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Revisiting Agency Problems in Public Private Partnerships

by

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ABSTRACT
This paper examines conditions under which the agency problem in PPP (expropriation of the private partner by the state) could be rectified. We propose that the issue lies not in a ‘complete contract’ (a more protective ex ante contract) in mitigating the agency problem in PPP, but rather the opposite! A least protective contract (no contract at all), coupled with the dynamics of an optimal “investment destruction” retaliatory strategy profile can solve the agency problem. We employ Selten’s idea of “trembles” in decision making process, that is, in real world PPP decisions are often ‘blurred’ by bounded rationality and other inconsistencies. These results provide a credible pre-commitment on the part of the state against expropriation in the future and a strategic rationale for PPP in which tacit collusion will always be sustained.

JEL Classification: C78, H54, L14

Keywords: Public Private Partnership, Agency problem, Trembles, In-complete contracts.

1. Introduction

1.1. What is PPP?
In recent years governments worldwide face budget gaps and are increasingly adopting Public-Private Partnerships (PPPs) models as a means to provide critical infrastructure service delivery. Some of the key areas in PPP are performance monitoring, contract compliance and dispute resolution. It has been estimated that over US$ 5 trillion in infrastructure investment is required to both maintain existing assets and construct new infrastructure globally. Most of these infrastructures particularly in transportation, water, energy, telecommunications, and environmental services have traditionally been under the purview of governments as they were considered vital to national interest. However, national, state, and local governments worldwide face tremendous budget gaps magnified by the pace of competitive globalization and are increasingly adopting Public-Private Partnerships (PPPs) models as a means to provide critical infrastructure service delivery. Timely, appropriate, well managed and efficient operated infrastructures are important prerequisites to ensure maximum benefits to the social and economic development of cities and regions it was intended to impact. Studies have suggested that an increase in infrastructure capacity per capita by 1% on average lead to a similar quantum increase in GDP per capita growth. Both investment and expansion of infrastructure can have a positive effect in poverty reduction, accumulating human capital, and increasing the welfare of the country. The reverse is true when infrastructure falls behind economic growth. This leads to delays in production and consumption and aggravates the social and economic imbalances in a country. Traffic congestion delays in cities, ports and airports lead to large economic losses in time and productivity. The timing in constructing new infrastructure and replacement of existing ones is extremely important. Failure to do so results in a widening “investment gap” observed in many developing economies. Developing economies on an average spend 3-3.5% of GDP on infrastructure. The estimated required amount is about 7% of GDP1. The resulting “bottleneck” prevents the optimal realization of potential economic

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growth. This coupled with the lack of funds in cautious global economy; private sector involvement in infrastructure development is appealing. Typically, the advantage of the private sector over the state is the ability of keeping borrowing off the public books\(^2\) and transferring the construction and operation of infrastructure onto a commercial basis\(^3\). Public-Private-Partnership (PPP) is essentially a form of privatization where the public sector and the private sector have a joint interest in the project through joint investment and sharing of risk. Other advantages include the private sector’s attitude towards an appropriate rate of return; achievement of efficiency through competition, meeting deadlines, risk mitigation and etc. In the EU it has been estimates that more than 1300 PPP contracts from 1990 to 2009, with a capital value of more than EUR 250 billion\(^4\). PPP can be realized through service contracts, Built-Operate-Transfer contracts and concessions.

1.2. Background to PPP

Among the three approaches, Built-Operate-Transfer (BOT) approach has been commonly utilized (Goetz et. al. 1997; Flyvbjerg et. al. 2003). Central to PPP arrangements is the process involving agency issues (Hall 1980) in particular the role of government agencies (the regulator) in holding-up and delaying the completion of infrastructure projects by the private sector partner or consortium (the operator). According to our limited knowledge, there is little or no theoretical research on the agency problem arising in PPP projects. A recent paper by Qiu and Wang (2011) is an exception but adopts the *ex ante complete* contracting mechanism in the analysis of the government’s (regulator) implementation of price regulation and a license extension policy (which is dependent on observed quality) mitigates the agency problem in BOT projects. The models suggest that a complete *ex ante* BOT contract with a price regulation during the concession period and a license extension after the concession period is capable of achieving full efficiency.

We argue otherwise in this paper. In reality, the large body of contract theory literature concludes that the outcome suggested by Qiu and Wang is rare! It is not possible to formulate a complete contract that specifies actions to be addressed in all possible contingencies (Ellingsen and Kristiansen 2011). Most view contracts to be incomplete due to unforeseen contingencies (Williamson 1985), bounded rationality (Grossman and Hart 1986), or certain transaction costs (Hart and Moore 1988). These make it impossible to have a complete contingent contract. An incomplete contract is one where by not all contractible terms are encompassed in the contract either because some dimensions of the investment are non-contractible, or some contingencies cannot be foreseen at the point of concluding the contract.

Specifically, we argue that PPP is prone to *ex ante* contractual incompleteness due to the following observations: 1) the unique cost structure of each PPP does not facilitate the drafting of a complete contract due to divergent interests of multiple parties involved to converge due to the lack of precedents (Feldmann 1985); 2) long duration nature of PPP may experience changes in government and administration which may have a new set of objectives (Ehrlich and Lui, 1999); 3) the plethora of

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\(^2\) The need to improve infrastructure and services has had to be balanced with the need to keep government borrowing under control, particularly as a requirement of the “Maastricht criteria”, which restrict the amount of debt a country can accumulate if it wishes to be a full member of the European Economic and Monetary Union.

\(^3\) The characteristics are the clearness of tasks and rules, financial and management autonomy, rigidity of budgetary restriction and responsibility before consumers and investors.

\(^4\) Commission of the European Communities, Green Paper on PPP.
opportunities for bribery and corruption at various levels of public procurement which escalate cost and diminish the commercial viability of the operating stage of the private operator (Karpoff et. al , 1999), leading to agency problems and moral hazard issues.

1.3. Incorporating agency issue into PPP

Given that a complete ex ante contract is rarely attainable, the central questions are therefore: How does one design a mechanism which mitigates agency problems? If such a mechanism exists, is this solution robust to account for “trembles” or “mistakes” made by the regulator and operator? Here we allow for some realism in modeling the strategic behaviors of the two main players in PPP arrangement by building a framework that accommodates the analysis of individuals with divergent beliefs with incomplete contracting so as to examine the conditions under which agency problems could be rectified. In order to model the bargaining mechanism and the decision making process between the regulator and operator, a game theoretic framework is developed with finite horizon to model a dynamic game. After initial investment into the project, a stream of payoffs will be generated, with the regulator being the proposer on how these payoffs are to be split in each bargaining period\(^5\) while the operator have to decide whether to ‘Accept’ or ‘Reject’. In this game the operator’s decision to invest or not is binary, and hence the game can be interpreted as purely cooperative in nature. Therefore we can deduce that the First Best solution is to invest, as it enhances the regulator’s valuation but does not reduce the operator’s cost of production. The game is thus assumed to have no under investment. When the regulator proposes an unacceptable “low” price to the operator, it means that he is playing a “selfish strategy\(^6\)” which expropriates the commercial interest of the operator, who then decides whether it is optimal for him to Accept or Reject the terms of proposal under the “selfish strategy”. This decision entails whether to Accept a favorable set of operating conditions, proxy by an “acceptable” price proposed by the regulator; or to Reject a set of unfavorable conditions, proxy by an “unacceptable” price proposed by the regulator. If the decision of Accept is chosen, the operator decides next whether to Perform or Underperform in the operating stages, proxy by PF or PF’ respectively. As a rational investor, the operator has to calculate the risk under “selfish strategy”. What he has to forecast is whether the regulator has sufficient incentive to expropriate. We inquire under what kind of “agreement” the regulator will not want to expropriate.

In reality the act of regulators proposing an unacceptable “low” price to operators in PPP is widely observed and well documented. It was noted that in the case of the German MAGLEV PPP (Feldmann 1995): “…key actors in the German federal government (regulator) are focused on increasing performance requirements regardless of operation costs, as well as in keeping the transparency of this cumulative process as low as possible”. Whereas for the private firm, Deutsche Bahn AG (operator),

\(^5\) Governments imposing price controls are well-known mechanisms that are often used to improve efficiency under monopoly condition. (See Laffont and Tirole 1993)

\(^6\) Not all of the regulator’s strategies are necessarily selfish in its original intention. The following list some circumstances in which convergence of interest might not materialize. 1) The unique cost structure of each project does not facilitate the drafting of a complete contract for interests to converge due to the lack of precedents. 2) Long duration nature of projects which leads to i) changes in administration in the public sector level which has its own new set of objectives ii) plethora of opportunities for bribery and corruption at various levels of procurement which escalate cost and makes the commercial viability of operating stage of the operator dire
whose commercial objective is that of “running the BOT scheme at the highest operating profits attainable”. Such was also the case of “deep creep” bringing the costs up during the “preferred Bidder” stage, in the Western High Speed Diameter, St Petersburg, Russia and Electronic road toll system, Czech Republic. Given these divergent objectives in the absence of complete contracting in PPP, many mega infrastructure PPP projects resulting in twice as costly as originally planned and result in lower income streams. Flyvbjerg et al (2003) and Skamris (1997) concluded cost overruns appear to be a global phenomenon this is more pronounced in developing nations than in developed ones. From the view of political lobbyist and promoters of PPP, cost overrun pays off. Cost escalation is a simple consequence of cost underestimation and underestimation is used tactically to get projects approved and built. Manifold political interests are nested and hidden in the process after a national infrastructure investment is approved by the government. Upon completion and operation it is not uncommon to encounter escalating operating and maintenance costs and unrealized ridership falling short of the forecast. The financial risks are already transferred to the operator, thus posing acute agency problems. Consequently the PPP’s commercial viability is diminished while the operational risk of the operator is escalated. These empirical observations highlighted examples in which regulators proposing an unacceptable “low” price to expropriate the operators for various self-seeking and politically objectives.

