Organizational Structure and Product Market Competition

by

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Abstract

We analyze an interaction between a firm’s choice of organizational structure and competition in the product-market. Two organizational structures are considered, namely a centralized-organization, whereby formal authority is retained by a principal, and a decentralized-organization, whereby formal authority is delegated to an agent. We show that the choice of organizational structure hinges on a trade-off between operating-profit and managerial effort. The principal may prefer to choose an organizational structure that generates lower operating-profit to motivate the agent to work hard. The choice of organizational structure may also determine whether the rival is active in the market or forced to exit the market.

Keywords: Formal and Real Authority, Delegation Structure, Product Market Competition

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1 Introduction

Ronald Coase’s seminal publication (Coase, 1937) provided an impetus to the burgeoning literature on the theory of the firm. In this literature, firms are no longer seen as merely a production function that transforms inputs into outputs, but instead as a complex organization that consists of self-centered agents who coordinate their activities in a hierarchy-of-command with a principal (CEO) acting as an ultimate decision-maker. Obviously, the principal faces some inevitable constraints such as limited span of control, attention, and ability, and thus may wish to delegate the decision-making authority to her subordinates to relax these constraints.

Aghion and Tirole (1997) develop a theoretical model that deals with this delegation of authority within a firm. In their paper, the principal has formal authority in the firm that gives her the right to make a decision which in turn affects the firm’s payoff. Obviously, she will only be able to make a decision if she is informed about all available choice of actions and their payoff consequences. In addition, having formal authority also enables her to overrule any decision proposed by her subordinate that is not in line with her interests. Nevertheless, she may decide not to overrule the agent’s decision when she is uninformed, and prefer to delegate the decision making authority to the agent instead, thereby empowering the latter. Thus, even though the principal has formal authority, it is the agent who has real authority in the firm due to his information superiority. The formal authority itself may also be delegated to the agent. Such a delegation of formal authority empowers the agent to make a decision that cannot be overruled by the principal. The optimality of the delegation of authority depends on a trade-off between loss of control and managerial initiative. Delegation is thus like a double-edged sword: It induces the agent to work harder, but it may result in a suboptimal decision by the agent.

In this paper we further extend the model of Aghion and Tirole (1997) to explicitly capture an interplay between the delegation of decision-making authority in a firm and the product-market competition between the firm and its rival. The firm must take into account of its rival’s optimal competitive-strategy when choosing its optimal competitive-strategy, and in turn this sort of strategy influences the firm’s choice of organizational structure, i.e. whether or not to delegate the

\(^1\)Throughout the paper we use ‘her’ for the principal and ‘him’ for the agent.
decision-making authority to an agent. Thus, a firm’s organizational structure is developed in response to its competitive strategy.\(^2\)

The following examples further illustrate the important link between organizational structure and business strategy.\(^3\) Prior to 1980, Eastman Kodak was virtually a monopolist in film production. However, during the early 1980s, Kodak’s market share was diminishing rapidly due to intense competition from its rival Fuji Corporation which produces a higher quality film than Kodak. As a response to this competitive pressure, Kodak decided to reorganize its structure of the decision-making authority. It shifted to a more decentralized structure that allows managers to make important decisions without approval from its CEO. Another example is the case of Honda Motor Company in 1991. Prior to 1991, the company adopted a decentralized decision making structure, in which important decision-making authority was delegated by the CEO to managers. However, by late 1980s, the company’s market share dropped to fourth position after Mitsubishi, Nissan and Toyota. In 1991, the new CEO of Honda, Nobuhiro Kawamoto, decided to centralize decision-making within the company. These two examples clearly demonstrate that competitive pressure can lead to either a more centralized or decentralized decision-making structure.

In our model, the action chosen by either the principal or the agent when formal authority is delegated influences the firm’s relative competitive-position vis-à-vis its rival, and hence the firm’s operating profit. Accordingly, under some settings, the principal may want to be the one choosing the decision, while under some other conditions she may want to delegate the choice to the agent instead. Our approach in modeling the principal’s decision and the corresponding trade-off follows that of Marin and Verdier (2003). However, their paper aims at explaining the link between firm structure, and international trade and globalization. For that purpose, they adopt a general-equilibrium macro framework. In contrast, our model aims at capturing the details of product-market competition and their role in shaping organizational structure, for which a partial-equilibrium Industrial Organization (IO) framework is more appropriate.\(^4\)

\(^2\)This is an influential idea that was put forward decades ago by Alfred Chandler in his seminal book (Chandler, 1962) that examines the historical evolution of the organizational structure of big US corporations like Du Pont, Standard Oil and General Motors. Chandler (1962) concludes that organizational structure is adapted to suit business strategy.

\(^3\)The following two examples are taken from Brickley, J., C. Smith, and J. Zimmerman (2006).

\(^4\)Another paper that uses the Aghion-Tirole framework to analyze the international trade and
The following hypothetical scenario depicts the setting of our model. Consider a firm with a principal and an agent. Suppose that the firm has to choose a production-process to employ from the various production processes available. A priori, both the principal and the agent do not really know the marginal-cost implications of these various production processes. They need to exert costly effort to enable them to learn these marginal-cost implications with some probability. Assume that there are exactly two production processes giving non-negative profits to both the principal and the agent that are worth pursuing, and one of them is preferred by the principal while the other one is preferred by the agent. When the principal’s preferred production-process is implemented, the agent gets no private benefits. The agent gets some private benefits only when his preferred production-process is implemented.

The principal’s most preferred production-process entails lower marginal cost than that of the agent. When the agent has a delegated authority (real or formal authority) and is entitled to choose a production-process, he will choose the one that generates private benefits for him. Accordingly, from the principal’s viewpoint, delegation of authority entails a trade-off between motivating the agent to exert effort using private benefits and employing an inefficient production-process.

The principal’s payoff is not only determined by the choice of production-process but also by the extent of product-market competition between the firm and its competitor. Thus, in contrast to the paper of Aghion and Tirole (1997) which considers the principal’s payoff as being exogenous, our paper considers it as being endogenously determined. Suppose that a firm engages in a Cournot duopoly competition with its rival whose magnitude of marginal cost is in between that of the principal’s and the agent’s. When the agent receives a delegated-authority, he chooses the less efficient production-process than that of the rival and this puts the firm at a disadvantageous position vis-à-vis its rival. Consequently, the firm’s market share shrinks. However, the delegation of decision-right and the presence of private benefits stimulate the agent to exert greater effort to acquire information on the marginal-cost parameters of all production processes. It will thus benefit the principal because it can save the principal’s effort costs and also induces the agent to exert greater effort. Accordingly, the optimality of delegation crucially depends on the marginal cost difference, product market competition, and managerial effort.

globalization issues is Puga and Treffler (2007).
We show that the optimal choice of organizational structure essentially depends on a trade-off between inducing the agent to exert high effort and the reduction in operating profit. When the product market competition intensifies because the firm’s marginal cost increases, or the rival’s marginal cost decreases, or the products become more substitutable, the principal becomes less inclined to delegate formal authority to the agent. Under some conditions, the principal may prefer to choose an organizational structure that gives lower operating-profit in order to motivate the agent to work hard. We also show that the choice of organizational structure may shape the prevailing market structure, in the sense that it determines whether the rival is active in the market or forced to exit.

Our paper is related to the IO literature on the strategic delegation and product market competition pioneered by Vickers (1985), Fershtman and Judd (1987) and Sklivas (1987). This literature focuses on the role of incentive-compensation contracts that owners of competing firms give to their managers and, on how these incentive-compensation contracts can affect the outcome of product market competition. Owners delegate the output decision to their managers, and in exchange managers receive a monetary compensation that is tied to the firms’ profits and sales. It is shown that by giving more emphasis on sales rather than on profits, a principal can induce her manager to act more aggressively by choosing a higher level of output. Our paper, although is done in an entirely different modeling framework, also deals with delegation by a principal to an agent. The principal may delegate not only the output decision but also the choice of production process to be implemented in the firm to the agent. In exchange, the managers may be allowed to choose a production process that gives him private benefits. Thus, we focus on non-monetary compensation instead of monetary compensation.

The rest of this paper is organized as follows. Section 2 presents the model, while Section 3 presents the solutions. Section 4 discusses several market structure configurations arising from these solutions. Section 5 summarizes the results and concludes.

\footnote{Subsequently, there have been extensive research that extends further this particular literature into various directions. The most recent one is Jansen, van Lier and van Witteloostuijn (2007).}

\footnote{To ensure participation, it is assumed that the agent in our paper receives his reservation wage which is normalized to zero.}
2 The Model

There are essentially two building blocks of the model, the first one is the choice of organizational structure and the second one is the product market competition.

2.1 Organizational Structure

This building block is largely based on Aghion and Tirole (1997). There are two firms, called 1 and 2. Firm 1 consists of a principal ($P$) and an agent ($A$). The principal must decide which organizational structure of the decision-making authority to adopt in the firm. There are two organizational structures available, namely a centralized decision-making authority ($P$-organization) in which the principal retains the formal decision-right (formal authority) in the firm, and a decentralized decision-making authority ($A$-organization) in which the principal delegates formal authority to the agent. Throughout this paper, we only focus on firm 1’s choice of organizational structure. We assume that firm 2 is owned and self-managed by its principal.\footnote{This set-up allows us to isolate the impact of competitive pressure from the ‘strategic’ impact of the rival’s (firm 2’s) choice of organizational structure on firm 1’s choice of organizational structure. Thus, it enables us to have a cleaner analysis on the interplay between competitive pressure and organizational structure and also simplifies our analysis considerably.}

Firm 1 must choose a production-process to employ out of $N$ available options. The marginal cost implications of all available production-processes are initially unknown, but they can be inferred by exerting effort. The principal’s effort is denoted by $E$ and the agent’s effort is denoted by $e$. Exerting effort is costly for both. The cost of effort is quadratic and takes the form of $E^2/2$ and $e^2/2$ for respectively the principal and the agent. Upon exerting effort they will be informed about the marginal cost parameters with probabilities $E$ and $e$.

