On Tariff, Quality Choice and Innovation in a Vertically Differentiated Monopoly with Discrete Preferences

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Abstract:
In a vertically differentiated monopolistic framework with discrete preferences we examine how protecting the low-quality segment raises the incentive for quality innovation. We show how the monopolist facing competitive imports, might fail to exert its complete monopoly power even if there is prohibitive tariff on both the high and low quality segment of the market. On the other hand, given non prohibitive tariff on the high quality segment, the potential gain for the monopolist exhausts at a level much below the prohibitive low-quality tariff level. Also a sufficiently low tariff on the high quality product can force the monopolist to produce the first best qualities irrespective of the tariff level on the low quality product.

Keywords: Protection; Quality Innovation; Discrete Types; Monopoly
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1. Introduction

Whether trade liberalization fosters innovation is a debated issue both theoretically and empirically. In the Indian context, Desai (1980), Lall (1984) and Marjit and Raychaudhuri (1997) argued quite strongly that tariff protection had eliminated innovation incentives whatsoever and made the domestic firms inward looking during the 1970s and 1980s. Whatever little innovation that took place were just minor innovations instead of at the frontiers of technology. A similar argument has often been put forward for other developing countries in the countless debates over trade liberalization as an appropriate export promotion strategy. But casual empiricism observes a mixed experience in this regard. Lowering of tariff has raised the R&D level in some countries whereas has lowered it in others. Aw and Roberts (1986) also found that the 1977-81 quota on footwear from Korea and Taiwan led to the quality upgrading of most important bundles throughout the period.¹

Theoretical analyses linking trade liberalization (or protection) and innovation have also remained inconclusive. That too much market power for the domestic firms is not conducive for innovation has also been argued by Porter (1990), White (1974) among others. Segerstorm et. al. (1990) put similar argument in a dynamic general equilibrium North-South model. On the other hand, Clemenz (1990) and Reitzes (1991) derived just the opposite result which essentially captures the Schumpeterian idea that market power and innovation are positively related [Kamien and Schwartz (1982)]. In a similar spirit, Rodrik (1992) demonstrated that in a dynamic set up, liberalization slows down the pace of the productivity increase and delays technological catch up since it shrinks the domestic monopolist’s sales and thus reduces incentives to invest in cost-reducing technology.

¹ Also Feenstra (1998) showed that the US-imposed VER between 1981 and 1985, affected both the price and quality of Japanese cars.
This paper provides a similar argument that protection fosters (quality) innovation in a vertically differentiated market. The present analysis, however, differs from the earlier ones in the assumption of a heterogeneous set of consumers with different preferences for the quality-differentiated good as in Mussa and Rosen (1978). It is sometimes realistic to assume holes in consumer preferences rather than continuous consumer preferences. The assumption of continuous preferences corresponds to no naturally occurring or laboratory economy, but has proved a fairly innocuous simplification in many economic exercises. But we in this exercise assume discrete consumer preferences with two types of consumers differing in their marginal willingness to pay for quality. In this framework, we examine the relationship between tariff protection quality up-grading and post innovative optimal quality choice by a quality constrained monopolist. The monopolist is quality constrained in the sense that it has the technological capability to produce qualities up to a certain level. It can gain the technological know how to produce qualities beyond that by spending a fixed sum in R&D. To put it differently, we conceive a situation where initial technological constraint does not allow him to offer the menu to the high-type that he would prefer. Only through innovation he can practice such discrimination and with competitive imports of similar varieties from abroad such gains are realized only under tariff protection. This, in essence, is similar to the Schumpeterian idea that protection increases the incentive for quality up-grading. Thus, pre-innovation, the monopolist was forced to produce a suboptimal (low) quality from a welfare point of view. We show that in our framework this tariff induced innovation helps mitigate quality distortion by the domestic monopolist to a large extent and thus this tariff protection is non-

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2 Recently several papers addressed optimal trade policy under vertical differentiation (see Herguera and Lutz (1998), Vandenbussche and Wauthy (2001), Herguera, Kujal and Petrakis (2002), etc.).

3 Of late, incentives for quality innovation and choice of innovation type have been examined for a closed economy by Bandyopadhyay and Acharyya (2003) and Lambertini and Orsini (2000).
distortionary (except for the cost of innovation).\textsuperscript{4} Obviously there will be the usual price distortion from monopoly pricing. Noteworthy is the fact that the monopolist is unable to exert its complete monopoly power even if there is prohibitive tariff on both the high and low quality varieties! To put it differently, in this model there will be no downward quality distortion by the monopolist at the lower end of the market even if both the high end and the low end of the market is completely protected.\textsuperscript{5} Moreover, the monopolists’ post-innovative potential gain increases if the low quality tariff is increased beyond the prohibitive tariff level. Whereas, on the contrary, when there is a non prohibitive tariff on the high end of the market the potential gain for the monopolist exhausts at a level much below than the prohibitive tariff level. We also show how a sufficiently low tariff on the high quality product can force the monopolist to produce the first best qualities irrespective of the tariff level on the low quality product. It will also be interesting to note the rates at which the innovation incentive increases for different protection levels.

The rest of the paper is organized as follows. In section 2 we lay down the framework. In section 3 we examine the innovation incentive and quality choice by the monopolist when the high-quality good segment is completely protected. Section 4 considers the case of non prohibitive tariff on high quality product. In section 5 we discuss very briefly the implication of a reduction in the tariff on high quality good given any tariff on the low quality product. Section 6 concludes the analysis.

\textsuperscript{4} Protection to the domestic monopolist can be motivated along the infant industry protection argument and also from a political economic point of view. We will touch upon these issues as we proceed.