1.4. “Expropriation” in the PPP

In general such acts of expropriations include cost overruns; the excessive transfer of operational risk from the state to the private sector, delays in payments and the spurious nature of the public sector comparator (PSC)

The rest of the paper is organized as follows: Section 2 lays out the preliminaries before the introduction of a stylized N period PPP model to derive an optimal strategy profile which is a Strongly Sequentially Stable Equilibrium (SSSE), Sub-game Perfect Equilibrium (SPE) and Subgame Consistent. Section 3 introduces another set of preliminaries before we extend the base model by discussing weak dominated strategies. We conclude that any SPE in which the operator invests is ‘Trembling Hand Perfect’ in the game’s agent strategic form. This ‘Trembling Hand Equilibrium’ is a Perfect Bayesian Equilibrium (PBE) and a Perfect Equilibrium (PE). Section 4 explores other extensions. Section 5 discusses the relevance of the results to real world PPP. Section 6 concludes. All

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7 This is an estimate of what it would have cost to undertake the project using traditional procurement methods. Public entities use the comparator as a benchmark to help decide whether an alternative procurement method using private finance in some form or another would offer better value for money.
2. The Basic Model

2.1. Preliminaries

The following concepts and Definitions in this sub-section apply to the rest of the paper. We begin
with the classical case where all players face a same game which is assumed to be with perfect recall
and common knowledge. We denote the game by \( G = (N, V, H, \{A(h)\}_{h \in H}, \{(\pi_i)\}_{i \in N} \) where \( N = \{1, 2, \ldots, n\} \) is the set of players, \( V \) is the set of nodes, \( H \) is the set of information sets, \( A(h) \) is the set
of pure actions available at information set \( h \), and \( \pi_i \) is player i’s payoff function. For simplicity, we
shall assume that the set of vertices is finite. A mixed action at information set \( h \) is a probability
distribution over the pure actions in \( A(h) \). We denote the set of mixed actions at \( h \) by \( \Delta(h) \), and the set
of information sets belonging to player \( i \) by \( H^i \). A behavioral strategy for player \( i \) is a function that
assigns to every \( h \in H^i \) a mixed action from \( \Delta(h) \). The set of strategies of player \( i \) is denoted by \( Y^i \),
and the set of strategy profiles is denoted by \( Y = \times_{i \in N} Y^i \). For \( y \in Y \) we denote by \( y(h) \) the mixed
action of \( y \) at \( h \) and by \( y(-h) \) the profile of mixed actions of \( y \) at all information sets other than \( h \);
\( \pi_i(y) \) denotes i’s expected payoff if \( y \) is followed from the root of the game. We can now formalize
the basic building blocks in our analysis: the notion of a course of action (CA) and the conditions under
which it will be mutually acceptable to all players.

DEFINITION 2. A CA is a mapping \( x : H \rightarrow \bigcup_{h \in H} \Delta(h) \cup \{0\}, \) with \( x(h) \in \Delta(h) \cup \{0\} \) for all \( h \in H. \)

Formally, the interpretation of \( x(h) = \{0\} \) is that the CA \( x \) does not specify which action from
\( \Delta(h) \) player \( i \) would take at \( h \), where \( h \in H^i \); otherwise \( x(h) \) specifies player \( i \)’s mixed action at \( h \).
In particular in this simple game the First Best solution is to invest, as it enhances the regulator’s
valuation but does not reduce the operator’s cost of production. A CA \( x \) is said to be complete if \( x(h) \neq \{0\} \) for all \( h \in H^i \). A complete CA in this case is therefore a strategy profile and the game is
thus assumed to have no under investment. Our solution concept in the PPP model, given the limitation
and constraints of incomplete contracting with divergent beliefs, is a CA that “makes sense” in every
player’s interpretation of the game and as such, it is a situation whereby no rational players would have
incentive to deviate from it given his belief. Observe that each player may rationalize his expectations in
a different way, as long as this does not violate the common knowledge of rationality as perceived by
each player.

Let \( (T | w) \) denote a subtree derived from \( T \) with its root as node \( w \), and let \( V_i (T | w) \) denote
the set of player \( i \)’s decision nodes in the subtree \( (T | w) \). Write the subgame with the subtree \( (T | w) \) as
\( G(T | w) \) and let \( x(T | w) \) represent the restriction of a strategy profile \( x \in X \) in subgame \( G(T | w). \)
For any \( Y \subseteq X \), let \( Y (T | w) \equiv \{x(T | w)| x \in Y \} \). Let \( \pi_i (x(T | w)) \) be player \( i \)’s payoff of
\( x(T | w) \) in subgame \( G(T | w). \) Define the conditional payoff \( \pi_i (x|w) \) as \( \pi_i (x(T | w)) \). Next let us
say a pair \((x, Y)\) is a “theory” endowed, where \( x \) is a strategy profile and \( Y \) is a subset of strategy
profiles. The first component of the theory is interpreted as a “proposal” or “recommendation” made
by the GA, while the second component is interpreted as a theory of “beliefs”—i.e., according to this
theory, a player believes those strategy profiles in \( Y \) to be plausible choices. Let \( Y_{-i} \equiv \{y_{-i} | (y_i, y_{-i}) \in Y \}. \) Formally, we define the following principle of sequential rationality.
DEFINITION 2.1. Player $i$ is sequentially rational given a theory $(x, Y)$ if for every $v \in V_i$ there exists $y_{-i} \in Y_{-i}$ such that $\pi_i(x|v) \geq \pi_i(y_i, y_{-i}|v)$ for all $y_i \in X_i$.

DEFINITION 2.2. A general system is defined as a pair $(X, \{\succ_Y\} _{Y \subseteq X})$, where $X$ is a nonempty set and $Y$ is an arbitrary binary relation on $X$. For any $Y \subseteq X$ and $x, y \in X$, $y \succ_Y x$ is interpreted as “$y$ dominates $x$ conditionally on $Y$”.

DEFINITION 2.3. A subset $K$ of $X$ is a (general) stable set in the general system $(X, \{\succ_Y\} _{Y \subseteq X})$ if the set $K$ is:

i. [internally stable] $\forall x \in K, \ y \succ^K x$ for all $y \in K$

ii. [externally stable] $\forall x \in X \setminus K, \ y \succ^K x$ for some $y \in K$.

Consider a Perfect Information game $G$. An alternative general system associated with $G$ is defined as $GS^* \equiv (X, \{\succ_Y\} _{Y \subseteq X})$ such that:

i. $X$ is the set of strategy profiles

ii. $\forall x, y \in X$, $\forall Y \subseteq X$, $y \succ_Y x$ if and only if $\exists i \in N, \exists v \in V_i, \pi_i(y'_i, y'_{-i}|v) > \pi_i(x|v)$ for some $y'_i \in X_i$ and for some $y'_{-i} \in Y_{-i}$.

DEFINITION 2.4. A strategy profile $x \in X$ is a Strongly Sequentially Stable Equilibrium (SSSE) if there exists a stable set $K$ in $GS^*$ such that $x \in K$.

That is, Player $i$ is strongly sequentially rational given a theory $(x, Y)$ if for every $v \in V_i$,

$$\pi_i(x|v) \geq \pi_i(y_i, y_{-i}|v)$$

for all $y_i \in X_i$ and all $y_{-i} \in Y_{-i}$.

2.1. The multi-period PPP model

The following figure shows the game tree of our theoretical framework. [Insert Figure 1]

In PPP, a regulator (RT) and an operator (OT), who are both assumed to be risk neutral and have zero rate of interest, play the following game in the simple two period model.\(^8\) When OT does not invest, denoted by $\{0\}$, they both get zero payoffs. When OT makes an irreversible investment denoted by $I$, the sunk investment will generate a total potential payoff denoted by $g$, where $g > c > 0$. Let $\alpha_1 \in [0, 1]$ denote the proportion of payoff generated in $t = 1$. Denote payoff dynamics by the vector $\alpha = (\alpha_1, 1 - \alpha_1)$. At the beginning of each $t = 1, 2$ RT proposes a price $p_t$. Let $P \in R$ has a minimum where $m(P) = \min\{p: p \in P\}$. OT then decides whether to accept ($Y_t$) or reject ($N_t$). Acceptance of $p_t$ will enable the game to continue. OT will operate as he should. But if OT decides to reject $p_t$ their relationship break down and the game ends. The net payoff in the game equals to the price $p_t$ proposed by RT minus OT’s cost of operation, which is assumed to be zero for now. After OT has accepted $p_t$ proposed by RT, he then decides whether to perform (PF) or not (PF').

