d_t}{\sqrt{T}} \frac{\left( \hat{\mu} - \mu \right)}{\sqrt{\frac{2}{T}}} \right)^2 \right] GQ^0 \right).
\end{align*} \]

from which we have \( A \overset{P}{\rightarrow} N(0; 2L_i) \) and \( B \overset{P}{\rightarrow} N(0; Q_i GQ_i^0) \). As \( B \equiv Q_i G(T; 1, \frac{\left( \hat{\mu} - \mu \right)}{\sqrt{\frac{2}{T}}}) \); the asymptotic covariance between \( A \) and \( B \) is given by
\[ E(AB) \mid i \in QGQ^0 \]
\[ \overset{P}{\rightarrow} P_i GQ_i^0. \]

Similarly, \( E(AB) \mid i \in QGQ_i^0 \). Note that unlike the univariate case, \( P_i \overset{6}{\not\rightarrow} Q_i \) in general, so that further simplification of the asymptotic variance of \( A + B \) is not obtainable.
Combining the above results, we conclude that the asymptotic variance of $P T \hat{T}$ is

$$- i = 2L_i + Q_i G Q_i^0 + P_i G Q_i^0 + Q_i G P_i^0,$$

completing the proof of Proposition 2.

For Proposition 3, we denote

$$v_{ij} = e_i e_j i \gamma_{ij}$$

so that $\hat{T}_j = ( P \hat{d}_{ij} d_{ij}^0 = T) i \gamma_{ij},$ from which, noting that $\hat{d}_{ij} \triangleq d_{ij} + \left( \Delta d_{ij} \right) ( \hat{\mu}_i \hat{\mu} ),$ we have

$$P T \hat{T}_j \triangleq \left( \frac{1}{T} X d_{ij} d_{ij}^0 \right) i \gamma_{ij} i \gamma_{ij}.$$

Now we have

$$P \frac{1}{T} X d_{ij} v_{ij} = P \frac{1}{T} X d_{ij} f (^3 u_i^3 i j l) + \left( e_i e_j i \gamma_{ij} i \gamma_{ij} \right) g$$

$$= P \frac{1}{T} X d_{ij} (^3 u_i^3 i j l) + P \frac{1}{T} X d_{ij} \left( e_i e_j i \gamma_{ij} i \gamma_{ij} \right)$$

$$\cdot A + B; \quad \text{say.}$$

Under Gaussian assumption, $E \left( ^3 u_i^3 i j l \right)^2 = 1 + \frac{1}{T_0}.$ As $^3 u_i^3 i j l$ are independently and identically distributed, and are contemporaneously uncorrelated with $d_{ij},$ we have

$$P \frac{1}{T} X d_{ij} \left( ^3 u_i^3 i j l \right) \triangleq P N (0; C_{ij});$$

Likewise, as $e_i e_j i \gamma_{ij} \triangleq ( ^3 u_i^3 i j l ) + \left( e_i e_j i \gamma_{ij} \right) = \hat{d}_{ij} (\hat{\mu}_i \hat{\mu}),$ we have

$$B \triangleq \left[ \frac{1}{T} X d_{ij} \left( ^3 u_i^3 i j l \right) \right] P \frac{1}{T} \hat{\mu}_i \hat{\mu};$$

so that $B \triangleq P N (0; Q_{ij} G Q_{ij}^0).$ Also, we can rewrite $B \triangleq Q_{ij} G i \gamma^2 @ (\hat{\mu}) = \hat{\mu},$ so that the asymptotic covariance between $A$ and $B$ is

$$E (A B)^0 = \frac{1}{T} E \left[ X d_{ij} \left( ^3 u_i^3 i j l \right) \right] P \frac{1}{T} \hat{\mu}_i \hat{\mu} i \gamma^2 Q_{ij}^0,$$

$$\cdot P_{ij} G i \gamma^2 Q_{ij}^0;$$

$$\hat{\mu}_i \hat{\mu}.$$

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Similarly, \( E(BA^0) \) = \( Q_{ij} G_{0}^{1} P_{ij}^{0} \). Combining the above results we conclude that the asymptotic variance of \( T_i \) \( = \sum_{j=2}^{P} \hat{d}_{ij} \nu_{ij} \) is equal to \( \sum_{ij} = C_{ij} + Q_{ij} Q_{ij}^{0} + P_{ij} G Q_{ij}^{0} + Q_{ij} G P_{ij}^{0} \), and the asymptotic variance of \( P_T \hat{\alpha}_{ij} \) is equal to \( L_{ij} \sum_{ij} L_{ij} \), completing the proof of Proposition 3.
References


Table 1: Empirical Size of Diagnostics for Univariate Conditional Heteroscedasticity

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<tr>
<th>DGP</th>
<th>EM</th>
<th>T</th>
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Notes: DGP is the data generating process. EM is the estimated model. RBM is the residual-based diagnostic using the correct variance matrix. POR is the Li-Mak portmanteau test. OLSM is the residual-based diagnostic using the OLS variance. We consider M = 1; 2; 3 and 4: The figures in the table are the empirical frequency of rejection in percentage. The nominal size of the tests is 5 percent. The estimation is based on Monte Carlo runs of 1000.
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**Notes:** DGP is the data generating process. EM is the estimated model. RB$M$ is the residual-based diagnostic using the correct variance matrix. POR$M$ is the Li-Mak portmanteau test. OLS$M$ is the residual-based diagnostic using the OLS variance. We consider $M = 1, 2, 3$ and $4$. The figures in the table are the empirical frequency of rejection in percentage. The nominal size of the tests is 5 percent. The estimation is based on Monte Carlo runs of 1000.
**Table 3:** Empirical Size and Power of Diagnostics for Multivariate Conditional Heteroscedasticity

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**Notes:** DGP is the data generating process. LL$M$ is the Ling-Li test based on $M$ autocorrelation coefficients. R1-$M$, R2-$M$ and R3-$M$ are the residual-based diagnostics with, respectively, $e_{t1}^2 - 1$, $e_{t2}^2 - 1$ and $e_{t1}e_{t2} - \hat{\rho}_t$ as the dependent variables in the artificial regression, where $M = 1, 2, 3$ and 4 stands for the order of the lagged terms taken in the regression. The figures in the table are the empirical frequency of rejection in percentage at the nominal size of 5 percent. The EM is CC-MGARCH. When the DGP is CC-MGARCH, the figures are the empirical size. Otherwise, the figures are the empirical power. The estimation is based on Monte Carlo runs of 1000.