\textsuperscript{5} These surprising results stems from the incentive structure of our model.
2. The Framework

Consider a vertically differentiated good with observable quality indexed by $q \in [0, \bar{q}]$. Suppose, domestic production is monopolized by a single firm. Though the present state of scientific knowledge makes it possible to produce the good elsewhere over such a range of qualities, the domestic monopolist is technologically constrained in the sense that the technology he has access to allows him to produce only qualities within the range $[0, \tilde{q}_i]$ but not beyond that, where $\tilde{q}_i < \bar{q}$. We shall later define quality innovation as a process whereby investing an exogenously given sum of $F_q$, he can learn the technical know-how to produce all $q \in [\tilde{q}_i, \bar{q}]^6$. In other words the cost can be defined as a function of $|\bar{q} - \tilde{q}_i|$ which is constant given $\bar{q}$ and $\tilde{q}_i$. With marginal cost of production invariant with respect to the output level but varying with the quality level, the cost function in the pre-innovation stage for this domestic monopolist can be defined as,

$$C = \bar{q}^2 \quad \forall q \in [0, \tilde{q}_i]$$

$$= \infty \quad \text{otherwise} \quad (1)$$

There are two types of consumers at home who differ in respect of their taste parameters: $\alpha_2 > \alpha_1$. The number of consumers of each type is $n_i$. Each consumer buys, if at all, only one unit of the good. The net utility that type-$\alpha_j$ consumer derives from the menu $(q_j, P_j)$ is assumed to be linear, which is just a simplification:

$$U_j = \alpha_j q - P, \quad j = 1, 2 \quad (2)$$

This preference function satisfies two important properties that are typically assumed in the literature: High-type consumers derive greater total as well as marginal utility than low-type

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6 In international trade theory there are models where quality affects variable production costs and there is no up-front R&D costs (See Krishna (1987), Bond (1988), Das and Donnenfeld (1987, 1989)). These are variable cost or quality models. Our model is also a variable cost of quality model (see later) with a positive fixed R&D investment cost. In this sense in essence our structure incorporates both the features of variable and fixed cost of quality improvement.
consumers from any quality. Thus, the (linear) indifference curves between price and quality of the two types cross each other *only once* [Cooper (1984)].

A typical type- \( \alpha_j \) consumer participates in the market if,

\[
\alpha_j q \geq P
\]

and selects the menu \( (q_2, P_2) \) if the following self-selection constraint is satisfied,

\[
\alpha_j q_2 - P_2 \geq \alpha_j q_1 - P_1
\]

for \( q_2 > q_1 \).

It is instructive at this point to look at the quality choice of the home firm had it not been technologically constrained, and faced no competition from abroad whatsoever. This exercise will help us understand the technological constraint and the potential gains from innovation. Under such a completely protected trade regime, at a separating equilibrium the unconstrained home firm would have chosen the following monopoly qualities that maximize his profit,

\[
\pi = n_1(\alpha_i q_1 - c q_1^2) + n_2(\alpha_2 q_2 - q_1) + \alpha_i q_1 - c q_1^2
\]

\[
\tilde{q}_1 = \frac{n_1 \alpha_i - n_2 (\alpha_2 - \alpha_1)}{2 n_1 \tilde{c}}, \quad \tilde{q}_2 = \frac{\alpha_2}{\tilde{c}}
\]

Of course, we must assume that the low end of the domestic market is sufficiently large, in the sense that \( \frac{n_1}{n_2} \geq \frac{\alpha_2 - \alpha_1}{\alpha_i} \), for this separating menu (i.e., \( \tilde{q}_1 > 0 \)) to be the profit maximizing menu\(^7\). Otherwise the monopolist will offer \( \tilde{q}_2 \) at price \( \alpha_2 \tilde{q}_2 \) to the \( \alpha_2 \)-consumers and will exclude the \( \alpha_i \)-type from the market. It is immediate that the technological constraint defined above, is assumed to be binding in the sense that without investing in R&D, it is possible for the home firm to offer at most \( \tilde{q}_1 \) level of quality. Pre-innovation, his problem then simply boils down to whether

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\(^7\) See Acharyya (1998).
to extract all surplus from the high-type by charging their reservation price which, of course, drives
low-type consumers out of the market, or to offer a pooling menu by charging the reservation price
of the low-types which leaves high-types with some surplus. The same restriction on the
distribution pattern mentioned above, however, imply that the pooling menu, \( q_1, P_1 = P_2 = \alpha q_1 \), is
relatively profitable. It is a standard wisdom that \( q_1 \) is suboptimal but \( q_2 \) is optimal from a welfare
point of view. In other words there is downward quality distortion in the low-quality segment of the
monopolistic market.

Suppose the world market for this quality-differentiated good is perfectly competitive and that
imports are subject to a tariff duty per unit. As long as the tariff duty is non-prohibitive, the
domestic consumers have the option to buy the domestic variety \( q_1 \) at some price that the home
firm would charge under such competition or the imported varieties at the tariff-inclusive (competitive) price. Suppose, producers abroad have identical cost of quality as the home firm except that they have no technological constraint. Thus, \( C^* = cq^2, \forall q \in [0, q] \). These simplifying
assumptions imply that competitive foreign producers will offer the quality \( q_j^* = \frac{\alpha_j}{2c}, j = 1, 2, \) to
the \( \alpha_j \)-type home consumers. It is important to point out that in this competitive equilibrium the \( \alpha_1 \)-
type consume \( q_1^* \) at price \( c q_1^* \) and will derive a net positive utility \( \alpha_1 q_1^* - c q_1^* = \frac{\alpha_1^2}{4c} > 0 \).

Similarly the \( \alpha_2 \) consumers purchase \( q_2^* \) at price \( c q_2^* \) derives a net positive utility
\( \frac{\alpha_2^2}{4c} > \frac{\alpha_1^2}{4c} > 0 \). This is different from the separating monopoly equilibrium where the \( \alpha_1 \)-type get 0
net utility and the \( \alpha_2 \) type derive positive net utility.