\(^8\) This paper use an additive approach \(FW = \sum_{t=1}^{N} X([\rho_t \ast (1 + r(t))] + [1 - \rho_t])\) where \(FW\) is the final wealth, $X$ is the endowment in each period, $\rho_t$ is the proportion invested in period $t$ with $\rho_t \in [0, 1]$, $r(t)$ is the return of the game in period $t$, and $N$ the planning horizon. One could however use a more realistic multiplicative approach, in which the returns of the periods are compounded \(FW = Y \prod_{t=1}^{N} ([\rho_t \ast (1 + r(t))] + [1 - \rho_t])\).
Suppose OT were to perform as he should, the overall payoff is still \((1 - \alpha_1)g\) for \(t = 2\) but if OT does not perform net payoff falls back to zero.

Typical for PPPs highlighted in the Introduction are their very long project duration, e.g. it can take up to several decades for the final product to be delivered (Capka, 2004; Haynes, 2002; Stough & Haynes, 1997; Merrow, 1988). We model this long project duration by \(L = \{(\alpha_1, \alpha_2, \ldots, \alpha_N) \in \mathbb{R}^N : \alpha_t > 0 \ \forall t \text{ and } \sum_{t=1}^{N} \alpha_t\}\) where \(\alpha\) refers to the share of total potential surplus in period \(t\). The original game corresponds to the case with \(N = 2\). The game tree for \(N > 2\) is a straightforward extension of Figure 1 to \(N\) periods. Intuitively, the strongest threat that OT can make is to destroy the investment immediately after the first period. Dividing the remaining into more periods simply implies OT may also threaten to destroy the investment in other periods as well. Hence the set of equilibrium payoff pairs is the same whether the game involves two period surplus dynamics or \(N\) period surplus dynamics. In order for the threat of retaliation to be most effective, the threat is stronger the larger the remaining potential surplus from the investment. Therefore we argue that the equilibrium outcomes of the game for \(N = 2\) holds true for \(N > 2\) as well.

**DEFINITION 2.4.** The following strategy profile in \(\mathcal{G}S^*\) is a retaliatory strategy profile \(r^*_p = r^*_pOT \times r^*_pRT\) where \(r^*_pOT\) and \(r^*_pRT\) is the mixed strategy profile for OT and RT respectively, which are defined as follows:

- \(r^*_pOT\):
  - For \(H = (I, p_t, Y_t)\), \(X^{OT}(h) = PF\) if \(p_t \in P\) \(\cup X^{OT}(h) = PF'\) if \(p_t \notin P\); For \(H = (I, p_1)\), \(X^{OT}(h) = Y_t\) if \(p_t \geq 0\) \(\cup X^{OT}(h) = N_t\) if \(p_t < 0\), where \(t = 1, 2\)
- \(r^*_pRT\):
  - For \(H = (I)\), \(X^{RT}(h) = p_t = m(P)\); For \(H = \{0\}\), \(X^{RT}(h) = p_1 = 0\)

Hence for \(r^*_p\), payoff for the game is \(\pi^T = \{m(P) - c, g - m(P)\}\).

**REMARKS.** OT will invest as this is the First Best solution to the game. In each period he accepts any price offer larger than or equal to his production cost. He does not destroy the investment as long as he has been offered an “acceptable price”. In the first period, RT offers the minimum “acceptable price”; in the second period, he offers a price equal to the OT’s production cost. The notation \(P\) denotes the set of “acceptable price”.

2.3. Notions of Equilibria in \(\mathcal{G}S^*\) - Strongly Sequentially Stable Equilibrium (SSSE), Subgame Perfect Equilibrium (SPE) and Subgame Consistent

**PROPOSITION 2.** The retaliatory strategy profile \(r^*_p\) in \(\mathcal{G}S^*\) is a Strongly Sequentially Stable Equilibrium (SSSE).

**Proof**

To show \(r^*_p\) is a SSSE we need to prove the following two claims first:

**Sufficiency**

**Claim 2.** \(r^*_p\) in \(\mathcal{G}S^*\) satisfies the “one deviation property”.

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9 One deviation property is satisfied if no player can increase his payoff by changing her action at the start of any subgame in which he is the first mover, given the other player’s strategies and the rest of her own strategy. \(r^*_p\) is SPE if and only if it
Proof of Claim 2.

By backward induction, after \( H = (I, p_1, Y_1, PF) \) and \((I, p_1, Y_1, PF')\), the equilibrium strategies are those given in \( r^*_p \). At the start of game, if OT switches from I to 0, his payoff will not increase since \( c \leq m(P) \). After \( H = (I) \), RT cannot benefit by offering a price \( p_1 \neq m(P) \) for the following reasons:

1) If RT increases \( p_1 \) to any \( p_1 \in P \), OT will accept his offer but his payoff decreases as he pays more now.

2) If RT increases \( p_1 \) to any \( p_1 \in P \), OT will accept his offer but destroy the investment after trade, resulting in smaller surplus in the subsequent period. Paying more in t=1 and getting less in the subsequent periods unambiguously decrease RT’s payoff.

3) If RT decrease \( p_1 \) to any \( p_1 \in [0, m(P)] \), OT will accept his offer. His payoff is at most \( \alpha g \) if he offers \( p_1 = 0 \). \( c \leq m(P) \leq (1 - \alpha_1)g \) ensures that such deviation does not benefit him as \( m(P) \leq (1 - \alpha_1)g \).

4) If RT decreases \( p_1 \) to any \( p_1 < 0 \) OT will reject his offer. The breakdown in relationship decreases his payoff. After \( H = (I, m(P), Y_1) \) OT is indifferent between choosing \( PF \) and \( PF' \) as both give him a payoff of \( m(P) - c \). For \( H = (I, p_1, Y_1, PF \) and \((I, p_1, Y_1, PF')\), RT’s payoff from the subsequent period is \( (1 - \alpha_1)g \) and 0 respectively. Deviating from \( r^*_p \) by offering \( p_1 < 0 \) results in rejection and a relationship breakdown which unambiguously lower RT’s payoff in either type of \( H \). Deviating by offering \( p_1 > 0 \) does not result in rejection but RT’s payoff is lower because he pays more.

Hence \( r^*_p \) in \( GS^* \) satisfies the “one deviation property” and is SSSE when \( c \leq m(P) \leq (1 - \alpha_1)g \) holds.

Necessity

Claim 2.1. A pure strategy SSSE with a strategy profile that is not a retaliatory strategy profile does not exist.

Proof of Claim 2.1.

Here we prove by contradiction. Assume in negation that \( S \) is such a strategy profile. We argue as follows that this is not possible. By backward induction, for \( H = (I, p_1, Y_1, PF) \) and \((I, p_1, Y_1, PF')\), the strategy in \( S \) cannot differ from those corresponding in \( r^*_p \) if \( S \) is subgame perfect. For \( H = (I, p_1) \), \( X^{OT}(h) = Y_1 \) if \( p_1 \in P' \) is subgame perfect as argued above. We now argue that \( r^*_p \) is not subgame perfect if \( P' \notin [0, \infty} \).

1) First it is not subgame perfect to accept any finite price offer \( p_1 \). Therefore the upper bound can only be infinity.
2) Second, setting a higher lower bound, say $\varepsilon$ for any $\varepsilon > 0$ is not subgame perfect. Since if OT is offered $p_1 = \omega$ for any $\omega \in (0, \varepsilon)$ his payoff from rejecting is $-c$, while accepting leaves him a payoff of at least $\omega - c$.

3) Third, setting a lower bound, say $-\varepsilon$ for any $\varepsilon > 0$ is not subgame perfect. Since if OT is offered $p_1 = -\omega$ for any $\omega \in (0, \varepsilon)$, his payoff from rejecting is $-c$ while accepting leaves him a payoff of $- (\omega + c)$.

4) At the start of game, OT chooses I both in $s^\ast$ and $r_p^\ast$ by the requirement that OT invests. Therefore, $s^\ast$ can only differ from $r_p^\ast$ for $H = (1, p_1, Y_1)$ for OT and for $H = (1)$ for RT.

Define a set $\bar{P} \in R$ such that if the strategy profile is $s$, $X^{OT}(h) = Y_1$ if $p_1 \in \bar{P}$. Suppose $\bar{P}$ has no minimum, there cannot be any SPE since it implies that RT can always offer lower price to increase his own payoff while such a deviation does not induce OT to choose PF' . Suppose $\bar{P}$ is an empty set, then by definition OT never chooses PF. Thus it is only subgame perfect if RT offers $p_1 = 0$. In turn this implies OT should not invest, a contradiction of the requirement that OT does invest. The set $\bar{P}$, therefore should be non-empty and has a minimum.