The principal has her most preferred production-process which, when implemented, gives marginal cost $c_p$, with $p$ indicating the principal’s production process. Likewise, the agent has his most preferred production-process which, when implemented, gives marginal cost $c_a = \gamma c_p$ with $\gamma \geq 1$ and $a$ indicating the agent’s production process. The rival (i.e. firm 2) is assumed to employ a production-process that gives marginal cost $c$. Essentially, the setting we are describing here is an example of the well-known managerial moral-hazard problem. When the manager has some discretion in choosing the production technology, he may not always select the
most efficient production-technology when choosing it forces him to forego substantial private benefits (see for instance Shleifer and Vishny 1997, Aghion and Tirole 1997, and Tirole 2006 for more detailed descriptions of managerial moral-hazard).

We rule out random-picking of production-process by assuming that at least one of the available production-processes has infinitely large marginal cost. This also implies that when none is informed, they will prefer to be inactive.

The party with formal authority, who could be either the agent or the principal, has the ultimate right to choose a production-process to employ. Obviously, she (he) will choose her (his) most preferred production-process when informed. The formal authority may be retained by the principal or delegated to the agent. When it is delegated, the agent has the power to choose a production-process to employ without being overruled by the principal. When the formal authority holder is uninformed, she (he) is willing to implement the other party’s most preferred production-process. Thus, in such a case this other party effectively has the real-authority in the firm. Throughout the paper we thus distinguish authority into formal and real authority. The later is the ’de-jure’ authority which is formally and/ or legally assigned to its holder, while the later is the ’de-facto’ authority which is obtained through a possession of superior information.

Similar to Aghion and Tirole (1997) we assume that the agent is solely motivated by private benefits. His most preferred production-process brings private benefits $b$, however it may not necessarily be the most economical one for the principal as $c_a \geq c_p$. On the contrary, the principal’s most preferred production-process is potentially more efficient than that of the agent, but when it is employed the agent receives no private benefits. Consequently, it reduces the agent’s incentive to exert effort.

The principal receives the firm’s profit $\pi_i$, with $i \in \{p, a\}$ denotes the chosen production-process, which could either be the one that is most preferred by the principal ($p$) or the one that is most preferred by the agent ($a$). The magnitude of $\pi_i$ depends on the relative marginal cost of the firm vis-à-vis its rival and also on the nature of product-market competition. Thus, in our framework the payoff of

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8We can also easily consider a less extreme case in which the agent receives some positive private benefits instead of no private benefits when the principal’s most preferred production-process is implemented. Such a consideration should not affect the qualitative aspects of our results.

9Our approach in modelling the trade-off faced by the principal is adapted from Marin and Verdier (2003).
the principal is going to be endogenously determined. When the principal’s most preferred production-process is implemented, she obtains $\pi_{1p}$, however when the agent’s most preferred production-process is implemented instead, she obtains only $\pi_{1a}$. Given that $c_p \leq c_a = \gamma c_p$, we therefore have $\pi_{1p} \geq \pi_{1a}$, and we can express the relationship between the two profits as, $\pi_{1a} = \alpha \pi_{1p}$ with $\alpha \in [0, 1]$. We interpret $\alpha$ as the degree of interest-congruence between the principal and the agent. When $\alpha = 1$, their interests are perfectly aligned, and accordingly the principal receives the same amount of profit no matter which production-process is implemented. For the purpose of our analysis, we thus define the degree of interest-congruence as,

$$\alpha = \frac{\pi_{1a}}{\pi_{1p}}$$

(1)

This endogenous determination of the interest-congruence parameter represents another important departure from the Aghion-Tirole’s framework.

### 2.2 Product Market Competition

Firms 1 and 2 produce differentiated goods. The demand side is characterized by the following quadratic representative consumer’s utility function,

$$U = q_{1i} + q_{2i} - \frac{1}{2} \left( q_{1i}^2 + q_{2i}^2 + 2\rho q_1 q_2 + z \right),$$

where $i \in \{p, a\}$ denotes the chosen production-process, $\rho \in [0, 1]$ is a parameter measuring the degree (strength) of product differentiation, $q_{1i}$ and $q_{2i}$ are the goods produced by respectively firms 1 and 2, and $z$ is a numeraire good. The two products are independent, i.e. completely differentiated, when the degree of product substitution equals to zero ($\rho = 0$), and they are homogenous when $\rho$ equals to one. This utility function is a variant of the one used in Singh and Vives (1984) and Zanchettin (2006).

From the utility maximization problem, we obtain the following linear-demand functions,

$$q_{1i} = \theta_0 - \theta_1 p_{1i} - \theta_2 p_{2i},$$

(2)

$$q_{2i} = \theta_0 - \theta_1 p_{2i} - \theta_2 p_{1i},$$

(3)

where $\theta_0 = \frac{1}{1+\rho}$, $\theta_1 = \frac{1}{1-\rho^2}$, and $\theta_2 = \frac{\rho}{1-\rho^2}$. Assuming that $\rho < 1$, these linear demand functions can be inverted to yield,
\begin{align*}
P_{1i} &= 1 - q_{1i} - \rho q_{2i} \\
P_{2i} &= 1 - q_{2i} - \rho q_{1i}
\end{align*}

Firms 1 and 2 engage in Cournot duopoly-competition.\textsuperscript{10}

\section*{2.3 The Time Line}

The time-line of the model is summarized in Figure 1 below. At \( t = 0 \), the principal chooses either a centralized decision-making structure (\( P \)-organization) or a decentralized decision-making structure (\( A \)-organization). In \( P \)-organization, the principal has formal authority over the choice of production-process, while in \( A \)-organization, the principal delegates formal authority to the agent. At \( t = 1 \), both the principal and the agent simultaneously exert costly effort to obtain information on the marginal-cost parameters of the production processes. When informed, the holder of formal authority chooses a production-process, otherwise, when she (he) is uninformed, the other informed party will choose a production-process. When both are uninformed, no production-process is chosen and the firm is inactive. At \( t = 2 \), the firm begins its production using the chosen production-process and competes with its rival in a Cournot fashion. Finally, at \( t = 3 \), all payoffs are realized. We solve the model using backward induction.

\section*{3 Solution of the Model}

We begin with the product-market competition. At this stage, we take as given the optimal organizational-structure chosen in an earlier stage.

\subsection*{3.1 Product-Market Competition}

The firms’ profit functions can be expressed as,

\begin{align*}
\pi_{1i} &= (1 - q_{1i} - \rho q_{2i})q_{1i} - c_i q_{1i} \\
\pi_{2i} &= (1 - q_{2i} - \rho q_{1i})q_{2i} - c q_{2i}
\end{align*}

\textsuperscript{10}The results obtained under Bertrand competition are qualitatively the same as those obtained under Cournot competition and are available upon request from the authors.
with $i \in \{p, a\}$. Recall that $c_a = \gamma c_p$ with $\gamma \geq 1$. Both firms will be active in the market when their optimal quantities are positive. Whether or not a firm’s optimal quantity is positive depends on the magnitude of the degree of product substitution ($\rho$) and the marginal-cost of the two competing firms ($c$ and $c_i$).

Under duopoly, given that the production-process $i$ is chosen, the optimal quantities can be derived as follows,

$$q_{1,i,d}^C = \frac{2 - \rho - 2c_i + c}{(4 - \rho^2)} \quad (8)$$

$$q_{2,i,d}^C = \frac{2 - \rho - 2c + c_i}{(4 - \rho^2)}. \quad (9)$$

Superscript $C$ denotes the Cournot case, and subscript $d$ denotes the duopoly case. Firms 1 and 2 will be active and produce positive quantities if and only if,

$$[2c - (2 - \rho)] < c_i < \frac{1}{2} \left[ c + (2 - \rho) \right]. \quad (10)$$

When $c_i > \frac{1}{2} [c + (2 - \rho)]$ prevails, it implies that the marginal cost of firm 1 resulting from production-process $i$ is so high that it is not possible for firm 1 to be active in the market. Consequently, firm 2 will be a monopolist. On the other hand, when $c_i < [2c - (2 - \rho)]$ prevails, it implies that the marginal cost of firm 1 resulting from
production-process \( i \) is sufficiently lower than its rival. Firm 2 cannot profitably compete with firm 1, and thus the latter will be a monopolist. Since the focus of our analysis is on firm 1’s choice of organizational structure, we only consider cases in which firm 1 is active in the market no matter which production-process \( i \in \{ p, a \} \) it chooses. We therefore impose the following assumptions.

**Assumption 1** For all \( i \in \{ p, a \} \), \( c_i < \frac{1}{2} \left[ c + (2 - \rho) \right] \).

This assumption implies that firm 1’s marginal cost \( c_i \) should not be too high relative to its rival’s marginal cost \( c \). Otherwise, firm 1 can never be active in the market.