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8 This assumption rules out export possibilities.
3. Prohibitive high-quality tariff:

Keeping with the observation that often imports of different quality are subject to differential tariff rates we assume that tariff rate $t_1$ is applied to all imports with qualities $q < q_2^*$ whereas the rate $t_2$ is applied to all imports of quality $q_2^*$ or higher. Moreover, for the time being let us assume that initially $t_1$ is set at some non-prohibitive level, $t_2$ is set at the prohibitive level in the sense defined below. Note that, with competitive imports from abroad, the high-end of the home market must be protected from import of similar high-quality varieties to ensure positive rents from innovation for the home firm. For example, if $t_2$ is zero, the home firm can gain nothing by offering a quality $q_2^*$ or higher, and therefore will have no incentive at all to innovate such qualities. This though does not mean that the high-end of the market should be completely protected through a prohibitive tariff, we assume so to focus solely on how increase in the tariff on low-quality imports raise the incentive for innovation despite raising the profit from selling just the low-quality variety. Later we will relax this assumption and consider the case of non-prohibitive tariff on both the high and low quality products.

Given such initial tariff regimes, the tariff inclusive domestic price of the imported quality $q_1^*$ equals $P_1^d = \bar{c}q_1^* + t_1$. But the home firm must charge a price $P_1(t_1)$ strictly less than this to induce the low-type consumers to buy the domestic variety $q_1$. A little manipulation of the self-selection constraint of the $\alpha_1$-type consumers yields such a price as,

$$P_1(t_1) = \alpha_1 \bar{q}_1 - \alpha_1 q_1^* + \bar{c}q_1^* + t_1 - \varepsilon = (\bar{c}q_1^* + t_1) - \alpha_1 (q_1^* - \bar{q}_1) - \varepsilon$$

(6)

9 Since the indifference curves (or self-selection constraints) are vertically parallel and the tariff shifts up the marginal cost curve of the foreign producers in a parallel fashion, their selection of profit-maximizing qualities for the domestic-country market remains the same at $q_j^*$. 
But the home firm will operate only if this price $P(t_i)$ covers marginal cost of producing $\tilde{q}_i$. From (6) it follows then that there is a strictly positive tariff $\tilde{t}_i = (g_1^* - \tilde{q}_i)\left[\alpha_i - \tilde{c}(q_1^* + \tilde{q}_i)\right]$ that must at least be offered to protect the domestic firm. That is, for all $t_i \in [0, \tilde{t}_i]$, domestic production is zero.

At the other extreme the domestic government can set a prohibitive tariff $t_{P1}$ on the low-quality import $q_1^*$ such that the home firm can charge the monopoly price along the individual rationality constraint of the $\alpha_1$-type consumers i.e. $P_1(t_i) = \alpha_i\tilde{q}_1$. Using (6) and for $\varepsilon$ sufficiently close to zero we get the prohibitive tariff on the low-quality import as $t_{P1} = \alpha_i q_1^* - \tilde{c} q_1^*$. Therefore, $\forall t \in [0, \tilde{t}_i]$, the low-type domestic consumers will buy the imported variety $q_1^*$ whereas $\forall t \in [\tilde{t}_i, t_{P1}]$ they will buy the variety $\tilde{q}_i$ offered by the domestic monopolist.

On the other hand, given a prohibitively high tariff on high-quality variety, the high-type domestic consumers must choose between the imported variety $q_1^*$ at price $P_1^d = \tilde{c} q_1^* + t_i$ and the domestic variety $\tilde{q}_i$ at $P_1(t_i)$ in the pre-innovation period. The following lemma specifies such a choice:

**Lemma 1:** In the pre-innovation stage, with a prohibitive tariff on the high-quality imported varieties, the high-type domestic consumers buy the imported variety $q_1^*$ $\forall t_i \in [0, t_{P1}]$ instead of the domestic variety, $\tilde{q}_i$.

**Proof:** Suppose, on the contrary that the high-type domestic consumers buy $\tilde{q}_i$. Then, by the self-selection constraint, the following must be true:

$$\alpha_2 \tilde{q}_i - P_1(t_i) > \alpha_2 q_1^* - P_1^d \quad \forall t_i$$

By (6), this boils down to,

$$(\alpha_2 - \alpha_1) \tilde{q}_i > (\alpha_2 - \alpha_1) q_1^* - \varepsilon$$
But $\tilde{q}_i < q_i^*$ and $\varepsilon > 0$. Hence a contradiction. ♦

Note that even a prohibitively high tariff on import of high-quality varieties cannot protect the home firm in the high-segment of the market$^{10}$. Thus such a tariff is essentially ineffective. But as we will see, after innovation, this will enable the home firm to extract surplus from the high-type consumers.

Given the above segmentation of the home market among the home firm and competitive foreign producers, in the pre-innovation stage, the home firm’s profit equals,

$$\Pi = n_i \left[ \alpha_i \tilde{q}_i - \alpha_i q_i^* + \tilde{c} q_i^{*2} + t_1 - \tilde{c} \tilde{q}_i^{*2} \right]$$

(7)

### 3.1 Quality Innovation

Consider an instantaneous and certain innovation process whereby investing a fixed sum $F$ the monopolist can develop all qualities over the range $[\tilde{q}_i, \tilde{q}]$. If innovation is undertaken at all, the home firm is no longer technologically constrained to choose the profit maximizing qualities, denoted by $q_1^I$ and $q_2^I$. But with the low-quality variety $q_i^*$ imported from abroad under a non-prohibitive tariff, he must ensure that low-type consumers purchase $q_1^I$ and high-type consumers buy $q_2^I$ instead of $q_i^*$. That is, he must charge $p_1$ and $p_2$ such that,

$$p_1(t_1) = \alpha_1 q_1^I - (\alpha_1 q_i^* - \tilde{c} q_i^{*2}) + t_1 - \varepsilon$$

(8a)

$$p_2(t_1) = \alpha_2(q_2^I - q_i^*) + (\tilde{c} q_i^{*2} + t_1)$$

(8b)

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$^{10}$ If tariff on low-quality imports is larger than $\tilde{t}_i = \alpha_2(q_i^* - \tilde{q}_i) - (\tilde{c} q_i^{*2} - \tilde{c} \tilde{q}_i^{*2})$, the home firm can charge a price $P_2(t_1) = (\tilde{c} q_i^{*2} + t_1) - \alpha_2(q_i^* - \tilde{q}_i)$ lower than $P_1(t_1)$ and induce even the high-types to buy the domestic variety $\tilde{q}_i$.