OT chooses I if and only if it is individual rational to invest. Thus for equilibrium there must exist a $\bar{P}_1 \subseteq \bar{P}$ such that $\bar{P}_1 \geq c$ : RT optimally offers and OT optimally accepts. As stated just now $m(P) = \min\{p: p \in P\}$, $s$ differs from a retaliatory strategy profile if and only if $\bar{P}_1 \neq m(\bar{P})$ . However this contradicts with the “one-deviation property”. This is because if $\bar{P}_1 \neq m(\bar{P})$, deviating by offering $m(\bar{P})$ gives RT a payoff of $y - m(\bar{P})$, which is larger than the payoff of not deviation, i.e. $y - \bar{P}_1$ by definition. Therefore $s$ cannot be subgame perfect if it differs from $r_p^\ast$ . There is no other SSSE other than a retaliatory strategy profile that can be subgame perfect with OT choosing I. Therefore $r_p^\ast$ in $G^\ast$ is a SSSE.

These two claims proved that $r_p^\ast$ must be a SSSE in $G^\ast$ and hence complete the proof of Proposition 2.

**Lemma 2.1.** The retaliatory strategy profile $r_p^\ast$ in $G^\ast$ is a Subgame Perfect Equilibrium (SPE).

**Proof.** See Appendix

**Lemma 2.2.** The retaliatory strategy profile $r_p^\ast$ in $G^\ast$ is Subgame Consistent.

**Proof.** See Appendix

The optimality of the retaliatory strategy profile in Proposition 2 solves the agency problem of RT expropriating OT by imposing a price lower than OT’s minimum acceptable price $m(P)$. This “low price” could capture the element of expropriation in the form of cost overruns or other forms of unfavorable operating conditions. $r_p^\ast$ in $G^\ast$ induces the correct incentive for OT to invest into PPP without fear of being expropriated. Since OT cannot really punish RT in the absence of a complete contract, the trick for such efficient mechanism is to give OT an ability to “destroy” the investment by underperforming if RT fails to offer the minimum acceptable price (i.e. $c \leq m(P) \leq (1 - \alpha_1)g$), but otherwise to give it a reward in the form of positive surplus sharing. However the most crucial rewarding element for RT under the optimal retaliatory strategy profile $r_p^\ast$ is not just about surplus sharing, but rather that “destruction” would mean the deepening of the “investment gap” and “bottleneck” or worse still, the lose of faith in PPP by the people as a mean to alleviate shortage of state financial resources and ameliorate insufficient efficiency of the state in many developing countries. Given that the state cannot cope this deficiency with its own capacity since it has little financial
resources and it is at an objective dilemma in increasing production efficiency in the public sector, intuitively it is not incentive compatible for him to deviate from the optimal retaliatory strategy profile $r^*_p$ in $GS^*$ as this is a credible threat by the OT to retaliate since $r^*_p$ is a SSSE, SPE and it is Subgame Consistent. We sum up this Section by noting that the mere possibility of $r^*_p$ sustains PPP, resulting in no destruction of the investment while minimizing the agency problem of expropriation.

3. Extension of the Model

3.1. Preliminaries

The following concepts and Definitions in this sub-section apply to the rest of the paper. In addition to the base model, in this section we reexamine the optimality of the retaliatory strategy profile $r^*_p$ in $GS^*$ of the base model when the unique cost structure, long duration and the possibility of bribery and corruption blur the decision making process of RT and OT. This is certain, that there cannot be any mistakes if the players are completely rational, signing a complete contract that specifies actions in all possible contingencies. Nevertheless, a satisfactory interpretation of equilibrium points in N period PPP seems to require that the possibility of mistakes is not completely excluded due to the occurrence of bounded rationality and certain transaction costs among others along the way. Worse still usually in PPP decisions do not occur in isolation, but repeatedly as part of a continuous broader strategy as the game progresses. Myopic loss aversion plaguing the human decision making process when more than two periods are involved is well documented (Samuelson 1963 and Kahneman and Lovallo 1993), hence the reliance of the potential use of weakly dominated strategies becomes spurious and Selten’s idea of “trembles” in modeling PPP is not only warranted but essential. In order to ensure that players’ actions are optimal in every contingency, including contingencies that might not actually arise, we follow Selten’s (1975) idea of “trembles” or “mistakes” by performing some modifications to the base model to better analyze the agency problem in real world PPPs.

In line with Selten (1975) we impose the rationality criterion through the requirement that actions chosen by a player be best responses to his beliefs about opponents’ actions. Specifically each player i associates with strategy $y^j \in Y^j$ of every player $j \neq i$ a “trembling sequence”$\{y^j_k\}_{k=1}^{\infty}$ of totally mixed strategies that converges to $y^j$. Let $y^j_k \Rightarrow y^j$ denote such a sequence. A strategy $y^i \in Y^i$ is player i’s perfect best response to $y^{-i} \in Y^{-i}$ if actions specified by $y^i$ remain optimal for i along the trembling sequences. However in most cases, player i does not know the precise strategy $y^j$ that player j might adopt. Rather, i is likely to have a set of strategies, $y^j \subseteq Y^j$ that i believes j might adopt. In that case, i assigns some probability distribution over $Y^j$. For any non-empty $Y^j \subseteq Y^j, \Delta(Y^j) = \Delta^0(Y^j)$ where $\Delta(Y^j)[\Delta^0(Y^j)]$ denotes the set of all strategies $y^j \in Y^j$ such that $y^j$ is outcome equivalent to a distribution over $Y^j$. Player i believes that player j will choose $y^j_t \in Y^j$ with probability $\lambda_t, t = 1, 2, \ldots, m$. Since perfection requires that player i associates $y^j_t$ with some sequence $y^j_{tk} \Rightarrow y^j_t$. Thus, $\lambda$ is uniquely associated with a sequence $\{y^j_k\}_{k=1}^{\infty}$ of totally mixed strategies of player j such that the kth
element $y^i_k$ along this sequence is outcome-equivalent to choosing $y^i_{t,k}$ with probability $\lambda_t$, $t = 1, 2, \ldots, m$. We denote by $y^i_{t,k} \Rightarrow^y y^i_t$ such a sequence with a limit of $y^i$. Our CA rationality criterion requires that every action chosen by a player be optimal along these “trembling” sequences. Specifically, we require that for a strategy $y^i \in Y^i$ of player $i$, there exist $y^i_k \Rightarrow y^i$ and $y^i_k \Rightarrow^y y^j$ for all $j \neq i$ such that, for all $h \in H^i$, and for all $k = 1, 2, \ldots$

$$\pi^i(y(h), y_k(h)) \geq \pi^i(a, y_k(h))$$

for all $a \in \Delta(h)$.

**Definition 3.** The CA defined in our extended model is that for every player $i$ and every $y^i_k \Rightarrow y^i$ and $y^i_k \Rightarrow^y y^j$ for all $j \neq i$ such that:

i. for all $h \in H$, $y(h) = x(h)$ whenever $x(h) \neq \{0\}$

ii. for all $h \in H^i$ and for all $k = 1, 2, \ldots$, $\pi^i(y(h), y_k(h)) \geq \pi^i(a, y_k(h))$ for all $a \in \Delta(h)$

3.2. The extended multi-period PPP model

The following figure shows the game tree for this section. [Insert Figure 2]

The fact that the two types of sub-game in the second period resemble an ultimatum game makes OT indifferent between choosing PF or PF’. It seems natural from a behavior point of view to think of OT as to break the tie by choosing PF’ if he has been offered an unacceptable price. Choosing PF’ gives OT a sure payoff of $m(P) - c$, while choosing PF gives him a payoff of at least $m(P) - c$. If for some reasons, RT actually offers him a second period price higher than zero, OT would have been better off performing. OT’s strategy of playing PF thus weakly dominates PF’ for $H = (I, p_1, Y_1)$. A retaliatory strategy profile therefore entails weakly dominated strategies. Nonetheless, if OT believes RT is sequentially rational, OT should never expect RT to offer any price higher than his production cost in the second period.

**Claim 3.** If there is a small possibility that either of the players make a mistake, the reliance of the potential use of weakly dominated strategies becomes spurious.

**Proof.** See Appendix

This concern leads us to ask the central question of the Section: Are the equilibria with a retaliatory strategy profile $r^*_p$ in $G^* S^*$ Trembling Hand Perfect or not? We carry out this checking through the notion of a Perturbed Games $G^* S^*$.

**Definition 3.1.** A Perturbed Game $G^* S^*$ is a pair $(G^* S^*, \eta)$ where $G^* S^*$ is an extensive game with perfect recall and $\eta$ is a function which assigns a positive probability $\eta_c$ to every personal choice $c$ in $G^* S^*$ such that:

i. $\sum_{c \text{ at } u} \eta_c < 1$

ii. $\tilde{P}_c = (1 - \epsilon_u)P_c + \epsilon_u q_c$ where $P_c$ is the rational choice at $u$ that selects $c$ in a local strategy and $\epsilon_u q_c = \eta_c$

iii. $\tilde{P}_c \geq \eta_c$ for every personal choice $c$
The probabilities $\eta_c$ are called minimum probabilities. For every choice $c$ at a personal information set $u$ define $u_c = 1 + \eta_c - \sum_{c' \in u} \eta_c$ where $u_c$ is called the maximum probability at $c$.