If only firm 1 is active, the optimal monopoly-quantity can be straightforwardly derived as,\(^{11}\)

\[
q_{1,m} = \frac{1 - c_i}{2}
\]

(11)

Subscript \( m \) denotes the monopoly case.

**Assumption 2** \( 0 \leq c < 2 - \rho \) and for all \( i \in \{ p, a \} \), \( 0 \leq c_i < 1 \).

The first part of the assumption ensures that \( [2c - (2 - \rho)] \) and \( \frac{1}{2} [c + (2 - \rho)] \) in (10) are, respectively, the lower and upper bounds of \( c_i \) that give us the duopoly case. The second part of the assumption implies that the case in which firm 1 is a monopolist is potentially viable. For notational simplicity we define,

\[
\Phi = [2c - (2 - \rho)] \quad \text{and} \quad \overline{\Phi} = \frac{1}{2} [c + (2 - \rho)].
\]

The prevailing market-structure depends on the relative magnitude of \( c_i, i \in \{ p, a \} \), vis-a-vis \( c \). The following lemma describes the prevailing market-structure configurations.

**Lemma 1** Given firm 1’s choice of production-process \( i \in \{ p, a \} \), its resulting marginal cost of either \( c_p \) or \( c_a = \gamma c_p \) (with \( \gamma \geq 1 \)), firm 2’s marginal cost \( c \), and also Assumptions 1 and 2, we have the following market-structure configurations;\(^{11}\)

\(^{11}\)Thus, when the rival’s marginal cost is too high relative to firm 1’s marginal cost, we have \((2 - \rho - 2c + c_i) < 0\). Firm 2 will produce nothing \((q_{2,i,d} = 0)\).
(i) For \( \Phi < c_p < \gamma c_p < \Phi \), firms 1 and 2 are duopolists no matter which production-process is chosen by firm 1.

(ii) For \( c_p < \Phi < \gamma c_p < \Phi \), firm 1 is a monopolist when production-process \( p \) is chosen by firm 1, whilst firms 1 and 2 are duopolists when production-process \( a \) is chosen by firm 1.

(iii) For \( c_p < \gamma c_p < \Phi < \Phi \), firm 1 is a monopolist no matter which production-process is chosen by firm 1.

It is interesting to note that as the degree of product substitution \( \rho \) approaches to one (zero), the range of \( c_p \) that allows for duopoly competition becomes smaller (larger). A higher \( \rho \) (i.e. closer to 1) implies a tougher competition since both products become less differentiated, consequently, a firm that has lower marginal cost will likely be a monopolist. The opposite prevails for a lower \( \rho \).

The resulting duopoly profits are,
\[
\pi_{1,1;d}^C = \frac{(2 - \rho - 2c_i + c)^2}{(4 - \rho^2)^2}
\]
\[
\pi_{2,1;d}^C = \frac{(2 - \rho - 2c_i + c_i)^2}{(4 - \rho^2)^2},
\]
while the resulting monopoly profit is,
\[
\pi_{1,1;m} = \frac{(1 - c_i)^2}{4}
\]

Using the above expressions for equilibrium profits, we can derive the following congruence parameters \( \alpha_j \), \( j \in \{1, 2, 3\} \) denotes various market-structure configurations stated in Points (i), (ii), and (iii) of Lemma 1.

**Lemma 2** The degree of interest-congruence between the principal and the agent under all market-structure configurations, \( \alpha_j \), \( j \in \{1, 2, 3\} \), can be derived as follows;
\[
\alpha_1 = \frac{\pi_{1,1,d}}{\pi_{1,1,d}} = \frac{(2 - \rho - 2c_p + c)^2}{(2 - \rho - 2c_p + c)^2}, \quad \alpha_2 = \frac{\pi_{1,1,d}}{\pi_{1,1,m}} = \frac{4(2 - \rho - 2c_p + c)^2}{(4 - \rho^2)^2(1 - c_p)^2}, \quad \text{and} \quad \alpha_3 = \frac{\pi_{1,1,m}}{\pi_{1,1,m}} = \frac{(1 - c_p)^2}{(1 - c_p)^2}.
\]

The congruence parameter \( \alpha \), to some extent, also captures the cost of delegation for the principal. Delegation of formal or real authority reduces firm’s profit due to the agent choosing an inefficient production-process. Nevertheless, despite the profit reduction, the principal may still be willing to delegate authority in order to
motivate the agent to work hard. To have $\alpha \in [0, 1]$, we require that $\pi_{1m,d}^C \leq \pi_{1p,d}^C$ in configuration 1 and $\pi_{1m,d}^C \leq \pi_{4p,m}$ in market-structure configuration 2. Since the agent’s most preferred production-process is clearly less efficient than the principal’s most preferred production process we obviously have $\pi_{1m,d}^C \leq \pi_{4p,d}^C$. It is also straightforward to verify that $\pi_{1m,d}^C \leq \pi_{4p,m}^C$.\[^{12}\]

Notice that when $\gamma$ approaches one, $\alpha_1$ and $\alpha_3$ in Lemma 2 should also approach one, implying that the interests of the principal and the agent become almost perfectly aligned. It is also obvious that $\alpha_3$ does not depend on the degree of product differentiation $\rho$ because no matter which production-process is chosen, firm 1 is always a monopolist under this market-structure.

Subsequently, we evaluate the impact of an increase in, among others; the degree of product-substitution $\rho$, the rival’s marginal cost $c$, firm 1’s marginal cost when the principal’s most preferred production-process is implemented $c_p$, and the cost-inefficiency of the agent’s most preferred production process $\gamma$ on congruence parameters $\alpha_1$ and $\alpha_2$.\[^{13}\] We have the following result.

**Proposition 1** The degree of interest-congruence between the principal and the agent under all market-structure configurations, $\alpha_j$, $j \in \{1, 2, 3\}$, are affected by the rival’s marginal cost $c$, firm 1’s marginal cost when the principal’s most preferred production-process is implemented $c_p$, and the cost-inefficiency of the agent’s most preferred production process $\gamma$, and the degree of product substitution $\rho$ in the following ways,

\[\frac{\partial \alpha_1}{\partial c} \geq 0; \quad \frac{\partial \alpha_1}{\partial c_p} \leq 0; \quad \frac{\partial \alpha_1}{\partial \gamma} < 0; \quad \text{and} \quad \frac{\partial \alpha_1}{\partial \rho} \leq 0.\]

\[^{12}\]The proof of $\pi_{1m,d}^C \leq \pi_{4p,m}^C$ can be sketched as follows. The f.o.c.s of (12), (13), and (14) with respect to own quantity variables are respectively $\partial \pi_{1m,d}^C / \partial q_{1m,d}^C = \left(1 - q_{1m,d}^C - \rho q_{2,d}^C - q_{1,d}^C - c_i\right) = 0$; $\partial \pi_{2m,d}^C / \partial q_{2m,d}^C = \left(1 - q_{2,d}^C - \rho q_{1,d}^C - q_{2,d}^C - c_i\right) = 0$; and $\partial \pi_{1,m} / \partial q_{1,m} = \left(1 - q_{1,m} - q_{1,1m} - c_i\right) = 0$. Next, we can express the monopoly and duopoly profits as $\pi_{1m} = \left(q_{1,m}\right)^2$ and $\pi_{1d}^C = \left(q_{1,d}^C\right)^2$. We know that if $q_{1,m} \geq q_{1,d}^C$ then $\pi_{1m} \geq \pi_{1d}^C$. Using the following f.o.c. $\partial \pi_{1m} / \partial q_{1,m} = 0$, we can simplify $\partial \pi_{1m} / \partial q_{1,m} = \left(1 - q_{1,m} - \rho q_{2,d}^C - q_{1,m} - c_i\right)$ into $\partial \pi_{1m} / \partial q_{1,m} = -\rho q_{2,d}^C$, which is clearly non-positive given that $0 \leq \rho \leq 1$. Hence, we can establish that $q_{1,m} \geq q_{1,d}^C$, which implies that $\pi_{1m} \geq \pi_{1d}^C$.

\[^{13}\]Only the congruence parameters under market-structure configurations 1 and 2, i.e. $\alpha_1$ and $\alpha_2$, are going to be affected by the degree of product substitution $\rho$. This is because duopoly competition can potentially exist only in these two configurations.
(ii) \( \frac{\partial \alpha_2}{\partial c} > 0; \frac{\partial \alpha_2}{\partial c_p} < 0; \frac{\partial \alpha_2}{\partial \gamma} < 0 \) and \( \frac{\partial \alpha_2}{\partial p} < 0 \).

(iii) \( \frac{\partial \alpha_3}{\partial c_p} \leq 0; \frac{\partial \alpha_3}{\partial \gamma} < 0 \).

**Proof.** See Appendix. ■

The non-negative signs of \( \frac{\partial \alpha_1}{\partial c} \) and \( \frac{\partial \alpha_2}{\partial c} \) can be explained as follows. An increase in the rival’s marginal cost \( c \), ceteris paribus, softens the product market competition for firm 1, and boosts its profit. We also know that \( \pi_{1n,d}^C \leq \pi_{1p,d}^C \leq \pi_{1p,m} \), and thus the positive impact of an increase in \( c \) will be larger when the profit is smaller. This implies that \( \pi_{1n,d}^C \) will increase the most, followed by \( \pi_{1p,d}^C \) and \( \pi_{1p,m} \) when \( c \) increases. Since \( \alpha_1 = \pi_{1a,d}^C / \pi_{1p,d}^C \) and \( \alpha_2 = \pi_{1a,d}^C / \pi_{1p,m} \), both \( \alpha_1 \) and \( \alpha_2 \) will therefore increase when \( c \) increases. All in all, the higher the cost advantage of firm 1 vis-a-vis firm 2 is, the higher the interest alignment between the principal and the agent will be.