But for $\alpha_2 > 2\alpha_1$, or $\frac{n_1}{n_2} \in \left[ \frac{\alpha_2 - \alpha_1}{\alpha_1}, \frac{\alpha_2 - \alpha_1}{2\alpha_1 - \alpha_2} \right]$ in the opposite case, such a price yields a lower profit for the home firm than $P_1(t_1)$ for all $t_1 \in \left[ 0, t_{p_1} \right]$. Thus, under these conditions, the home firm charges $P_1(t_1)$ and the high-type consumers buy the low-quality imported variety as stated in Lemma 1 above.
Given such prices, it is straightforward to check that the home firm’s choices of \(q^1_i\) that maximize 
\[
\pi(t) = n_1[p_1(t) - cq^1_i - \bar{c}q^1_i] + n_2[p_2(t) - cq^2_i - \bar{c}q^2_i]
\] are the same as the imported qualities: \(q^1_1 = q^*_1, q^2_1 = \bar{q}_2 = q^*_2\). This result is, of course, due to the specific tariff that does not alter the marginal cost of quality and the vertically parallel property of indifference curves between price and quality.

Thus the monopolist’s profit in the post innovative situation is,
\[
\pi(IN) = (n_1 + n_2)t_1 + n_2[\alpha_2(q^*_2 - q^*_1) - (\bar{c}q^*_2 - \bar{c}q^*_1)]
\]

(9)

The gain from innovation is simply the gain from quality discrimination. With heterogeneous set of consumers, \(\alpha_1 \neq \alpha_2\), and \(\frac{n_1}{n_2} > \frac{(\alpha_2 - \alpha_1)}{\alpha_1}\), quality discrimination or a separating menu defined in (5) maximizes the home firm’s profit. But without innovation, this was simply not feasible and the home firm was compelled to offer a pooling menu. Innovation allows him to discriminate between the two types and therefore to extract greater surplus from the high-type. However, competition from foreign producers under non-prohibitive tariff on low-quality imports restricts the ability of the home firm to discriminate to the extent it would have done under complete protection as indicated by the menu defined in (5). Formally, for \(\forall t \in [\bar{t}, t_{p1}]\)

\[
RG(IN) = \pi(IN) - \pi = n_2\alpha_1 + n_2[(\alpha_2q^*_2 - \alpha_1q^*_1) - (\bar{c}q^*_2 - \bar{c}q^*_1)] + n_1[\alpha_1(q^*_1 - \bar{q}_1) + (\bar{c}q^*_1 - \bar{c}\bar{q}_1^2)]
\]

(10)

whereas for \(t \in [0, \bar{t}]\), since pre-innovation profit is zero, so \(RG(IN) = \pi(IN)\). In any case the gain from innovation increases with the rate of tariff on low-quality imports, initially at the rate \((n_1 + n_2)\) and then at a smaller rate \(n_2\). Thus,
Proposition 1:

*Given prohibitive tariff on the high-quality variety, any specific or per unit tariff on import of low-quality variety, regardless of whether it is protective or not, raises the incentive for quality innovation.*

A higher tariff on low quality imports enables the domestic firm to raise price of such variety, and consequently to raise the price of the high-quality variety as well along the higher self-selection constraint of the high-type consumers. It is because of this scope of extracting a greater surplus from the high-type that a higher tariff on low-quality imports increases the gain from quality innovation and consequently the incentive for quality innovation. But does the potential to extract greater surplus for the monopolist exhaust at $t_{p1}$? Interestingly the answer is no. The monopolist cannot charge the monopoly price on $q_1^*$ even if $t$ is set at $t_{p1}$. The following lemma states exactly that:

Lemma2: At $t_1 = t_{p1}$ the monopolist cannot charge the monopoly price on $q_2^*$, it can at most charge $c q_2^* + t_{p1} = \alpha_1 q_1^* + c(q_2^* - q_1^*)$.

Proof: Monopoly price of $q_2^*$ when the monopolist offers $\tilde{q}_1$ to the low-type is

\[ P_m(q_2^*) = \alpha_2 q_2^* - \alpha_2 \tilde{q}_1 + \alpha_1 \tilde{q}_1. \]  

(11)

Now $P_m(q_2^*) - P_2(t = t_{p1}) = (\alpha_2 - \alpha_1) \left[ \frac{q_2^*}{2} + \frac{q_1^*}{2} - \tilde{q}_1 \right] > 0$. \[ \blacklozenge \]

In other words the above lemma points to the fact that $t_{p1} < t_{p2}$ where $t_{p2} = \alpha_2 q_2^* - \tilde{c} q_2^* - (\alpha_2 - \alpha_1) \tilde{q}_1$. But most interesting is what happens when $t_1 > t_{p1}$. The monopolist finds it optimal to adjust the low quality offered downward along the IR constraint of
the $\alpha_1$ type consumers and in the process charges a higher price on the high quality product. This continues up to $\hat{t}_1 = t_{p1} + \frac{n_1(\alpha_2 - \alpha_1)^2}{2n_1c}$ where the price charged on the high quality is exactly equal to that charged by a discriminating monopolist, i.e. according to the IC constraint of the $\alpha_2$ type consumers. The proposition below formally states the result:

**Proposition 2:**

*The monopolist finds it optimal to modify the low quality $\forall t_1 \in (t_{p1}, \hat{t}_1]$ where the optimal low quality offered will be $\hat{q}_1 = q_1^* - \frac{(t_1 - t_{p1})}{(\alpha_2 - \alpha_1)}$ where $\hat{t}_1 = t_{p1} + \frac{n_1(\alpha_2 - \alpha_1)^2}{2n_1c}$ and all $t_1 \in (t_{p1}, \hat{t}_1]$ is distortionary.*

**Proof:** Let $\hat{q}_1$ be the low quality offered by the monopolist at price $\alpha_1\hat{q}_1$ to the $\alpha_1$-type. We know that the monopolist offers $q_2^*$ at price $P_2(t_1) = \alpha_2(q_2^* - q_1^*) + c q_1^* + t_1$ to the $\alpha_2$-type consumers. From the self-selection constraint of the $\alpha_2$-type we get

$$\alpha_2\hat{q}_1 - \alpha_1\hat{q}_1 = \alpha_2 q_2^* - \alpha_2(q_2^* - q_1^*) - c q_1^* - t_1$$

$$\Rightarrow \hat{q}_1 = q_1^* - \frac{(t_1 - t_{p1})}{(\alpha_2 - \alpha_1)}.$$  \hspace{1cm} (12)

The maximum possible value of $t_1$ can be found when $P_2(t_1)$ is equal to the monopoly price of $q_2^*$ i.e. when $P_2(t_1) = \alpha_2 q_2^* - \alpha_2\tilde{q}_1 + \alpha_1\tilde{q}_1$. From this we can easily derive $\hat{t}_1 = t_{p1} + \frac{n_2(\alpha_2 - \alpha_1)^2}{2n_2c}$. Needless to point out, that all $t_1 > t_{p1}$ leads to downward quality distortion of the low-quality segment of the market since $\hat{q}_1 < q_1^*$ where $q_1^*$ is the socially optimal low quality. ■

What is evident from the above proposition is the fact that the monopolist is not fully protected even if there is prohibitive tariff in both the low and high quality products. The incentive structure
is such that $t_i$ needs to be higher than $t_{p1}$ so that the monopolist is completely protected. The relative gain in this situation will be

$$RG(Q) = n_1[\alpha_i \tilde{q}_1 - \tilde{c} \tilde{q}_1^2] + n_2 t_1 + \frac{n_2 (\alpha_2 - \alpha_i)^2}{4\tilde{c}} - n_1[\alpha_i \tilde{q}_1 - \tilde{c} \tilde{q}_1^2] \tag{13}$$

But what will be the relative gain look like $\forall t_i \in (t_{p1}, \hat{t}_i]$? The following proposition provides an answer:

**Proposition 3:**

_The relative gain from innovation reaches maximum at $t_i = \hat{t}_i$ and $\forall t_i \in (t_{p1}, \hat{t}_i]$ the relative gain increases at a decreasing rate._

**Proof:** Put $\hat{q}_i = \frac{q_i}{\alpha_i} - \frac{(t_i - t_{p1})}{(\alpha_2 - \alpha_i)}$ into (13) and maximize the expression with respect to $t_i$. It is easy to check that (13) is maximized at $\hat{t}_i = t_{p1} + \frac{n_2 (\alpha_2 - \alpha_i)^2}{2n_1 \tilde{c}}$. Note also that $\frac{\partial^2 RG}{\partial t_i^2} = -\frac{2n_1 \tilde{c}}{(\alpha_2 - \alpha_i)^2} < 0$.

This completes the proof.

The curvature can be explained by the fact that as $t_i$ gets increased beyond $t_{p1}$ there is a gain from the high end of the market as well as a loss from the low-end of the market. The former outweighs the latter and the gain increases but at a falling rate. One result that’s immediately become apparent is that in this model given the implicit threat of entry and given prohibitive tariff on the high quality segment downward quality distortion at the lower end of the market by the monopolist can only happen for very high low quality tariff ranges: in this situation for more than prohibitive tariff ranges. This surprising implication is of course due to the incentive structure of the model and also points out how strong the threat of potential entry might be in mitigating quality distortion. But of

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11 Note that the pre-innovation profit for all $t_i > t_{p1}$ is $n_1[\alpha_i \tilde{q}_1 - \tilde{c} \tilde{q}_1^2]$. 
course there will be the usual price distortion. More interesting results of this kind follow where we
discuss the case of uniform tariff protection and non-prohibitive high quality tariff case. But before
that we examine the innovation decision by the monopolist in this situation.

Given that innovation involves a (fixed) cost, $F_q$, which it can save upon by not undertaking
innovation. Thus such gains from innovation must cover the cost of innovation. For all non-
protective tariff, $t_i \in [0, \hat{t}_i]$, it is immediate that all viable innovations (i.e., innovations for which
$\pi(IN) > F_q$) will be undertaken. But for protective tariff rates, only those innovations for which
$RG(IN) > F_q$ will be undertaken. The innovation decision is illustrated in Figure 1.

**Figure 1 Goes Here**

Given the positively sloped RG(IN) curve, we can define a level of (fixed) innovation cost, $\tilde{F}_q$ such
that the relative (net) gain from quality innovation (for $t_{p1}$) is exactly zero. For all $F_q$ greater than
this the domestic monopolist will only innovate for $t_i > t_{p1}$. But, for $F_q \in [0, \tilde{F}_q]$, the innovation
decision depends on the level of protection. For example, for $F_q = F_q^0$, the monopolist undertakes
innovation only for a $t_i \in [t_i^0, \hat{t}_i]$. That is, given an initial situation where the tariff on low-quality
import is smaller than $t_i^0$, the government can induce the domestic monopolist to innovate by
raising the tariff rate beyond $t_i^0$.