That is, “noise” is introduced to both players’ actions, by restricting their probability of choosing each of their optimal actions to strictly less than 1. It is possible that OT accidentally accepts an undesirable price offer, accidentally plays PF or PF’ where he should not. For RT, with probability greater than 0, he offers a price slightly “off-equilibrium” by $\pm \epsilon$, where $\epsilon > 0$. In any strategic form game, no weakly dominated strategy can be trembling hand perfect. Therefore in the first game any subgame perfect equilibrium in which OT invests is not trembling hand perfect in the game’s strategic form. However as suggested in Osborne and Rubinstein (1994), a trembling hand perfect equilibrium of a finite extensive game should be a strategy profile that corresponds to a trembling hand perfect equilibrium of the agent strategic form of the game. Since the strategic form restricts any player to consider only other players’ possibilities of making mistakes, while ignoring the possibility of he himself may also make mistakes at some points of the game. The agent strategic form captures the idea that players also take into account their own mistakes at other information sets before making a move.

3.3. Notions of Equilibria in $G^S$ - Trembling Hand Perfect Equilibrium, Perfect Bayesian Equilibrium (PBE) and Perfect Equilibrium (PE)

In the agent strategic form of the game, there is one player for each information set in the extensive game: each player in the extensive game is split into a number of agents, one for each of his information sets, all agents of a given player having the same payoffs. Denote the following strategy profile as a retaliatory strategy profile $\sigma^n$ that consists of mixed strategies:

$$r_p^{OT} = r_p^{OT} \times r_p^{RT}$$

where $r_p^{OT}$ and $r_p^{RT}$ denote the mixed strategy profile for OT and RT respectively, where

$r_p^{OT}$ is defined as:

- Assign probability $\left(1 - \frac{1}{n}\right)$ to I and $\frac{1}{n}$ to 0 at start of game
- For $H = (I, p_1,.)$:
  - Assign probability $\left(1 - \frac{1}{n} - \frac{1}{n^2}\right)$ to $Y_1PF$, $\frac{1}{n}$ to $Y_1PF'$ and $\frac{1}{n^2}$ to $N_1$ if $p_1 \in P$ and $p_1 \geq 0$
  - Assign probability $\frac{1}{n}$ to $Y_1PF$, $\left(1 - \frac{1}{n} - \frac{1}{n^2}\right)$ to $Y_1PF'$, and $\frac{1}{n^2}$ to $N_1$ if $p_1 \notin P$ and $p_1 \geq 0$
  - Assign probability $\frac{1}{n}$ to $Y_1PF$, $\frac{1}{n}$ to $Y_1PF'$, and $\left(1 - \frac{2}{n}\right)$ to $N_1$ if otherwise
- For $H = (I, p_1,Y_1PF, p_2)$:
  - Assign probability $\left(1 - \frac{1}{n}\right)$ to $Y_2$ and $\frac{1}{n}$ to $N_2$ if $p_2 \in P$ and $p_2 \geq 0$
  - Assign probability $\frac{1}{n}$ to $Y_2$ and $\left(1 - \frac{1}{n}\right)$ to $N_2$ if $p_2 \in P$ and $p_2 < 0$
\( r_p^{RT} \) is defined as:

- For \( H = I \)
  - Assign probability \((1 - \frac{2}{n})\) to \( p_1 = m(P), \frac{1}{n} \) to \( p_1 = m(P) + \varepsilon, \frac{1}{n} \) to \( p_1 = m(P) - \varepsilon \), and 0 to all other possible \( p_1 \).
- \( H = (I, m(P), Y_1 PF) \):
  - Assign probability \((1 - \frac{2}{n})\) to \( p_2 = 0, \frac{1}{n} \) to \( p_2 = \varepsilon, \frac{1}{n} \) to \( p_2 = -\varepsilon \), and 0 to all other possible \( p_2 \).
- \( H = (I, m(P) + \varepsilon, Y_1 PF) \):
  - Assign probability \(1 - \frac{1}{n} - \frac{1}{n^2}\) to \( p_2 = 0, \frac{1}{n} \) to \( p_2 = \varepsilon, \frac{1}{n} \) to \( p_2 = -\varepsilon \), and 0 to all other possible \( p_2 \).
- \( H = (I, m(P) - \varepsilon, Y_1 PF) \):
  - Assign probability \(1 - \frac{1}{n} - \frac{1}{n^2}\) to \( p_2 = 0, \frac{1}{n} \) to \( p_2 = \varepsilon, \frac{1}{n} \) to \( p_2 = -\varepsilon \), and 0 to all other possible \( p_2 \).

**DEFINITION 3.2.** Denote the mixed strategy profile \( \sigma^* \) as the limit of the sequences of mixed strategy profile \( \sigma^n \) when \( n \to \infty \).

Note that \( \sigma^* \) is also a pure strategy profile and is equivalent to \( x(h) \).

The following proves that there exists a sequence of mixed strategy profiles \( \sigma^n \to \sigma^* \) such that \( \pi_i(\sigma^*_h, \sigma'^{n}_{-h}) \geq \pi_i(\sigma_h, \sigma'^{n}_{-h}) \) for \( i = OT, RT \)

**PROPOSITION 3.** The retaliatory strategy profile \( r_p^* \) in \( G^* \) is Trembling Hand Perfect.

**Proof.**

1) \( H = (I, p_1, Y_1, PF) \):

The mixed strategy profile \( \sigma^n \) suggests that OT’s own trembles restricts him from correctly accepting or rejecting RT’s price offer only with probability \(1 - \frac{1}{n}\). RT’s strategy in \( \sigma^* \) for \( H = (I, p_1, Y_1, PF) \) is to offer \( p_2 = 0 \). The following show that \( p_2 = 0 \) satisfies \( \pi_i(\sigma^*_h, \sigma'^{n}_{-h}) \geq \pi_i(\sigma_h, \sigma'^{n}_{-h}) \) for all \( n \).

   a) For \( H = (I, m(P), Y_1, PF) \), RT’s best response is \( p_2 = 0 \) as proven by the following expected payoffs:

\[
\begin{align*}
\pi_{RT}(p_2 = 0 | \sigma^n, H) &= \left(1 - \frac{1 - \alpha_1}{n}\right)g - m(P) \\
\pi_{RT}(p_2 = \varepsilon | \sigma^n, H) &= \left(1 - \frac{1 - \alpha_1}{n}\right)g - m(P) - (1 - \frac{1}{n})\varepsilon \\
\pi_{RT}(p_2 = -\varepsilon | \sigma^n, H) &= \left(\alpha_1 + \frac{1 - \alpha_1}{n}\right)g - m(P) + \frac{\varepsilon}{n}
\end{align*}
\]
b) For $H = (I, m(p) + \varepsilon, Y_1PF)$, RT’s best response is $p_2 = 0$ as proven by the following expected payoffs:

\[
\pi_{RT}(p_2 = 0|\sigma^n, H) = \left(1 - \frac{1 - \alpha_1}{n}\right)g - m(p) - \varepsilon
\]
\[
\pi_{RT}(p_2 = \varepsilon|\sigma^n, H) = \left(1 - \frac{1 - \alpha_1}{n}\right)g - m(p) - (2 - \frac{1}{n})\varepsilon
\]
\[
\pi_{RT}(p_2 = -\varepsilon|\sigma^n, H) = (\alpha_1 + \frac{1 - \alpha_1}{n})g - m(P) - (1 - \frac{1}{n})\varepsilon
\]

c) $H = (I, m(p) - \varepsilon, Y_1, PF)$, RT’s best response is $p_2 = 0$ as proven by the following expected payoffs:

\[
\pi_{RT}(p_2 = 0|\sigma^n, H) = \left(1 - \frac{1 - \alpha_1}{n}\right)g - m(p) - \varepsilon
\]
\[
\pi_{RT}(p_2 = \varepsilon|\sigma^n, H) = (\alpha_1 + \frac{1 - \alpha_1}{n})g - m(P) + (1 + \frac{1}{n})\varepsilon
\]

2) $H = (I, p_1)$

a) After history $H = (I, m(p) + \varepsilon)$

The mixed strategy profile $\sigma^n$ suggests that RT’s own trembles restricts him from correctly offering $p_2 = 0$ with probability $\left(1 - \frac{1}{n} - \frac{1}{n^2}\right)$, while mistakenly offering $p_2 = \varepsilon$ with probability $\frac{1}{n^2}$, and offering $p_2 = \varepsilon$ with probability $\frac{1}{n^2}$. Suppose $m(p) + \varepsilon \in P$, which means a price offer slightly higher than the minimum of the set $P$ is also in the set $P$. OT’s strategy in $\sigma^*$ after $H = (I, m(p) + \varepsilon)$ is to choose $Y_1PF$. The following show that $Y_1PF$ satisfies $\pi_i(\sigma^*_h, \sigma^*_b) \geq \pi_i(\sigma^*_h, \sigma^*_b)$ for all $n$.