The explanation behind the non-positive signs of \( \frac{\partial \alpha_3}{\partial c_p} \) and \( \frac{\partial \alpha_3}{\partial c} \) is analogous to the one above. An increase in firm 1’s marginal cost \( c_p \), ceteris paribus, toughens the product market competition for firm 1, and reduce its profit regardless of whether the firm is a monopolist or a duopolist. Given that we have \( \pi_{1a,d}^C \leq \pi_{1p,d}^C \leq \pi_{1p,m} \), the reduction in \( \pi_{1a,d}^C \) will be the largest, followed by the reduction in \( \pi_{1p,d}^C \) and \( \pi_{1p,m} \). Consequently, the higher is the cost disadvantage of firm 1 vis-à-vis firm 2, the lower the interest alignment between the principal and the agent will be.

The degree of product substitution \( \rho \) only influences \( \alpha_1 \) and \( \alpha_2 \) but not \( \alpha_3 \). An increase in \( \rho \) toughens the product market competition for both firms, and squeezes their profits. The reduction in \( \pi_{1a,d}^C \) dominates the reduction in \( \pi_{1p,d}^C \), and consequently \( \alpha_1 \) falls. However, in market-structure configuration 2, only the numerator of \( \alpha_2 \) is negatively affected by an increase in \( \rho \). As a result, \( \alpha_2 \) falls, implying that, when \( \rho \) increases, it becomes increasingly more attractive for the principal to implement her most preferred production process than that of the agent.

Finally, an increase in \( \gamma \) implies that the duopoly profit \( \pi_{1a,d}^C \) and the monopoly profit \( \pi_{1a,m} \) decrease, while the duopoly profit \( \pi_{1p,d}^C \) and the monopoly profit \( \pi_{1p,m} \) remain unchanged. Accordingly, it is straightforward to see that \( \alpha_1 \), \( \alpha_2 \), and \( \alpha_3 \) will thus fall.
3.2 The Optimal Choice of Production Process and Effort Levels

Next, we move backward to the earlier stage. In $P$-organization, the principal retains formal authority and chooses the production-process whenever she is informed. If she is uninformed, she is willing to delegate the choice to an informed agent. The principal and the agent’s expected payoffs can be expressed as,

$$U_P = E\pi_{1p} + (1 - E) e\alpha\pi_{1p} - \frac{E^2}{2}$$  \hspace{1cm} (15)

$$U_A = (1 - E)eb - \frac{e^2}{2}$$  \hspace{1cm} (16)

The above expected payoffs are constructed in the following way. With probability $Ee$, the principal and the agent are both informed, and the principal overrules the agent’s choice of production-process and ask the agent to implement the principal’s most preferred production-process instead. The principal obtains $\pi_{1p}$ and the agent obtains no private benefits. The magnitude of profits $\pi_{1p}$ depends on the market-structure configurations derived previously. Recall that firm 1 may become either a monopolist and obtain $\pi_{1p} = \pi_{1p,m}$, or a duopolist and obtain $\pi_{1p} = \pi_{1p,d}^C$.

With probability $E(1 - e)$, only the principal is informed, and thus she would prefer to implement her most preferred production-process that yields profits $\pi_{1p}$ for the principal and no private benefits for the agent.

With probability $(1 - E)e$, only the agent is informed, and thus it is optimal for the principal to let the agent implement his most preferred production-process. Thus, here the agent has the real-authority. The principal obtains $\alpha\pi_{1p}$, $\alpha \in [0, 1]$, while the agent obtains $b$. Recall that $\alpha$ is endogenously determined and its value depends on the prevailing market-structure configuration $j \in \{1, 2, 3\}$ that was derived in the previous section.

Finally, with probability $(1 - E)(1 - e)$, both the principal and the agent are uninformed, status-quo prevails and payoffs are normalized to 0.

Exerting effort is costly for both the principal and the agent and the costs are assumed to be increasing at an increasing rate in the amount of effort exerted, i.e. $E^2/2$ and $e^2/2$.

The principal and the agent choose respectively $E$ and $e$ to maximize their expected payoffs. The effort best-response functions can be derived as $E^*$ =
$(1 - e^*) \pi_1p$ and $e^* = (1 - E^*) b$. We require $0 \leq \pi_1p \leq 1$ and $0 \leq \alpha b \leq 1$ for stability and to ensure that the equilibrium effort levels are non-negative. We can also observe that effort levels are strategic substitutes, $E$ is decreasing in $e$, and vice-versa. Solving the best-response functions simultaneously yields the following optimal effort levels,

\begin{align*}
  E^* &= \frac{(1 - \alpha b) \pi_1p}{1 - \alpha b \pi_1p} \quad (17) \\
  e^* &= \frac{(1 - \pi_1p) b}{1 - \alpha b \pi_1p}. \quad (18)
\end{align*}

It can be straightforwardly verified that $\frac{\partial E^*}{\partial \pi_1p} > 0$, $\frac{\partial E^*}{\partial b} < 0$, and $\frac{\partial E^*}{\partial \alpha} < 0$ for the principal; and $\frac{\partial e^*}{\partial \pi_1p} < 0$, $\frac{\partial e^*}{\partial b} > 0$, and $\frac{\partial e^*}{\partial \alpha} > 0$ for the agent. These are essentially the results obtained by Aghion and Tirole (1997). Higher profits induce higher monitoring-effort by the principal, and less effort by the agent. Higher private benefits motivate the agent to work harder and lower the monitoring-effort by the principal. Finally, when the interests of the principal and the agent become more aligned, the principal lowers her monitoring-effort and the agent exerts more effort.

In $A$-organization, the principal delegates the formal decision-right to the agent. The agent will then choose the production-process whenever he is informed and the principal cannot overrule the agent’s decision. However, when the agent is uninformed while the principal is informed, the principal will instead choose the production-process. Thus, in this case the principal has the real decision-right. The principal and the agent’s expected payoffs (denoted by $V_A$ and $V_P$) can be written as,

\begin{align*}
  V_P &= e\alpha \pi_1p + E(1 - e)\pi_1p - \frac{E^2}{2} \quad (19) \\
  V_A &= eb - \frac{e^2}{2}. \quad (20)
\end{align*}

The above expected payoffs are constructed in a similar fashion as the expected payoffs under $P$-organization. The effort best-response functions can be derived respectively as, $E^{**} = (1 - e^{**}) \pi_1p$ and $e^{**} = b$. We require that $0 \leq \pi_1p \leq 1$ and $0 \leq b \leq 1$ for stability and to ensure that the equilibrium effort levels are non-negative. The optimal effort levels under $A$-organization can be derived as,
\[ E^{**} = (1 - b)\pi_1p \]  
\[ e^{**} = b \]  

It can be easily verified that under A-organization we have \( \frac{\partial E^{**}}{\partial \pi_1p} > 0 \), \( \frac{\partial E^{**}}{\partial b} < 0 \), and \( \frac{\partial e^{**}}{\partial \pi_1p} = 0 \) for the principal; and \( \frac{\partial e^{**}}{\partial b} = 0 \), \( \frac{\partial e^{**}}{\partial \alpha} > 0 \), and \( \frac{\partial e^{**}}{\partial \alpha} = 0 \) for the agent.

### 3.3 The Choice of Organizational Structure: P-organization v.s. A-organization

We now evaluate the principal’s choice of organizational structure. We begin with the principal’s expected payoffs under P-organization. By plugging back the solutions for the optimal effort levels (17) and (18) into (15) we can express the principal’s expected payoffs as \( U_P(\alpha, b, \pi_1p) \), in which \( \pi_1p \) is the optimal profit derived in the product-market competition stage. We can then show that,

**Lemma 3** The principal’s expected payoffs under P-organization \( U_P(\alpha, b, \pi_1p) \), have the following characteristics,

1. \( U_P(\alpha, b, \pi_1p)\big|_{\pi_1p=0} = 0 \) and \( U_P(\alpha, b, \pi_1p)\big|_{\pi_1p=1} = 1/2 \).
2. \( \frac{\partial U_P(a, b, \pi_1p)}{\partial \pi_1p} \big|_{\pi_1p=0} = ab \), and \( \frac{\partial U_P(a, b, \pi_1p)}{\partial \pi_1p} \big|_{\pi_1p=1} = 1 \).
3. For \( 0 \leq ab \leq 1/3 \) we have \( \frac{\partial^2 U_P(a, b, \pi_1p)}{\partial \pi_1p^2} \geq 0 \).
4. For \( 1/3 < ab < 1 \) we have: \( \frac{\partial^2 U_P(a, b, \pi_1p)}{\partial \pi_1p^2} < 0 \) if \( \pi_1p < \frac{3ab-1}{2ab} \), \( \frac{\partial^2 U_P(a, b, \pi_1p)}{\partial \pi_1p^2} > 0 \) if \( \pi_1p > \frac{3ab-1}{2ab} \), and \( \frac{\partial^2 U_P(a, b, \pi_1p)}{\partial \pi_1p^2} = 0 \) if \( \pi_1p = \frac{3ab-1}{2ab} \).