### 3.2: Some Welfare Results:

It is interesting to check welfare gains from tariff protections which induce innovation as discussed
above. Here the domestic welfare is defined as the total surplus: sum of consumers surplus and
profit of the domestic firm.
First consider the welfare property of tariff $t_i$ on the low-quality imports given prohibitive tariff on the high-quality good. Following lemma makes a definite statement in this regard:

**Proposition 4:**

a) *Post innovation, given prohibitive tariff on the high-quality good, any low-quality tariff $t_i \in [0, t_{p_1}]$ is non-distortionary.*

b) *All $t_i \in (t_{p_1}, \hat{t}_i]$ is distortionary.*

**Proof:**

a) Given tariff $t_i \in [0, t_{p_1}]$ on the low-quality good, the total surplus that the $\alpha_1$ consumers derive is given by $n_1[\alpha_1 q_{i_1}^* - \bar{c} q_{i_1}^* - t_1]$ and the total surplus of the $\alpha_2$ consumers is $n_2[\alpha_2 q_{i_2}^* - \alpha_2 (q_{i_2}^* - q_{i_1}^*) - \bar{c} q_{i_1}^* - t_1]$. The monopolist’s gross post-innovative profit is $(n_1 + n_2)t_1 + n_2[\alpha_2 (q_{i_2}^* - q_{i_1}^*) - (\bar{c} q_{i_2}^* - \bar{c} q_{i_1}^*)]$. Adding consumers’ surplus and the producers’ surplus and rearranging terms we get the total welfare as $n_1[\alpha_1 q_{i_1}^* - \bar{c} q_{i_1}^*] + n_2[\alpha_2 q_{i_2}^* - \bar{c} q_{i_2}^*]$ which is nothing but the perfectly competitive total consumers surplus from both $\alpha_1$ and $\alpha_2$ types. This completes the proof.

b) The consumer surplus of the $\alpha_1$-type $\forall t_i \in (t_{p_1}, \hat{t}_i]$ is zero since the monopolist will extract all surplus from the $\alpha_1$-type. The optimal low-quality offered is given in equation (12). The price charged to the $\alpha_2$-type consumers will be $\alpha_2 q_{i_2}^* - \alpha_2 \hat{q}_1 + \alpha_1 \hat{q}_1$. Therefore the total surplus of the $\alpha_2$-type consumers after some calculations turn out to be $n_2(\alpha_2 - \alpha_1)\hat{q}_1$. The total producer’s surplus is $n_1(\alpha_1 \hat{q}_1 - \bar{c} \hat{q}_1^2) + n_2[\alpha_2 q_{i_2}^* - \alpha_2 \hat{q}_1 + \alpha_1 \hat{q}_1 - \bar{c} q_{i_2}^*]$. Summing total consumer surplus and producer surplus we get the total welfare as $n_1(\alpha_1 \hat{q}_1 - \bar{c} \hat{q}_1^2) + n_2[\alpha_2 q_{i_2}^* - \bar{c} q_{i_2}^*]$ which is less than the competitive welfare since $\hat{q}_1 < q_{i_1}^*$. ■
The above results can be explained as follows. For all $t_i \in [0, t_{P1}]$, the domestic firm acts like a price-taker since, facing competitive imports from abroad, it cannot raise the price of its product more than the tariff-inclusive marginal cost of production for any given quality. The tariff enables it to raise price of its low-quality variety above the corresponding marginal cost only to the extent it raises the foreign import price. By the vertically parallel property of consumer preferences, such tariff-inclusive higher prices, however, leaves optimal choice of qualities by the domestic firm the same as the imported qualities. Consequently, tariff just redistributes total surplus among domestic consumers and producer without generating any quality distortion and, therefore, any dead-weight loss.

But when $t = t_{P1}$, the tariff-inclusive price of imported low-quality good become so high as to leave domestic consumers with no surplus. For all tariffs higher than this low-type domestic consumers are better-off by not consuming the imported variety. This creates scope for the domestic firm to offer them a lower quality than the imported variety at their reservation price and to extract more surplus from the high-type through consequent increased quality differentiation and discrimination. Therefore, all such tariffs put a dead-weight loss and lower total surplus.

The above result is important in the sense that in this structure, post innovation, even a prohibitive tariff on the high-quality variety and a sufficiently high tariff on the low-quality product induce no deadweight loss except for the cost of product innovation. This follows from the fact that there is no quality distortion both in the low and high quality segment of the market. There will only be the usual price distortion from monopoly pricing. Thus this protection can be justified along the infant industry protection argument and one can also argue from a political economic point of view that there are evidences where the domestic industry had a greater lobbying power to
influence government policies\textsuperscript{12}. Since our objective in this paper is somewhat different we abstract from these issues. It can also be pointed out that quality innovation in a sense removes any distortion that might have existed pre-innovation even for sufficiently high tariffs on the low and high quality products. But if $t_1$ is increased beyond the prohibitive level there will be distortion in the economy stemming from the downward quality distortion from the low-quality segment of the market.

Next we consider the case where there is a non prohibitive tariff on the high-quality product and we examine the post innovative quality choice.

4. Non-prohibitive tariff on high quality:

We now consider the situation when there is a non-prohibitive tariff on the high quality product. The pre-innovation situation is more or less the same except for the fact that the $\alpha_2$ type consumers might consume $q_1^*$ or $q_2^*$ depending on the level of $t_1$ and $t_2$. But the pre-innovative profit for the monopolist capable of producing upto $\tilde{q}_1$ remain the same.

To proceed with the analysis further we state the following lemma.