OT’s best response is $Y_1PF$ as proven by the following expected payoffs:

\[
\pi_{OT}(Y_1PF|\sigma^n, H) = m(P) + \varepsilon - c + \frac{1}{n} \left(1 - \frac{1 - \alpha_1}{n}\right)g - m(P) - \frac{1}{n} \left(1 - \frac{1 - \alpha_1}{n}\right)\varepsilon
\]
\[
\pi_{OT}(N_1|\sigma^n, H) = -c
\]

b) $H = (I, m(p))$

The mixed strategy profile $\sigma^n$ suggests that RT’s own trembles restricts him from correctly offering $p_2 = 0$ with probability $\left(1 - \frac{2}{n}\right)$, while mistakenly offering $p_2 = \varepsilon$ with probability $\frac{1}{n}$, and offering $p_2 = \varepsilon$ with probability $\frac{1}{n}$. OT’s strategy in $\sigma^*$ for $H = (I, m(p))$ is to choose $Y_1PF$.

The following show that $Y_1PF$ satisfies $\pi_i(\sigma^*_h, \sigma^*_b) \geq \pi_i(\sigma^*_h, \sigma^*_b)$ for all $n$.

OT’s best response is $Y_1PF$ as proven by the following expected payoffs:

\[
\pi_{OT}(Y_1PF|\sigma^n, H) = m(P) - c + \varepsilon \left(1 - \frac{2}{n}\right)
\]
\[
\pi_{OT}(N_1|\sigma^n, H) = -c
\]
c) \( H = (I, m(p) - \varepsilon) \)

The mixed strategy profile \( \sigma^n \) suggests that RT’s own trembles restricts him from correctly offering \( p_2 = 0 \) with probability \( \left( 1 - \frac{1}{n} - \frac{1}{n^2} \right) \), while mistakenly offering \( p_2 = \varepsilon \) with probability \( \frac{1}{n^2} \) and offering \( p_2 = -\varepsilon \) with probability \( \frac{1}{n} \). OT’s strategy in \( \sigma^* \) for \( H = (I, m(p)) \) is to choose \( Y_1PF' \). The following show that \( Y_1PF' \) satisfies \( \pi_i(\sigma_{h}^*, \sigma_{-h}^*) \geq \pi_i(\sigma_{h}, \sigma_{-h}^*) \) for all \( n \).

OT’s best response is \( Y_1PF' \) as proven by the following expected payoffs:

\[
\pi_{OT}(Y_1PF'|\sigma^n, H) = m(P) - \varepsilon - c
\]
\[
\pi_{OT}(Y_1PF'|\sigma^n, H) = m(P) + \varepsilon - c
\]
\[
\pi_{OT}(N_1|\sigma^n, H) = -c
\]

3) \( H = (I) \)

The mixed strategy profile \( \sigma^n \) suggests that after \( H = (I, m(p)) \) and \( H = (I, m(p) + \varepsilon) \) OT’s own trembles restricts him from correctly choosing \( Y_1PF \) with probability \( \left( 1 - \frac{1}{n} - \frac{1}{n^2} \right) \), while mistakenly choosing \( Y_1PF' \) with probability \( \frac{1}{n^2} \), and choosing \( N_1 \) with probability \( \frac{1}{n^2} \).

Similarly for \( H = (I, m(p) - \varepsilon) \) OT’s own trembles restricts him from correctly choosing \( Y_1PF \) with probability \( \left( 1 - \frac{1}{n} - \frac{1}{n^2} \right) \), while mistakenly choosing \( Y_1PF' \) with probability \( \frac{1}{n} \), and choosing \( N_1 \) with probability \( \frac{1}{n^2} \).

RT’s strategy in \( \sigma^* \) after \( H = (I) \) is to offer \( p_1 = m(P) \).

The following show that \( p_1 = m(P) \) satisfies \( \pi_i(\sigma_{h}^*, \sigma_{-h}^*) \geq \pi_i(\sigma_{h}, \sigma_{-h}^*) \) for some \( n \). The argument consists of four steps.

a) RT’s best response is \( p_1 = m(P) \) if

\[
K_1(n)(1 - \alpha_1)g \geq \varepsilon \geq K_2(n)(1 - \alpha_1)g
\]

Where

\[
K_1(n) = \left( 1 - \frac{4}{n} + \frac{5}{n^2} - \frac{2}{n^3} - \frac{2}{n^4} \right) \left/ \left( 1 + \frac{1}{n} - \frac{4}{n^2} + \frac{1}{n^3} + \frac{3}{n^4} \right) \right.
\]
\[
K_2(n) = \left( \frac{1}{n} - \frac{4}{n^2} + \frac{4}{n^3} + \frac{1}{n^4} - \frac{2}{n^5} \right) \left/ \left( 1 - \frac{2}{n^3} + \frac{1}{n^5} \right) \right.
\]

Consider the following expected payoffs

i) \( \pi_{RT}(p_1 = m(P)|\sigma^n, H) = g - m(P) + \frac{m(P)}{n^2} - \frac{3g}{n} + \frac{3g}{n^2} - \frac{2g}{n^3} - \frac{\varepsilon}{n} - \frac{3\varepsilon}{n^2} - \frac{\varepsilon}{n^3} - \frac{2\varepsilon}{n^4} + \frac{3g\alpha_1}{n} - \frac{4g\alpha_1}{n^2} + \frac{2g\alpha_1}{n^4} \)

ii) \( \pi_{RT}(p_1 = m(P) + \varepsilon|\sigma^n, H) = g - m(P) - \varepsilon + \frac{m(P)}{n^2} - \frac{2g}{n} - \frac{g}{n^2} + \frac{4g}{n^3} - \frac{g}{n^4} - \frac{2g}{n^5} - \frac{\varepsilon}{n} + \frac{3\varepsilon}{n^2} + \frac{\varepsilon}{n^3} - \frac{2\varepsilon}{n^4} - \frac{\varepsilon}{n^5} + \frac{4g\alpha_1}{n} - \frac{2g\alpha_1}{n^2} + \frac{g\alpha_1}{n^4} + \frac{2g\alpha_1}{n^5} \)
iii) \( \pi_{RT}(p_1 = m(P) - \varepsilon|\sigma^n, H) = \varepsilon - m(P) + g\alpha_1 + \frac{m(P)}{n^2} + \frac{\varepsilon}{n} - \frac{2\varepsilon}{n^2} + \frac{\varepsilon}{n^3} - \frac{\varepsilon}{n^4} - \frac{4\varepsilon}{n^6} \)

\[ \frac{g\alpha_1}{n^2} - \frac{2g\alpha_1}{n^3} \]

\( \pi_{RT}(p_1 = m(P)|\sigma^n, H) \geq \pi_B(p_1 = m(P) + \varepsilon|\sigma^n, H) \) if the RHS of \( K_1(n)(1 - \alpha_1)g \geq \varepsilon \geq K_2(n)(1 - \alpha_1)g \)

\[ \pi_{RT}(p_1 = m(P)|\sigma^n, H) \geq \pi_B(p_1 = m(P) - \varepsilon|\sigma^n, H) \] if the LHS of \( K_1(n)(1 - \alpha_1)g \geq \varepsilon \geq K_2(n)(1 - \alpha_1)g \)

There exists a \( \tilde{n} \) such that for any \( n \geq \tilde{n} \), the inequality \( K_1(n)(1 - \alpha_1)g \geq \varepsilon \geq K_2(n)(1 - \alpha_1)g \) holds. Since \( K_1(n) \) is monotonically increasing to 1 as \( n \to \infty \) while \( K_2(n) \) is monotonically decreasing to 0 as \( n \to \infty \)

4) \( H = \{\emptyset\} \)

The mixed strategy profile \( \sigma^n \) suggests that RT’s own trembles restricts him from correctly offering \( p_1 = m(P) \) with probability \( \left(1 - \frac{2}{n}\right) \), while mistakenly offering \( p_1 = m(P) + \varepsilon \) with probability \( \frac{1}{n} \) and offering \( p_1 = m(P) - \varepsilon \) with probability \( \frac{1}{n} \). OT’s strategy in \( \sigma^* \) at start of game is to choose I. The following show that I satisfies \( \pi_i(\sigma^*_h, \sigma^*_{-h}) \geq \pi_i(\sigma_h, \sigma^*_{-h}) \) for some \( n \).