**Proof.** See Appendix. □

Thus, \( U_P \) is increasing in \( \pi_1p \), and given that \( \pi_1p \in [0, 1] \), the lower and upper bounds of \( U_P \) are equal to those stated in Point (i). Also, \( U_P \) is concave in \( \pi_1p \) for \( \pi_1p < \frac{3ab-1}{2ab} \) and convex in \( \pi_1p \) for \( \pi_1p > \frac{3ab-1}{2ab} \), and \( \pi_1p = \frac{3ab-1}{2ab} \) is the inflection point of \( U_P \). It can be easily verified that there exists a non-negative inflection point only if \( 1/3 < ab \leq 1 \). Consequently, when \( 0 \leq ab \leq 1/3 \) prevails, \( U_P \) is convex in \( \pi_1p \).
Next, we evaluate the principal’s expected payoffs under $A$-organization. By plugging in (21) and (22) into (19), we can express the principal’s expected payoffs as $V_P(\alpha, b, \pi_{1_p})$, where $\pi_{1_p}$ is the optimal profit derived in the product-market competition stage. We can then show that,

Lemma 4  The principal’s expected payoffs under $A$-organization $V_P(\alpha, b, \pi_{1_p})$ have the following characteristics,

(i) $V_P(\alpha, b, \pi_{1_p})\big|_{\pi_{1_p}=0} = 0$ and $V_P(\alpha, b, \pi_{1_p})\big|_{\pi_{1_p}=1} = \alpha b + \frac{(1-b)^2}{2}$.

(ii) $\frac{\partial V_P(a, b, \pi_{1_p})}{\partial \pi_{1_p}} \bigg|_{\pi_{1_p}=0} > 0$; $\frac{\partial V_P(a, b, \pi_{1_p})}{\partial \pi_{1_p}} \bigg|_{\pi_{1_p}=1} = \alpha b + (1 - b)^2$

(iii) $\frac{\partial^2 V_P(a, b, \pi_{1_p})}{\partial \pi_{1_p}^2} \geq 0$.

Proof. See Appendix. ■

Thus, $V_P$ is increasing and convex in $\pi_{1_p}$, and given that $\pi_{1_p} \in [0, 1]$, the lower and upper bounds of $V_P$ are equal to those stated in Point 1.

Lemmas 3 and 4 can be used to compare the magnitude of $U_P(\alpha, b, \pi_{1_p})$ and $V_P(\alpha, b, \pi_{1_p})$ for all of the admissible range of $\alpha$, $b$, and $\pi_{1_p}$ to determine under what conditions $P$-organization is superior to $A$-organization, and vice versa. The result is stated in the following proposition.

Proposition 2  The principal prefers $P$-organization to $A$-organization, i.e. $U_P(\alpha, b, \pi_{1_p}) > V_P(\alpha, b, \pi_{1_p})$, when $0 < \alpha b < \frac{1}{3}$ and $b < 2(1 - \alpha)$ are satisfied simultaneously. She prefers $A$-organization to $P$-organization, i.e. $U_P(\alpha, b, \pi_{1_p}) < V_P(\alpha, b, \pi_{1_p})$, when either $0 < \alpha b < \frac{1}{3}$ and $b > 2(1 - \alpha)$ are satisfied simultaneously, or $\frac{1}{3} < \alpha b < 1$ is satisfied.

Proof. See Appendix. ■

Figure 2 depicts the above proposition graphically. In $x$-axis, we have the interest-congruence parameter $\alpha$ and in $y$-axis we have private benefits $b$. When the degree of interest-congruence $\alpha$ is sufficiently small, it is always better for the principal to retain formal authority by adopting $P$-organization. However, when the congruence parameter $\alpha$ is sufficiently high, the attractiveness of $A$-organization...
is increasing in the level of private benefits $b$. This is because when the principal and the agent have sufficiently aligned interests and private benefits $b$ are sufficiently high, delegation of formal authority to the agent will motivate the agent to exert higher effort. This will increase the probability of the agent being informed. Accordingly, there is a higher chance that the agent’s most preferred production-process is implemented. Since the congruence parameter $\alpha$ is sufficiently high, the principal will not suffer much from being forced to choose the agent’s most preferred production process.

![Figure 2: The Optimal Choice of Organizational Structure](image)

**Proposition 3** Under $A$-organization, the agent’s most preferred production-process ($a$) is more likely to be chosen than the principal’s most preferred production-process ($p$). The latter is more likely to be chosen under $P$-organization.

**Proof.** See Appendix.

In the next section we will evaluate what happen to the optimal choice of organizational structure when the intensity of product market competition changes.
4 Discussions

4.1 Market-Structure Configuration 1: The Duopoly Case

Under this configuration, the assignment of real authority does not really matter for the market structure. No matter who has real authority, the selected production-process will result in duopoly competition. However, the principal’s and the agent’s expected payoffs will depend on who has formal authority in the firm. The principal will therefore choose an organizational structure that gives her the highest expected payoff.

Figure 2 depicts the principal’s choice of organizational structure. A-organization dominates P-organization for all pairs of \((\alpha; b)\) located in the \(V_P\) area, while P-organization dominates A-organization for all pairs of \((\alpha; b)\) located in the \(U_P\) area. Note that private benefits \(b\) are exogenously determined, while the degree of interest-congruence \(\alpha_1\) depends on among others; the rival’s marginal cost \(c\), the firm’s marginal cost under the principal’s most preferred production-process \(c_p\), the inefficiency-parameter of the agent’s most preferred production-process \(\gamma\), and the degree of product substitution \(\rho\). The following proposition summarizes the impacts of a change in exogenous variables \((c, c_p, \gamma, \rho)\) affecting \(\alpha_1\) and the level of private benefits \(b\) on the optimal choice of organizational-structure. As a benchmark, we consider the case in which the principal is indifferent between the two organizational structures.

**Proposition 4** Suppose that initially the principal is indifferent between retaining formal authority, i.e. choosing P-organization, and delegating formal authority, i.e. choosing A-organization, and the relative marginal-costs configuration is given by \(\Phi < c_p < c < \gamma c_p < \Phi\).

(i) Holding \(b\) constant, the principal delegates (retains) formal authority if any of these changes prevails; \(c\) increases (decreases), \(c_p\) decreases (increases), \(\gamma\) decreases (increases), or \(\rho\) decreases (increases).

(ii) Holding \(\alpha\) constant, the principal delegates (retains) formal authority if the agent’s private benefits \(b\) increase (decrease).

Points (i) and (ii) above are derived using the results stated in Propositions 1 and 2. Choosing A-organization implies that the principal is willing to give up
formal authority. This also implies that, when the agent is informed, the principal will never be able to force the agent to implement her most preferred production-process, which is more efficient. Nonetheless, the principal’s expected payoff under $A$-organization may still be higher than that under $P$-organization. Under $A$-organization, the agent will be more motivated to exert effort. Since effort levels are strategic substitutes, an increase in the agent’s effort levels should decrease the principal’s effort levels, thereby allowing the principal to gain from a reduction in the cost of effort and an increase in the probability of implementing a production-process other than the status quo. As long as the gains outweigh the reduction in the firm’s operating profit ($\pi_{1,a,d}^C - \pi_{1,p,d}^C$) due to the adoption of a less efficient production-process, choosing $A$-organization is indeed optimal for the principal. All in all, we have shown that the optimal choice of organizational structure crucially depends on a trade-off between operating profit and managerial effort.

It is also worth noting that our results point to a negative relationship between the toughness of product-market competition and the incentive of the principal to delegate formal authority. Competition becomes tougher for firm 1 when its marginal cost increases, or its rival’s marginal cost decreases, or the inefficiency parameter of the agent’s most preferred production-process increases. In such cases, firm 2 becomes relatively more efficient than firm 1. Its profit will increase at the expense of firm 1. Competition also becomes tougher when products are more substitutable. However, it negatively affects both firms in a similar fashion. We show in this paper that a tougher competition makes the principal less inclined to delegate formal-authority to the agent.

The case of delegation of formal authority (or $A$-organization) can also be loosely interpreted as outsourcing or divestiture, while the case of no-delegation of formal authority (or $P$-organization) can be loosely interpreted as merger or integration. On the basis of this loose interpretation, our results argue that a more intense competition should result in a higher prevalence of integrated firms.

4.2 Market-Structure Configuration 2: The Mixed Case

Under this configuration, firm 1 is a duopolist whenever the agent has real authority and selects his most preferred production process, but instead firm 1 is a monopolist whenever the principal has real authority and selects her most preferred production process. Similar to the previous market-structure configuration, the assignment
of formal authority determines the magnitude of the agent’s and the principal’s expected payoffs.

The following proposition summarizes the impacts of a change in the exogenous variables \((c, c_p, \gamma, \rho)\) affecting \(\alpha_2\) and the level of private benefits \(b\) on the optimal choice of organizational-structure. The case in which the principal is indifferent between the two organizational structures is used as our benchmark.

**Proposition 5** Suppose that initially the principal is indifferent between retaining formal authority, i.e. choosing \(P\)-organization, and delegating formal authority, i.e. choosing \(A\)-organization, and the relative marginal-costs configuration is given by \(c_p < \Phi < \gamma c_p < \Phi\).

(i) Holding \(b\) constant, the principal delegates (retains) formal authority if any of these changes prevails; \(c\) increases (decreases), \(c_p\) decreases (increases), \(\gamma\) decreases (increases), or \(\rho\) decreases.

(ii) Holding \(\alpha\) constant, the principal delegates (retains) formal authority if the agent’s private benefits \((b)\) increase (decrease).