Lemma 3: \( \exists t_2^* \in (t_{p_1}, t_{p_2}) \) such that $\alpha_2$-type is indifferent between $q_2^*$ at price $\bar{c}q_2^2 + t_2^*$ and $q_1^*$ at price $\alpha_i q_1^*$ where $t_2^* = \alpha_2 (q_2^* - q_1^*) + \alpha_i q_1^* - \bar{c}q_2^2$.

\textbf{Proof:} From the self-selection constraint of the $\alpha_2$-type we get

\[
\alpha_2 q_1^* - \alpha_i q_1^* = \alpha_2 q_2^* - \bar{c}q_2^2 - t_2^*
\]

\[
\Rightarrow t_2^* = \alpha_2 (q_2^* - q_1^*) + \alpha_i q_1^* - \bar{c}q_2^2 = \frac{1}{4\bar{c}} \left[ (\alpha_2 - \alpha_i)^2 + \alpha_i^2 \right] > 0.
\]

\textsuperscript{12}For example in India, the Reliance industry historically had close relationship with the Congress government.
Also \( t_2^* - t_{p_1} = (q_2^* - q_1^*) (\alpha_2 - \alpha_1) > 0 \) and \( t_2^* - t_{p_2} = - (\alpha_2 - \alpha_1) (q_1^* - \tilde{q}_1) < 0 \) completes the proof. □

From the above lemma we can get the following 2 subcases:

**Case 1:** \( t_2 \in (0, t_2^*) \)

We consider first the situation when \( t_2 = t^0 < t_2^* \). Price of \( q_2^* \) in the domestic market will be \( \bar{c} q_2^* + t_2^0 \). As usual let \( t_1 \) be the tariff on the low-quality good. Thus the price of the lower quality variety in the domestic market will be \( \bar{c} q_1^* + t_1 \). From the IC constraint of the \( \alpha_2 \) type consumers it can be easily checked that if \( t_1 \leq t_2^0 - \frac{(\alpha_2 - \alpha_1)^2}{4\bar{c}} = t_1' \) the monopolist will be better-off offering the bundle \((q_1^*, \bar{c} q_1^* + t_1)\) to the \( \alpha_1 \)-type and \((q_2^*, P_2(t_1))\) to the \( \alpha_2 \) type where \( P_2(t_1) = \alpha_2 (q_2^* - q_1^*) + \bar{c} q_1^* + t_1 \) and this comes directly from the IC constraint of the \( \alpha_2 \) type consumers. The gross gain for the monopolist in this situation will be \( n_1 t_1 + n_2 t_1 + \frac{(\alpha_2 - \alpha_1)^2}{4\bar{c}} \). But \( \forall t_1 \in (t_1', t_{p_1}] \) the monopolist will offer \((q_1^*, \bar{c} q_1^* + t_1)\) to the \( \alpha_1 \)-type but it cannot increase the price of \( q_2^* \) any more and will offer the bundle \((q_2^*, \bar{c} q_2^* + t_2^0)\) to the \( \alpha_2 \) type and the gross gain will turn out to be \( n_1 t_1 + n_2 t_2^0 \). It is evident that in this situation there is no potential gain for the monopolist if \( t_1 \) is increased beyond \( t_{p_1} \). Again interesting to note is that in this situation there will be no quality distortion in the market whatever be the level of \( t_1 \). There will be the usual price distortion associated with monopoly pricing.
Proposition 5:

*With competitive producers abroad, there will be no quality distortion in the market* $\forall \ t_2 \in (0,t^*)$ whatever be the level of $t_1$.

**Proof:** Follows from the above discussion.

In other words any $t_1$ is non-distortionary in this situation. It is immediately apparent from the above proposition that tariff on the high-quality variety needs to be sufficiently higher for any quality distortion at the lower end of the market to exist. The relative gains for the monopolist in this situation will look like the following:

$\forall t_1 \in [0,\tilde{t}]$, $RG = n_1t_1 + n_2t_1 + \frac{(\alpha_2 - \alpha_1)^2}{4\bar{c}}$.

$\forall t_1 \in (\tilde{t},t_1')$, $RG = n_2t_1 + \frac{(\alpha_2 - \alpha_1)^2}{4\bar{c}} - n_1(q_1^* - \tilde{q}_1)[\alpha_1 - \bar{c}(q_1^* + \tilde{q}_1)]$.

$\forall t_1 \in (t_1',t_m)$, $RG = n_2t_2^0 - n_1(q_1^* - \tilde{q}_1)[\alpha_1 - \bar{c}(q_1^* + \tilde{q}_1)]$.

Note that $\forall t_1 \in (t_1',t_m)$ the relative gain is invariant with respect to a change in $t_1$. This is mainly due to the fact that as $t_1$ is increased beyond $t_1'$ the monopolist will increase the price of $q_1^*$ by the same amount of $t_1$ but the price of $q_2^*$ cannot increase since $t_2$ is fixed at $t_2^0$. This coupled with the fact that the pre-innovation gain depends on $t_1$ makes the relative gain independent of $t_1$ since it simply cancels out. Thus $\forall t_1 \in (t_1',t_m)$ the relative gain will remain invariant to an increase in $t_1$.

This is illustrated in the following figure:

**Figure 2 Goes Here**
Case 2: $t_2 \in (t_2^*, t_{p2}]$.

Equally interesting case would be to consider $t_2^* < t_2 = t_2^0 < t_{p2}$. In this case in the post innovative situation $\forall t_i \in [0, t_{p1}]$ the monopolist will offer the bundle $(q_i^*, \bar{c}q_i^* + t_i)$ to the $\alpha_i$-type and $(q_2^*, P_2(t_i))$ to the $\alpha_2$ type where $P_2(t_i)$ is same as in the previous section and the gross gain will also be the same i.e. $n_1t_i + n_2t_i + \frac{(\alpha_2 - \alpha_i)^2}{4\bar{c}}$. But in this situation there is a potential gain for the monopolist if $t_i$ is increased beyond $t_{p1}$. Quite similar to the prohibitive tariff case the monopolist will start adjusting downward the low quality offered in the market along the IR constraint of the $\alpha_i$-type and in the process will raise the price of $q_2^*$. The optimal low-quality offered will be

\[
\hat{q}_i = q_i* - \frac{(t_i - t_{p1})}{(\alpha_2 - \alpha_i)}.
\]

This can be found directly from the IC constraint of the $\alpha_2$ type and the computation is similar to the previous section. Note that if $t_i$ is increased beyond $t_{p1}$, $\hat{q}_i < q_1^*$.