Consider the following payoffs

i) \( \pi_{OT}(I|\sigma^n, H) = \left(1 - \frac{1}{n^2}\right)m(P) - c + \left(\frac{1}{n} - \frac{4}{n^2} + \frac{5}{n^3} - \frac{1}{n^4} - \frac{3}{n^5} + \frac{1}{n^6}\right)\varepsilon \)

ii) \( \pi_{OT}(0|\sigma^n, H) = 0 \)

iii) \( \pi_{OT}(I|\sigma^n, H) \geq \pi_{OT}(0|\sigma^n, H) \) if \( \left(1 - \frac{1}{n^2}\right)m(P) - c + \left(\frac{1}{n} - \frac{4}{n^2} + \frac{5}{n^3} - \frac{1}{n^4} - \frac{3}{n^5} + \frac{1}{n^6}\right)\varepsilon \geq 0 \)

Since \( \varepsilon > 0 \) by definition there exist a \( \tilde{n} \) such that \( n \geq \tilde{n} \), then

\[ \left(1 - \frac{1}{n^2}\right)m(P) - c + \left(\frac{1}{n} - \frac{4}{n^2} + \frac{5}{n^3} - \frac{1}{n^4} - \frac{3}{n^5} + \frac{1}{n^6}\right)\varepsilon \geq 0 \] will approximate \( m(P) - c + \frac{\varepsilon}{n} \geq 0 \) which is implied by \( m(P) - c \geq 0 \), the original IR constraint of the CSM(OT).

REMARKS. The strategy profile \( \sigma^* \) which is equivalent to \( r^*_P \) is Trembling Hand Perfect in \( G\hat{S}^* \) since the sequence of mixed strategy profile \( \sigma^n \to \sigma^* \) is such that \( \pi_i(\sigma^*_h, \sigma^*_{-h}) \geq \pi_i(\sigma_h, \sigma^*_{-h}) \) holds, where \( \tilde{n} = \max (\tilde{n}, \tilde{\tilde{n}}) \).

LEMMA 3. The retaliatory strategy profile \( r^*_P \) in \( G\hat{S}^* \) is a Perfect Bayesinan Equilibrium (PBE).

Prove. See Appendix

LEMMA 3.1. The retaliatory strategy profile \( r^*_P \) in \( G\hat{S}^* \) is a Perfect Equilibrium (PE).

Prove. See Appendix

The motivation for this Section is the observation of many real world PPPs in which creating and operating them is fraught with significant risks. Uncertainty of conditions in project implementation is
bound up with fluctuations of macroeconomic conditions, difficulties in forecasting demand, possible changes in legislation, deviations of construction costs and operation from projected values and so forth. Meanwhile, boundary position of PPP between private and public sectors is plagued with corruption, and other abusing at a choice of the private partner, which can take place at conclusion of contracts, definition payments mode, and so forth. Large scale state investment is often accompanied by serious miscalculations related to expected costs, terms of building, needs for facilities being created and etc.

Under these realistic observations of “mistakes” or “noise” in the actions by RT and OT the reliance of the potential use of weakly dominated strategies becomes spurious and Selten’s idea of “trembles” in modeling PPP projects is not only warranted but essential. We have proven the optimal retaliatory strategy profile \( r^*_p \) in \( GS^* \) is robust in the environment of a perturbed game \( GS^* \) by showing it to be a Trembling Hand Perfect Equilibrium, Perfect Baysian Equilibrium (PBE) and Perfect Equilibrium (PE).

4. Other Extensions

In this section we reexamine the main results (Proposition 2 and 3) but with some modifications so as to better fit some realistic situations. By varying the degree of completeness of the acceptable course of action, the notion of a CA has potential usefulness in a wide range of applications.

4.1. Share of the surplus [Insert Fig. 3]

This point generalizes the result to cases with players sharing surplus differently. Denote \( \beta \in [0,1] \) as the share of surplus that goes to RT. The payoff in \( GS^* \) can be modified accordingly. \( GS^* \) is a special case of this modified game when \( \beta = 1 \). The question we address in this sub-section is the following: Is a retaliatory strategy profile in which OT chooses I is subgame perfect for \( \beta \in [0,1] \)?

With similar analysis, one can show that in this modified game, OT’s IR constraint becomes \( m(P) \geq c - (1 - \beta)\alpha_1 g \). RT’s IC constraint becomes \( m(P) \leq (1 - \alpha_1)y - (1 - \beta)\alpha_1 g \). OT’s IR constraint becomes less restrictive as \( \beta \) gets smaller. This is intuitive because the smaller the \( \beta \), the larger is OT’s share, and therefore the more valuable is the trading relationship to OT. He therefore has a higher incentive to invest. RT’s constraint is more restrictive as \( \beta \) gets smaller. This is also intuitive because a smaller \( \beta \) means RT gets less surplus. He demands lower first period price offer in order to refrain from fully expropriating OT. Combining the IR and IC gives \( (1 - \alpha_1)g \geq c \), the condition for a retaliatory strategy profile to be SSSE, which is the same as the result obtained under Proposition 2. It implies that whether or not a retaliatory strategy profile in the modified game is SSSE is independent of \( \beta \). Intuitively this result is based on the fact that RT always assumes all the bargaining power. Whatever share of trade surplus player OT has, RT is able to completely expropriate by offering a corresponding set of prices. For a smaller \( \beta \), RT can simply offer a lower, possibly negative offers as well.

4.2. Relationship specificity [Insert Fig. 4]

This point argues that the ability to redeploy an asset from the investment makes the right to destroy more likely to mitigate expropriation. Let \( s_0, s_1 \) and zero denote the scrap values of the investment at \( t = 0,1,2 \) respectively, where \( s_0 \in [0, c) \) and \( s_0 \geq s_1 \). The bigger is \( s_0 \) the less relationship-specific is the investment. The condition \( s_0 < c \) ensures that reducing asset specificity alone does not solve the agency problem. \( GS^* \) is a special case of this modified game when \( s_0 = s_1 = 0 \).
The presence of scrap values result in less restrictive IR and IC constraints: OT now demands a second stage price at least equal to the scrap value; RT would have to offer a higher \( p_1 \) anyway when there is a scrap value. Offering \( m(P) \) therefore does not make that much of a difference relatively to the case with no scrap value. With similar analysis OT’s IR and the RT’s IC constraints are \( m(P) \geq c - s_1 \) and \( m(P) \leq (1 - \alpha_1)g - (2s_1 - s_0) \) respectively. Combining the two constraints gives \( (1 - \alpha_1)g - s_1 \geq c - s_0 \) which has a familiar interpretation: the second stage surplus that is subject to OT’s manipulation has to be larger than the amount of sunk investment, which is the gross investment cost minus the scrap value. Since \( s_0 \geq s_1 \) comparing \( (1 - \alpha_1)g \geq c \) with \( (1 - \alpha_1)g - s_1 \geq c - s_0 \) indicates that any \( \alpha_1 \) that supports an equilibrium in Proposition 2 also does so in this modified game.

**REMARKS:** Varying the degree of asset specificity does not invalidate the basic result. The set of surplus dynamics patterns, as denoted by \( \alpha \), that supports an equilibrium with investment, is larger in this modified game than that in Proposition 2.

4.3. Partial destruction [Insert Fig. 5]

Suppose OT can only destroy a fraction, \( \tau \in [0,1] \) of the remaining surplus at the end of the first period. The second period surplus then becomes \( (1 - \tau)(1 - \alpha)g \). \( G S^* \) is a special case of this modified game when \( \tau = 1 \). By similar analysis RT’s IC constraint is \( m(P) \leq \tau(1 - \alpha_1)g \). OT’s payoff under such a retaliatory profile is \( m(P) - c \). Therefore his IR is \( m(P) \geq c \). Combining the two give the condition for a retaliatory strategy to be SSSE: \( (\tau)(1 - \alpha)g \geq c \)

[Insert Fig. 6] This Figure illustrates the set of \( (\alpha_1, \tau) \) such that a retaliatory strategy profile is SSSE for a given \( (c, g) \). It shows that as OT’s ability to manipulate the surplus weakens (\( \tau \) decreases) the set of \( \alpha_1 \) for a retaliatory strategy profile to be SSSE shrinks. The weaker is the threat, the smaller is the set of surplus dynamics patterns that accommodates a retaliatory strategy profile as SSSE. The merit of this section is to tell us precisely when the ability to destroy ceases to be powerful enough to mitigate expropriation. When \( \tau \leq \frac{c}{g} \) there is no two stage surplus dynamics such that a retaliatory strategy profile is SSSE and hence OT never invests.

5. Discussion

An evaluation among PPP projects has often been referred to as a debate between case studies! This is because no two PPP projects are the same and while it is tempting to generalize, this remains futile. However, one can safely conclude that generally the outcomes have not been encouraging not without exceptions. For example, there have been headline failures such as the M1/M15 motorway in Hungary, the Trakia Highway in Bulgaria and the Horgos-Pozega motorway in Serbia. Many other PPP projects have been poor value for money. Agency problems are costly and are evidently endemic in PPPs globally due to the manifold vested interests and divergent beliefs of the regulator who is likely to expropriate the operator by transferring operational risk to it. Consequently the PPP’s commercial viability is diminished while the operational risk of the operator escalates. The particular time schedule of investments and payments in PPP contracts-with payments typically begin after the completion of infrastructure, several years after signature of contracts. This means that if the contracts is improperly dealt and addressed, the contract is a powerful instrument for keeping public expenses out of the books, for under-evaluating them and for bias decisions in favor of PPP schemes that accelerate investment and
delay payments by the public sector to the private sector. Even where the PSC calculations are carried out, such as in the UK, the outcomes cannot be relied on. The UK’s PSC calculation has been widely criticized for being ‘rigged’ in favor of PPP schemes\textsuperscript{10}.