Thus, when firm 1 faces less intense product-market competition, for instance because of the rival’s marginal cost increases, or its own marginal cost decreases, or the products become less substitutable, it is more likely that the principal will choose \(A\)-organization than \(P\)-organization. Otherwise, when firm 1 faces a tougher competition, it is more likely that the principal will choose \(P\)-organization than \(A\)-organization.

From proposition 3 we know that in \(A\)-organization, production-process \(a\) has a higher probability of being chosen than production process \(p\). We also know that under this market-structure configuration, duopoly competition prevails when \(a\) is being implemented, while monopoly prevails when \(p\) is being implemented instead. Therefore, we can establish the following proposition.

**Proposition 6** In market-structure configuration 2, duopoly competition is more likely to occur when \(A\)-organization is chosen, while monopoly is more likely when \(P\)-organization is chosen instead.
All in all, our results presented in this sub-section point to interesting implications. First, we show that when the product-market competition becomes softer, an-organization is more likely to be chosen, and this implies that duopoly competition is more likely to occur. We know that duopoly profit is less than monopoly profit, and yet firm 1 would rather choose an-organization than p-organization. Thus, firm 1 would prefer to accommodate rather than to drive its rival out of the market. By sacrificing operating profit, the principal can motivate the agent to greater effort and this allows the principal to economize on her effort cost. Since the product-market competition is softening, the degree of interest congruence between the principal and the agent increases, and therefore the reduction in operating profit does not dominate the benefit of effort-cost reduction.

Second, when the product-market competition becomes tougher, p-organization is more likely to be chosen, and this implies that monopoly is more likely to occur. By choosing p-organization, firm 1 is able to drive its rival out of the market. Here, the reduction in operating profit dominates the benefit of effort-cost reduction. Thus, our results show that the choice of organizational structure may have an important implication on market structure.

The above result, to some extent, also gives an interesting empirical interpretation. We show that under an-organization it is likely that there is more competition and less market concentration than under p-organization. Further, in an an-organization, the manager is powerful and possesses formal control right over important decisions. The power conferred to the manager typically arises from the fact that the firm has a dispersed ownership-structure in which no single shareholder can dominate the firm. On the contrary, when the ownership structure is concentrated in the hand of a large shareholder, the formal control right is usually retained by the controlling shareholder. We can thus use ownership concentration as a proxy for the organizational structure. An organization with low ownership concentration resembles more of an an-organization than a p-organization. Our result thus points to a positive relationship between ownership concentration and market concentration; higher ownership concentration implies higher market concentration.

4.3 Market-Structure Configuration 3: The Monopoly Case
Under this configuration, the assignment of real authority does not really matter for the market structure. Firm 1 is always a monopolist irrespective of who has real
authority. However, the assignment of formal authority determines the magnitude of the agent’s and the principal’s expected payoffs.

The following proposition summarizes the impacts of a change in the exogenous variables \((c_p, \gamma)\) affecting \(\alpha_3\) and the level of private benefits \(b\) on the optimal choice of organizational-structure. Notice that \(c\) and \(\rho\) do not influence \(\alpha_3\). As a benchmark, we consider the case in which the principal is indifferent between the two organizational structures.

**Proposition 7** Suppose that initially the principal is indifferent between retaining formal authority, i.e. choosing \(P\)-organization, and delegating formal authority, i.e. choosing \(A\)-organization, and the relative marginal-costs configuration is given by \(c_p < \gamma c_p < \Phi < \Phi\).

(i) Holding \(b\) constant, the principal delegates (retains) formal authority if either \(c_p\) decreases (increases) and \(\gamma\) decreases (increases).

(ii) Holding \(\alpha\) constant, the principal delegates (retains) formal authority if the agent’s private benefits \((b)\) increase (decrease).

It is interesting to note that the above results are in contrast to the results of Marin and Verdier (2003). In their paper, an increase in the degree of product market competition induces the principal to move from \(P\)-organization to \(A\)-organization. Our paper shows that even in the absence of product-market competition the principal may choose \(A\)-organization instead of \(P\)-organization.

## 5 Conclusion

This paper explores the link between a firm’s choice governing organizational structure of the decision-making process and product market competition. We consider a firm that consists of a principal and an agent. The firm engages in a Cournot duopoly competition. Two types of organizational structure are considered, namely \(P\)-organization in which formal authority is retained by the principal and \(A\)-organization in which formal authority is delegated to the agent. The holder of formal authority is entitled to decide which production process to implement. If the principal is the holder, she prefers to choose the most economical production process. If the agent is the holder he prefers to choose a production process.
that gives him private benefits even though it may not necessarily be the most economical one.

Apriori, the marginal cost implications of all the available production process are unknown to both the principal and the agent. However, they can be learned when costly information acquisition effort is exerted. The greater the exerted effort, the higher the cost of effort will be, but also the higher is the probability of getting informed. The presence of private benefits motivates the agent to exert higher effort, which is what the principal wants. Delegation of formal authority to the agent may thus be actually beneficial for the principal in terms of effort elicitation, although it may give the principal a lower operating profit.

In the paper, we distinguish authority into formal and real authority. The former is defined as the authority that is legally assigned to the holder, while the latter is defined as the authority that is obtained due to the information superiority.

We show that there is a negative relationship between the toughness of product market competition and the incentive of the principal to delegate formal authority to the agent. Competition gets tougher for various reasons such as an increase in own marginal cost, a decrease in its rival’s marginal cost, an increase in the inefficiency of the production process, and an increase in the degree of product substitution. Our paper thus argues that a delegation of formal authority will be less likely to occur when the product market competition intensifies.

We also have other interesting implications. Under some conditions, firm 1 would prefer to choose an organizational structure that will accommodate its rival and gives the firm a lower operating profit. Had the firm chosen an alternative organizational structure, its rival would have been driven out of the market. Essentially, by sacrificing its operating profit, the principal can motivate the agent to exert higher effort and this allows the principal to economize on her effort cost. The reduction in operating profit is dominated by the benefit of effort-cost reduction. On the contrary, under some other conditions, firm 1 may instead prefer to choose an organizational structure that will drive its rival out of the market. This happens when the reduction in operating profit dominates the benefit of effort-cost reduction. To conclude, we show that the choice of organizational structure may shape the prevailing market structure.

Appendix
Proof of Proposition 1  Taking the derivatives of $\alpha_1$ with respect to $c$, $c_p$, $\gamma$, and $\rho$, 

\[
\frac{\partial \alpha_1}{\partial c} = 4 (\gamma - 1) \frac{c_p (2 - \rho - 2c_p \gamma + c)}{(2 - \rho - 2c_p + c)^3}
\]

\[
\frac{\partial \alpha_1}{\partial c_p} = -4 (\gamma - 1) \frac{(2 - \rho + c) (2 - \rho - 2c_p \gamma + c)}{(2 - \rho - 2c_p + c)^3}
\]

\[
\frac{\partial \alpha_1}{\partial \gamma} = -4 c_p (2 - \rho - 2c_p \gamma + c)
\]

\[
\frac{\partial \alpha_1}{\partial \rho} = \frac{4 (2 - \rho - 2c_p \gamma + c)}{(2 - \rho - 2c_p + c)^3} c_p (1 - \gamma)
\]

Evaluating their signs gives us the followings $\frac{\partial \alpha_1}{\partial c} \geq 0$, $\frac{\partial \alpha_1}{\partial c_p} \leq 0$, $\frac{\partial \alpha_1}{\partial \gamma} < 0$, and $\frac{\partial \alpha_1}{\partial \rho} \leq 0$.

Next, evaluating the derivatives of $\alpha_2$ with respect to $c$, $c_p$, $\gamma$, and $\rho$ yields,

\[
\frac{\partial \alpha_2}{\partial c} = \frac{8 (2 - \rho - 2c_p \gamma + c)}{(\rho^2 - 4)^2 (c_p - 1)^2}
\]

\[
\frac{\partial \alpha_2}{\partial c_p} = -\frac{8 (2 - \rho - 2c_p \gamma + c)}{(\rho^2 - 4)^2 (c_p - 1)^2} [2 - 2\gamma - \rho + c]
\]

\[
\frac{\partial \alpha_2}{\partial \gamma} = -16 \frac{c_p (c - \rho - 2c_p \gamma + 2)}{(\rho^2 - 4)^2 (c_p - 1)^2}
\]

\[
\frac{\partial \alpha_2}{\partial \rho} = \frac{32 c_p (2 - 2\gamma c_p + c - \rho)^2}{(1 - c_p)^2 (4 - \rho^2)^3} \left[ \frac{c}{2} - \frac{(2 - \rho)^2}{4\rho} - c_p \right]
\]

We can easily verify that $\frac{\partial \alpha_2}{\partial c} > 0$ and $\frac{\partial \alpha_2}{\partial \gamma} < 0$.