This can go upto $t_2^0 - \frac{(\alpha_2 - \alpha_i)^2}{4\bar{c}} = t_i^{''}$. Thus $\forall t_i \in (t_{p1}, t_i^{''})$ there will be downward quality distortion in the market and the monopolists’ gross gain will be

\[
\text{Gross}(Q) = n_1\left[\alpha_i\hat{q}_i - \bar{c}\hat{q}_i^2\right] + n_2\left[\alpha_2(q_2^* - q_1^*) + \bar{c}q_1^* + t_i - \bar{c}q_2^*\right].
\]

Relative gain as usual can be found by subtracting $n_1[\alpha_i\hat{q}_i - \bar{c}\hat{q}_i^2]$ from the above expression and the relative gain will be increasing and concave in $t_i$ up to $t_i^{''}$. Thus the relative gain expressions will look like the following:

$\forall t_i \in [0, \hat{t}_i], \quad RG(Q) = n_1t_i + n_2t_i + \frac{(\alpha_2 - \alpha_i)^2}{4\bar{c}}$..

$\forall t_i \in (\hat{t}_i, t_{p1}], \quad RG(Q) = n_2t_i + \frac{(\alpha_2 - \alpha_i)^2}{4\bar{c}} - n_1(q_1^* - \hat{q}_i)\left[\alpha_1 - \bar{c}(q_1^* + \hat{q}_i)\right]$. 

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\[ \forall t_1 \in (t_{p1}, t'_1], \ RG(Q) = n_1(\alpha_1 \bar{q} - \bar{c}q_1^2) + n_2[\alpha_2(q_2^* - q_1^*) + \bar{c}q_1^2 + t_1 - \bar{c}q_2^* - t_1 - \bar{c}q_1^2]. \]

The difference between this situation and case 1 is that in this case the monopolist gains from an increase in \( t_1 \) beyond \( t_{p1} \) whereas in the earlier case monopolists gain exhausted for low quality tariff ranges below \( t_{p1} \). Also in the previous case there was no possibility of downward quality distortion but in this case there is a possibility if \( t_1 \) is increased beyond \( t_{p1} \).

5. Increasing/reducing \( t_2 \) given any tariff on low-quality: A Discussion

Before we wind this off we discuss very briefly the impact of a change in the tariff on high-quality good given any low quality tariff whether prohibitive or non-prohibitive. Let the low quality tariff is set at \( t'_1 \). Given a tariff \( t_2 \) on the high quality product from we know that if

\[ P_2(t_1) = \alpha_2(q_2^* - q_1^*) + \bar{c}q_1^2 + t_1' > \bar{c}q_2^* + t_2, \quad \text{i.e.} \quad t_2 \leq \tilde{t}_2 = \frac{(\alpha_2 - \alpha_1)^2}{4\bar{c}} + t_1' \]

then there will be a separating equilibrium where the \( \alpha_1 \)-type buys \( q_1^* \) at price \( \bar{c}q_1^2 + t_1' \) and the \( \alpha_2 \)-type buys \( q_2^* \) at price \( \bar{c}q_2^* + t_2 \). But if \( t_2 \) is increased beyond \( \tilde{t}_2 \) the monopolist cannot charge a higher price for \( q_2^* \) because the \( \alpha_2 \)-type will then start buying the low quality product and the monopolist will be worse-off. Thus profit of the monopolist will remain fixed even if \( t_2 > \tilde{t}_2 \). The gross gain \( \forall t_2 \in [0, \tilde{t}_2] \) will be \( n_1t_1' + n_2t_2 \) and \( \forall t_2 \in (\tilde{t}_2, t_{p2}] \) is \( (n_1 + n_2)t_1' \). Thus given \( t_1 \) if \( t_2 \) is increased the gross gain will increase at the rate \( n_2 \). This is because the monopolist will be able to charge higher price on the high quality product. But if \( t_2 \) is increased beyond that then the monopolists’ profit will remain invariant.
6. Conclusion

In a vertically differentiated domestic market with a single domestic firm facing discrete demand and competitive producers abroad we have demonstrated for different tariff ranges how tariff protection can raise the incentive for quality innovation. We have also examined how a monopolist might fail to exert complete monopoly power in the market even if both the high quality and the low quality segment of the market are completely protected. To put it differently, even if there is prohibitive tariff on the higher and lower segment of the market the monopolist will offer the first best qualities to both the high and low type consumers. Downward quality distortion at the lower end will occur only if the tariff on low quality product is increased beyond the prohibitive level. We have also seen that with competitive producers abroad the high quality tariff should be sufficiently higher for the possibility of downward quality distortion at the lower segment of the market whatever the tariff on the low quality product is.

This analysis differs from the earlier analyses on the assumption of holes in consumer preferences that often lead to some interesting and counter-intuitive results. The earlier analyses do not focus explicitly on the innovation incentives of a quality constraint monopolist with respect to trade policy. Although the discrete consumer types with full market coverage rules out any demand effect whatsoever that may arise due to tariff protection, this analysis helps to capture the supply side arguments and optimal innovation decision by a quality constraint domestic monopolist equally efficiently.
References:


Figure 1