These empirical observations provide the impetus for our models to accommodate the analysis of individuals with divergent beliefs under incomplete contracting so as to examine the conditions under which the agency problem of expropriation could be rectified. As mentioned in the preceding paragraph, such acts of expropriations include cost overruns; the excessive transfer of operational risk from the state to the private sector, delays in payments and the spurious nature of PSC. Worse still, these acts of expropriations are exacerbated by the predominance of bounded rationality, manifold vested interests and divergent beliefs of multiple parties, long project duration and hence cost recovery, and rampant bribery/corruption in public procurement. All in all, they result in the diminished commercial viability of the operating stage of PPPs and ultimately lead to their collapse. In this setting, incomplete contracts emerge as rational responses by individuals with different beliefs and views of the world, even without considering any transaction costs. Thus, our framework also serves to unify “divergent beliefs” and “incomplete contracts” viewpoints, which are the hallmarks of real world PPP.

This paper has provided a rudimentary theoretical framework in examining the conditions under which the agency problem of expropriation could be rectified. Consequently the objective is to derive an optimal retaliatory strategy profile in PPP where tacit collusion will always be sustained. This optimal retaliatory strategy profile provides a credible pre-commitment on the part of the regulator against expropriation of the operator in the future. This also provides a strategic rational for the formation of PPP, and to a lesser extent serve as a check and balance to restrain and govern a process that otherwise tends to become an anarchic and self-serving means for rent seeking by special interest groups worldwide. Baker, Gibbons and Murphy (2002) have shown that the future profitability may act as an effective check for short-term opportunistic behavior. This aim is also in the same spirit as Ng (2011) and Strahilevitz (2005), who argues that “the right to destroy” helps restore investment incentive.

In Proposition 2 we solved the agency problem in PPP theoretically. The trick for such efficient mechanism is to give OT an ability to “destroy” the investment by underperforming if RT fails to offer the minimum acceptable price but otherwise to give it a reward in the form of positive surplus sharing. However the most crucial rewarding element for RT is not just about surplus sharing, but rather that “destruction” would mean the deepening of the “investment gap” and “bottleneck” or worse still, the lose of faith in PPP by the people as a mean to alleviate shortage of state financial resources and ameliorate insufficient efficiency of the state in many developing countries. Given that the state cannot cope this deficiency with its own capacity since it has little financial resources and it is at an objective dilemma in increasing production efficiency in the public sector, intuitively it is not incentive compatible for him to deviate from the optimal retaliatory strategy profile. The end result is that the PPP is sustained, resulting in no destruction of the investment while minimizing the agency problem of expropriation.

\textsuperscript{10} The National Audit Office’s deputy comptroller and auditor-general Jeremy Coleman has been one such critic, dismissing some calculations as “utter rubbish” and “pseudo-scientific mumbo-jumbo.” Nicholas Timmins: “Warning of ‘spurious’ figures on value of PFI”, Financial Times, 05.06.2002
The motivation behind Proposition 3 is that we observed in the real world creating and operating PPP is fraught with significant risks both to the state and the private sector. Uncertainty of conditions in project implementation is bound up with fluctuations of macroeconomic conditions, difficulties in forecasting demand, possible changes in legislation, deviations of construction costs and operation from projected values and so forth. Meanwhile, boundary position of PPP between private and public sectors is plagued with corruption, and other abusing at a choice of the private partner, which can take place at conclusion of contracts, definition payments mode, and so forth. Large-scale state investment is often accompanied by serious miscalculations related to expected costs, terms of building, needs for facilities being created and etc. Under these realistic observations of “mistakes” or “noise” in the actions by RT and OT the reliance of the potential use of weakly dominated strategies becomes spurious and Selten’s idea of “trembles” in modeling PPP projects is not only warranted but essential. We have proven the optimality of Proposition 2 under the real world “trembles”. Proposition 2 and 3 are therefore a set of robust results for real world PPP which could provide a credible pre-commitment on the part of the state against expropriation in the future and a strategic rationale for PPP in which tacit collusion will always be sustained. They are the main findings of our paper.

6. Conclusion

The objective of this paper is not to advocate a more protective ex ante contract to mitigate the agency problem when the private sector is expropriated by the state in PPP, but rather to suggest the least protective contract (no contract at all), coupled with the dynamics of the surplus stream that does so. Such dynamics lie in the incentive compatibility constrains of the state and the private sector in their optimal strategy profile. However we have not further discussed the cases highlighted in Section 4, for example in 4.1 where the private sector has the upper hand in bargaining power through its status as a de-facto monopoly under the settings of “divergent beliefs” and “incomplete contracting”. This opposing end of the spectrum will add new dimension and favor to the existing models.

Appendix

A. Proof of Lemma 2

Let $x$ be an SSSE. Then, there is a stable set $K$ in $G^*$ such that $x \in K$. Therefore, $y >^k x$ for all $y \in K$. Since $x \in K$, it follows that, $\forall i \in N, \forall u \in V_i, \pi_i(x|v) \geq \pi_i(y_i', y_{-i}|v)$ for all $y_i' = x_i$. Therefore $x$ is an SPE. Since every SPE is SSE and every SSSE is SSE, every SSSE is SPE.

B. Proof of Lemma 2.1

By Claim 2.1, $G^*$ associated with $G$ satisfies the one deviation property of the general system. Therefore, a strategy profile $x$ is not dominated given a set $Y$, iff, for each subgame the player who makes the first move cannot obtain a better outcome by changing only his initial action, with respect to any of the opponents’ strategy profiles in $Y_{-i}$. By the construction of $K$, $x \in K$ iff $x$ is not dominated given $K$. $K(T|w)$ must be contained in the largest stable set in the $G$ associated with the subgame $G(T|w)$. Since $x(T|w) \in K(T|w)$, it follows that $x(T|w)$ is an SSE in $G^*$ associated with the subgame $G(T|w)$. The retaliatory strategy profile $r^*_p$ in $G^*$ is subgame consistent.

C. Proof of Claim 3
For $y^i \in Y^i$ let $y^i_k \Rightarrow y^i$ and $y^j_k \Rightarrow y^j$ for all $j \neq i$ such that $\pi^i(y(h), y_k(-h)) \geq \pi^i(a, y_k(-h))$ for all $a \in \Delta(h)$. By the one deviation property (Rubinstein 1994), $y^i$ is $i$’s best response to $y^{-i}$. Thus $y^i$ is not a strictly dominated strategy. In this game where every player has only one information set, $y^i$ is $i$’s best response to $y^{-i}_k$. As for all $j \neq i$, $y^i_k$ is a totally mixed strategy, it follows that $y^i$ is not a weakly dominated strategy.

**D. Proof of Lemma 3.**

Define the distribution, $d^i(h)$, over the vertices in an information set $h \in H^i$ as follows: $d^i(h) \equiv \lim_{k \to \infty} d^i_k(h)(h)$, where $d^i_k(h)$ is the unique distribution over these vertices derived from $y^i_k[i]$ using Bayes’ rule. By Claim 2.1, it follows that player $i$’s strategy $x^i$ is a sequential best response to $x^{-i}$ when $i$’s beliefs over the vertices in $h \in H^i$ are given by $d^i(h)$. Hence, $x$ is a Perfect Bayesian Equilibrium.

**E. Proof of Lemma 3.1**

By the continuity of $\pi^i$ we have that for all players $i$ and for all $h \in H^i$

$\pi^i(x(h), x(-h)) \geq \pi^i(a, x(-h))$ for all $a \in \Delta(h)$ Since $x$ is a totally mixed strategy, the sequence $y_k \equiv x, k = 1, 2, ...$, can be chosen as the trembling sequence for every player $i$. Hence, $x$ is a Perfect Equilibrium.

**References**


Commission of the European Communities, Green Paper on PPP.

Financial Times, 05.06.2002
Fig. 1: Game tree of the base model

Fig. 2: Game tree of the extended model
Fig. 3: Share of trade surplus game tree

Fig. 4: Relationship specificity game tree
Fig. 5: Partial destruction game tree

\[ \begin{align*}
\text{OT} & \quad p_1 - c, \alpha_1 g - p_1 \\
\text{RT} & \quad p_1 + p_2 - c, g - p_1 - p_2 \\
\text{PF} & \quad [\alpha_1 + (1 - \tau)(1 - \alpha_1)]g - p_1 - p_2 \\
\text{PF'} & \quad p_1 - c, \alpha_1 g - p_1 \\
\text{OT} & \quad p_1 - c, \alpha_1 g - p_1 \\
\text{OT} & \quad -c, 0 \\
\text{OT} & \quad 0, 0
\end{align*} \]

Fig. 6: Partial destruction