It can also be shown that $\frac{\partial \alpha_2}{\partial c_p} > 0$. The proof can be outlined as follows. We know that $\alpha_2 = \frac{\pi_{1a,d}^c}{\pi_{1p,m}}$, and

\[
\frac{\partial \alpha_2}{\partial c_p} = \frac{\frac{\partial \pi_{1a,d}^c}{\partial c_p} \pi_{1p,m} - \frac{\partial \pi_{1p,m}}{\partial c_p} \pi_{1a,d}}{(\pi_{1p,m})^2}
\]

\[
= -\frac{8 (2 - \rho - 2c_p \gamma + c)}{(c_p - 1)^3 (\rho^2 - 4)^2} (c - 2\gamma - \rho + 2).
\]

Thus, $\text{sign} \left( \frac{\partial \alpha_2}{\partial c_p} \right) = \text{sign} (c - 2\gamma - \rho + 2)$. We have,

\[
\frac{\partial \alpha_2}{\partial c_p} \begin{cases} 
> 0 \text{ if } \gamma < \frac{1}{2} (c + 2 - \rho) \\
= 0 \text{ if } \gamma = \frac{1}{2} (c + 2 - \rho) \\
< 0 \text{ if } \gamma > \frac{1}{2} (c + 2 - \rho)
\end{cases}
\]
Let us define $\gamma = \frac{1}{2}(c + 2 - \rho)$, and suppose that

$$
\gamma = \frac{1}{2}(c + 2 - \rho) - \varepsilon,
$$

with either $\varepsilon < 0$ or $\varepsilon \geq 0$. We know that the monopoly profit is larger than the duopoly profit, $\pi_{1,m} > \pi_{1,d}^C$ or,

$$
\frac{(1 - c_p)^2}{4} > \frac{(2 - \rho - 2\gamma c_p + c)^2}{(4 - \rho^2)^2}.
$$

Substituting $\gamma$ into the above expression and simplifying the resulting expression we obtain,

$$
0 > 2c - \rho(2 - \rho) + \frac{4\varepsilon c_p}{(1 - c_p)}.
$$

We know that in order to have market-structure configuration 2, we require $c_p < [2c - (2 - \rho)] < \gamma c_p < \frac{1}{2}[c + (2 - \rho)]$. We also know that $0 \leq c_p < 1$. Consequently, we should have, $2c - (2 - \rho) > 0$. Next, since $0 \leq \rho \leq 1$, we have $\rho(2 - \rho) \leq (2 - \rho)$, which implies that,

$$
2c - \rho(2 - \rho) > 0.
$$

Suppose that we have $\varepsilon \geq 0$, then,

$$
0 > 2c - \rho(2 - \rho) + \frac{4\varepsilon c_p}{(1 - c_p)}\quad \text{with } \varepsilon \geq 0,
$$

which is a contradiction. Thus, in order to satisfy Assumption 2, we should have $\varepsilon < 0$ where $\varepsilon$ is sufficiently high, and hence

$$
\gamma = \gamma - \frac{\varepsilon}{(1 - c_p)} \quad \text{and } \gamma > \gamma.
$$

Accordingly, since $\gamma > \gamma$ we thus have $\frac{\partial \pi_{1,d}}{\partial \rho} < 0$. This completes the proof.

Next, we proof that indeed $\text{sign} \left( \frac{\partial \pi_{2,d}}{\partial \rho} \right) < 0$. First, we derive the following f.o.c.s for the maximization of (12) and (13),

$$
\frac{\partial \pi_{1,d}^C}{\partial q_{1,d}^C} = \left( 1 - 2q_{1,d}^C - \rho q_{2,d}^C - c_i \right) = 0
$$

$$
\frac{\partial \pi_{2,d}^C}{\partial q_{2,d}^C} = \left( 1 - 2q_{2,d}^C - \rho q_{1,d}^C - c_i \right) = 0.
$$

Totally differentiating the above f.o.c.s, while holding $c_i$ constant, yields

$$
\frac{\partial^2 \pi_{1,d}^C}{\partial \left( q_{1,d}^C \right)^2} \left( \frac{d_1}{d_{1,d}} \right)^2 + \frac{\partial^2 \pi_{1,d}^C}{\partial q_{1,d}^C \partial q_{2,d}^C} \left( \frac{d_{2,d}}{d_{2,d}} \right) + \frac{\partial^2 \pi_{1,d}^C}{\partial q_{1,d}^C \partial \rho} \frac{d\rho}{d_{1,d}} = 0
$$

$$
\frac{\partial^2 \pi_{2,d}^C}{\partial \left( q_{2,d}^C \right)^2} \left( \frac{d_2}{d_{2,d}} \right)^2 + \frac{\partial^2 \pi_{2,d}^C}{\partial q_{2,d}^C \partial q_{1,d}^C} \left( \frac{d_{1,d}}{d_{1,d}} \right) + \frac{\partial^2 \pi_{2,d}^C}{\partial q_{2,d}^C \partial \rho} \frac{d\rho}{d_{2,d}} = 0.
$$
We know that \( \frac{\partial^2 \pi_{1,d}^C}{\partial (q_{1,i,d}^C)^2} = \frac{\partial^2 \pi_{2,d}^C}{\partial (q_{2,i,d}^C)^2} = -2; \frac{\partial^2 \pi_{1,d}^C}{\partial q_{1,i,d}^C \partial q_{2,i,d}^C} = \frac{\partial^2 \pi_{2,d}^C}{\partial q_{2,i,d}^C \partial q_{1,i,d}^C} = -\rho; \frac{\partial^2 \pi_{1,d}^C}{\partial q_{1,i,d}^C \partial \rho} = \frac{\partial^2 \pi_{2,d}^C}{\partial q_{2,i,d}^C \partial \rho} = -q_{2,i,d}^C \); and \( \frac{\partial^2 \pi_{2,d}^C}{\partial q_{2,i,d}^C \partial \rho} = -q_{1,i,d}^C \). Hence,\[
\begin{bmatrix}
-2 & -\rho \\
-\rho & -2
\end{bmatrix}
\begin{bmatrix}
dq_{1,i,d}^C \\
dq_{2,i,d}^C
\end{bmatrix}
= \begin{bmatrix}
q_{1,i,d}^C \\
q_{2,i,d}^C
\end{bmatrix}
\begin{bmatrix}
\rho
\end{bmatrix}
\]
Thus,
\[
\frac{dq_{1,i,d}^C}{d\rho} = \frac{1}{4 - \rho^2} \left(-2q_{2,i,d}^C + \rho q_{1,i,d}^C\right)
\]
\[
\frac{dq_{2,i,d}^C}{d\rho} = \frac{1}{4 - \rho^2} \left(\rho q_{2,i,d}^C - 2q_{1,i,d}^C\right),
\]
using (8) and (9) we can re-write,
\[
-2q_{2,i,d}^C + \rho q_{1,i,d}^C = \frac{1}{4 - \rho^2} \left[-(2 - \rho)^2 - (4 - \rho)c - 2c_i - 2\rho c_i\right] < 0.
\]
Thus, we know that \( \frac{dq_{1,i,d}^C}{d\rho} < 0 \). Hence, we can establish the following,
\[
\frac{\partial \alpha_2}{\partial \rho} = \frac{\partial}{\partial \rho} \left(\frac{\pi_{1,a,d}^C}{\pi_{1,p,m}^1}\right) = \frac{1}{\pi_{1,p,m}^1} \frac{\partial \left(q_{1,a,d}^C\right)^2}{\partial \rho}
\]
\[
\frac{\partial \alpha_2}{\partial \rho} = \frac{1}{\pi_{1,p,m}^1} 2q_{1,a,d}^C \frac{\partial q_{1,a,d}^C}{\partial \rho} < 0.
\]
Thus, we proof that \( \frac{\partial \alpha_2}{\partial \rho} < 0 \).

Finally, evaluating the derivatives \( \alpha_3 \) with respect to \( \gamma \) yields,
\[
\frac{\partial \alpha_3}{\partial \gamma} = -2(\gamma - 1) \frac{(c_p\gamma - 1)}{(c_p - 1)^3}
\]
\[
\frac{\partial \alpha_3}{\partial \gamma} = \frac{2c_p(c_p\gamma - 1)}{(c_p - 1)^2}.
\]
It can be straightforwardly verified that we have \( \frac{\partial \alpha_3}{\partial c_p} \leq 0 \) and \( \frac{\partial \alpha_3}{\partial \gamma} < 0 \). Thus, we complete our proof of Proposition 1.■

Proof of Lemma 5 Point 1 can be verified easily by substituting \( \pi_{1,p} = 0 \) and \( \pi_{1,p} = 1 \) into \( U_P(\alpha, b, \pi_{1,p}) \). Next, taking the first derivative of \( U_P(\alpha, b, \pi_{1,p}) \) with respect to \( \pi_{1} \), yields,
\[
\frac{\partial U_P(\alpha, b, \pi_{1,p})}{\partial \pi_{1,p}} = 1 - (1 - \alpha b) \frac{\left(1 - \pi_{1,p}\right)}{(1 - \alpha b \pi_{1,p})^3} > 0.
\]
Recall that we impose $0 \leq \pi_{1p} \leq 1$, $0 \leq \alpha b \leq 1$, and $0 \leq b \leq 1$, hence it can be verified that the above derivative has a positive sign as stated in Point 2. It is also straightforward to show that $\frac{\partial U_P(\alpha, b, \pi_{1p})}{\partial \pi_{1p}}|_{\pi_{1p}=0} = \alpha b$ and $\frac{\partial U_P(\alpha, b, \pi_{1p})}{\partial \pi_{1p}}|_{\pi_{1p}=1} = 1$.

Next, taking the second derivative of $U_P(\alpha, b, \pi_{1p})$ with respect to $\pi_{1p}$ and simplifying the resulting expression we obtain,

$$\frac{\partial^2 U_P(\alpha, b, \pi_{1p})}{\partial \pi_{1p}^2} = \frac{1 - \alpha b}{(1 - \alpha b \pi_p)^3} (1 - 3\alpha b + 2\alpha b \pi_p).$$

Notice that $\text{sign} \frac{\partial^2 U_P(\alpha, b, \pi_{1p})}{\partial \pi_{1p}^2} = \text{sign} (1 - 3\alpha b + 2\alpha b \pi_p)$, implying that $\frac{\partial^2 U_P(\alpha, b, \pi_{1p})}{\partial \pi_{1p}^2} < 0$ if $\pi_{1p} < \frac{3\alpha b - 1}{2\alpha b}$, $\frac{\partial^2 U_P(\alpha, b, \pi_{1p})}{\partial \pi_{1p}^2} > 0$ if $\pi_{1p} > \frac{3\alpha b - 1}{2\alpha b}$, and $\frac{\partial^2 U_P(\alpha, b, \pi_{1p})}{\partial \pi_{1p}^2} = 0$ if $\pi_{1p} = \frac{3\alpha b - 1}{2\alpha b}$. However, since $0 \leq \pi_{1p} \leq 1$ we know that when $1/3 \leq \alpha b \leq 1$ then $\pi_{1p} > 0$, and thus we can have $\frac{\partial^2 U_P(\alpha, b, \pi_{1p})}{\partial \pi_{1p}^2} < 0$ and $\frac{\partial^2 U_P(\alpha, b, \pi_{1p})}{\partial \pi_{1p}^2} > 0$ depending on whether we have $\pi_{1p} < \pi_{1p}$ or $\pi_{1p} > \pi_{1p}$. Otherwise, when $0 \leq \alpha b < 1/3$, we only have $\frac{\partial^2 U_P(\alpha, b, \pi_{1p})}{\partial \pi_{1p}^2} > 0$.

**Proof of Lemma 6**  Point 1 can be verified easily by substituting $\pi_{1p} = 0$ and $\pi_{1p} = 1$ into $V_P(\alpha, b, \pi_{1p})$. Taking the first derivative of $V_P(\alpha, b, \pi_{1p})$ with respect to $\pi_{1p}$, yields,

$$\frac{\partial V_P(\alpha, b, \pi_{1p})}{\partial \pi_{1p}} = \alpha b + (1 - b)^2 \pi_{1p} > 0$$

Since we have $0 \leq \pi_{1p} \leq 1$, $0 \leq \alpha b \leq 1$, and $0 \leq b \leq 1$, therefore the above derivative has indeed a positive sign as stated in Point 2. It is also straightforward to show that $\frac{\partial V_P(\alpha, b, \pi_{1p})}{\partial \pi_{1p}}|_{\pi_{1p}=0} = \alpha b$ and $\frac{\partial V_P(\alpha, b, \pi_{1p})}{\partial \pi_{1p}}|_{\pi_{1p}=1} = \alpha b + (1 - b)^2$.

Finally, taking the second derivative of $V_P(\alpha, b, \pi_{1p})$ yields

$$\frac{\partial^2 V_P(\alpha, b, \pi_{1p})}{\partial \pi_{1p}^2} = (1 - b)^2 \geq 0.$$  

Given that $0 \leq b \leq 1$, the sign of the above derivative is positive.

**Proof of Proposition 2**  The proof consists of three parts.

First, we begin with the case of $0 \leq \alpha b \leq 1/3$. We know from Lemma 5 (Points 2 and 3) that $U_P(\alpha, b, \pi_{1p})$ is increasing at an increasing rate in $\pi_{1p}$. We also know from Lemma 6 (Points 2 and 3) that $V_P(\alpha, b, \pi_{1p})$ is also increasing at an increasing rate in $\pi_{1p}$. Further, from Lemma 5 (Point 1) and Lemma 6 (Point 1) we know that
Since \(\partial \pi_{1p} = 0\) and \(V_P (\alpha, b, \pi_{1p}) |_{\pi_{1p}=0} = 0\). Thus, both functions start at the same origin. Both functions will have another intersection, other than \(\pi_{1p} = 0\), at \(\hat{\pi}_{1p} = \frac{1}{ab} \left[ 1 + \frac{(1-ab)(\sqrt{a^2 + (a-b)^2})}{(1-b)^2} \right] \), and given that the permissible value of \(ab\) is \([0, 1]\) we can establish that \(\hat{\pi}_{1p} > 1\). Accordingly, for the admissible value of \(\pi_{1p} \in [0, 1]\), we know that \(U_P (\alpha, b, \pi_{1p}) \) and \(V_P (\alpha, b, \pi_{1p}) \) only intersect at \(\pi_{1p} = 0\).

Second, we now evaluate the relative magnitude of \(U_P (\alpha, b, \pi_{1p}) \) vis-à-vis \(V_P (\alpha, b, \pi_{1p}) \). From Lemma 5 (Point 1) and Lemma 6 (Point 1) we know that \(U_P (\alpha, b, \pi_{1p}) |_{\pi_{1p}=1} = 1/2\) and \(V_P (\alpha, b, \pi_{1p}) |_{\pi_{1p}=1} = ab + (1-b)^2 / 2\). Thus, we have \(ab + (1-b)^2 / 2 > 1/2\) if \(b > 2 - 2\alpha\), and \(ab + (1-b)^2 / 2 > 1/2\) if \(b < 2 - 2\alpha\). Thus, we can conclude that when \(b > 2 - 2\alpha\) prevails, then \(\alpha\)-organization dominates \(P\)-organization, i.e. \(U_P (\alpha, b, \pi_{1p}) < V_P (\alpha, b, \pi_{1p})\). \(\alpha\)-organization is dominated by \(P\)-organization, i.e. \(U_P (\alpha, b, \pi_{1p}) > V_P (\alpha, b, \pi_{1p})\) if \(b < 2 - 2\alpha\). Thus, we partly confirm Proposition 3.

Third, we now evaluate the case of \(1/3 < ab \leq 1\). From the first part of the proof, we know that for the admissible value of \(\pi_{1p} \in [0, 1]\), \(U_P (\alpha, b, \pi_{1p})\) and \(V_P (\alpha, b, \pi_{1p})\) will only have one intersection at \(\pi_{1p} = 0\). Also we know that \(\pi_{1p} = \frac{3ab-1}{2ab}\) is the inflection point of \(U_P (\alpha, b, \pi_{1p})\). Let us denote this inflection point by \(\tilde{\pi}_{1p}\). For \(\pi_{1p} \in [0, \tilde{\pi}_{1p}]\), since \(U_P (\alpha, b, \pi_{1p})\) is increasing at a decreasing rate in \(\pi_{1p}\) and \(V_P (\alpha, b, \pi_{1p})\) is increasing at an increasing rate in \(\pi_{1p}\) and there is no other intersection point between \(U_P (\alpha, b, \pi_{1p})\) and \(V_P (\alpha, b, \pi_{1p})\) except \(\pi_{1p} = 0\), then we know that \(U_P (\alpha, b, \pi_{1p}) < V_P (\alpha, b, \pi_{1p})\). Similarly, it is straightforward to see that for \(\pi_{1p} \in [\tilde{\pi}_{1p}, 1]\) we also have \(U_P (\alpha, b, \pi_{1p}) < V_P (\alpha, b, \pi_{1p})\). To conclude, whenever \(1/3 < ab \leq 1\), then \(\alpha\)-organization dominates \(P\)-organization, i.e. \(U_P (\alpha, b, \pi_{1p}) < V_P (\alpha, b, \pi_{1p})\). Thus, we complete the proof.

**Proof of Proposition 3** Let us denote the probability that production-process \(a\) is chosen in \(\alpha\)-organization by \(Pr (a | A\text{-org})\) and the probability that production process \(a\) is chosen in \(P\)-organization by \(Pr (a | P\text{-org})\). Using (17), (21), (18), and (22) we can express,

\[
Pr (a | A\text{-org}) = e^*(1 - E^*) \frac{1 - \pi_{1p}}{1 - ab\pi_{1p}} b
\]

\[
Pr (a | P\text{-org}) = e^*E^* + e^* (1 - E^*) = e^* b
\]

Since \(0 \leq \pi_{1p} \leq 1\), \(0 \leq \alpha \leq 1\) and \(0 \leq b \leq 1\), we thus have \(Pr (a | A\text{-org}) \geq Pr (a | P\text{-org})\).

Next, we denote the probability that production-process \(p\) is chosen in \(\alpha\)-organization by \(Pr (p | A\text{-org})\) and the probability that production process \(p\) is cho-
sen in $P$-organization by $Pr(p|P\text{-org})$. We can then write,

$$
Pr(p|A\text{-org}) = (1-e^*)E^* \\
= (1-b)^2 \pi_1p, \\
Pr(p|P\text{-org}) = e^*E^* + (1-e^*)E^* = E^* \\
= \frac{(1-\alpha b)}{(1-\alpha b \pi_1p)} \pi_1p.
$$

We will show that $Pr(p|A\text{-org}) \geq Pr(p|P\text{-org})$. Suppose, on the contrary, we have $Pr(p|A\text{-org}) < Pr(p|P\text{-org})$. This implies that $\frac{(1-\alpha b)}{(1-\alpha b \pi_1p)} < (1-b)^2$ or $\frac{(1-\alpha b)}{(1-b)} < (1-b)\left(1-\alpha b \pi_1p\right)$. We have $\frac{(1-\alpha b)}{(1-b)} \geq 1$ because $\alpha b \leq b$. We also know that $(1-b)\left(1-\alpha b \pi_1p\right) \leq 1$. Hence, we have a contradiction. Consequently, we have indeed $Pr(p|A\text{-org}) > Pr(p|P\text{-org})$.

**References**


