An empirical test for the identifying assumptions
of the Blanchard and Quah (1989) model

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Abstract
In their vector autoregression model, Blanchard and Quah (1989) employed as identifying assumptions uncorrelatedness between aggregate supply and aggregate demand shocks and the long-run output neutrality condition. We examine the empirical consistency of these assumptions with actual data. To derive a testable form, the Blanchard and Quah model is transformed into a cointegration representation that produces identical results. This alternative setup is extended to allow for a test of uncorrelatedness and long-run output neutrality. Empirical results indicate that the two assumptions, when taken together, are accepted by the data for Japan and the U.S. but rejected for Germany. Further analysis suggests that the imposition of long-run output neutrality is responsible for such rejection in Germany, and this can be explained in association with hysteresis effects.

Key words: Blanchard and Quah; Cointegration; Shocks; Identifying assumptions; Hysteresis
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1. Introduction
In an influential study, Blanchard and Quah (BQ, 1989) developed a vector autoregression (VAR) model that identifies the effects of aggregate supply (AS) and aggregate demand (AD) shocks on real output and the unemployment rate. Two assumptions are employed for identification of structural shocks. The first assumption is that AS and AD shocks are mutually uncorrelated. The rationale for this is that structural shocks are primitive, exogenous forces and hence have no common causes. The second assumption is that an AD shock has no long-run effects on real output. This is based on the long-run output neutrality condition, according to which the AS curve is vertical and independent of AD factors in the long run, so that only AS shocks can have long-run effects on real output.¹

Both assumptions are not without criticism, however. Cover et al. (2006) suggest reasons why AS and AD shocks may be correlated with each other. Shiller (1986) and Pesaran and Smith (1998) argue that assuming uncorrelatedness between shocks may not be reasonable in many econometric applications of the structural VAR methodology.² Cooley and Dwyer (1998) and Giordani (2004) provide examples in which this assumption produces specification errors and

¹ Uncorrelatedness between structural shocks is a common assumption that most structural VAR studies impose in the identification process. The absence of this assumption entails n(n–1)/2 additional restrictions for exact identification in an n-variable model. The long-run output neutrality assumption has also been extensively adopted for identifying AS and AD shocks individually.
² The structural shocks are generally permitted to correlate with each other in the simultaneous equation macroeconometric model (e.g. the Cowles Commission approach). The dynamic structure is restricted, instead, for identification, typically by excluding contemporaneous and lagged regressors. See Pagan and Robertson (1995) and Jacobs and Wallis (2005) for a comparison between this traditional approach and the VAR modeling.
biased impulse response estimators. Cross (1995) and Ball (1999) show that monetary policy can have long-term real effects. Campbell and Mankiw (1987) and Keating (2005) summarize a variety of possible mechanisms through which AD shocks may have long-run effects on real output. In fact, BQ acknowledged some of these effects, such as hysteresis (à la Blanchard and Summers, 1986) and endogenous models of growth. The hysteresis hypothesis suggests that the natural rate of unemployment depends on the past levels of the unemployment rate, allowing AD shocks to influence the natural rate and hence, real output in the long run. In endogenous growth models, an AD shock can have long-run effects on real output as long as it produces temporary changes in the amount of resources allocated to growth (Stadler, 1986; King et al., 1988).

We examine the extent to which the uncorrelatedness and long-run output neutrality assumptions of the BQ model are consistent with actual data. This issue is important because the model implications are dependent on the adequacy of the assumptions in use. Where the assumptions are inconsistent with data, their imposition may result in misrepresentation of the true dynamic structure of the model. Nevertheless, the uncorrelatedness and long-run output neutrality assumptions have rarely been evaluated for their empirical relevance. The main reason is that the two assumptions are exact-identifying restrictions, and hence they cannot be tested.

To derive a testable form, the present paper casts the BQ model in a cointegration framework. The starting point is that real output is an $I(1)$ process and the unemployment rate is
stationary in levels. This implies that the two variables are cointegrated where the unemployment rate itself is the cointegration relation. The corresponding vector error correction model is structurally identified by using a procedure that decomposes the shocks into those with permanent effects and those with transitory effects. The results are exactly the same as those from the BQ model.\textsuperscript{3}

This cointegration representation is extended to allow for relaxation of the identifying assumptions underlying the BQ model. The correlation between AS and AD shocks and the long-run response of real output to an AD shock are not restricted to being zero. They are allowed to be determined by the data. Thus, comparison of the results with those from the BQ model can be used as a joint test for evaluating the empirical relevance of the two assumptions with respect to the data. The paper also introduces two companion representations. One allows for correlation between AS and AD shocks under the long-run output neutrality assumption. The other admits that an AD shock can have potentially long-run effects on real output under the uncorrelatedness assumption between AS and AD shocks. These models are utilized for examining the question of which assumption, i.e. uncorrelatedness of the shocks or long-run output neutrality, has greater impact when the joint test of the assumptions is rejected by the data.

The remainder of this paper is organized as follows. Section 2 presents a cointegration
\textsuperscript{3} Englund \textit{et al.} (1994) and Fry and Pagan (2005) gave a general indication of this possibility, but they did not prove equivalence between the BQ model and its cointegration representation, due to a different focus.
representation of the BQ model and an extension is made to allow for relaxation of the uncorrelatedness and long-run output neutrality assumptions. Empirical applications to Germany, Japan, and the U.S. are given in Section 3. Section 4 offers a further analysis for Germany, as the results turn out to be different from those for Japan and the U.S. The German unemployment rate appears to exhibit hysteresis. Section 5 summarizes the major findings of the paper, together with concluding remarks. Appendix A formally proves equivalence between the BQ model and its cointegration representation. Appendix B provides detailed derivations of the two companion models.

2. A joint test of uncorrelatedness and long-run output neutrality

2-1. The Blanchard and Quah model

The Blanchard and Quah (BQ) model begins with estimation of a reduced-form VAR model given as:

\[ \Delta y_t = \sum_{i=1}^{p} f_{yy,i} \Delta y_{t-i} + \sum_{i=1}^{p} f_{yu,i} u_{t-i} + e_{1t} \]  \hspace{1cm} (1)

\[ u_t = \sum_{i=1}^{p} f_{uy,i} \Delta y_{t-i} + \sum_{i=1}^{p} f_{uu,i} u_{t-i} + e_{2t} \]  \hspace{1cm} (2)

where \( y_t \) is real output, \( u_t \) is the unemployment rate, \( \Delta \) is the first difference operator, \( e_t = (e_{1t}, e_{2t})' \) is a vector of reduced-form shocks and is iid with a mean of zero and a
covariance matrix of \( E(\varepsilon_t \varepsilon_t') = \Omega \). Let \( z_t \) be the vector \( (\Delta y_t, u_t)' \). The vector moving average (VMA) representation for (1) and (2) can be expressed in compact form as:

\[
z_t = C(L)e_t
\]

where \( C(L) = C_0 + C_1 L + C_2 L^2 + \cdots, \ C_0 = I, \ C(1) = \sum_{i=0}^{\infty} C_i, \) and \( L \) is the lag operator.

Subject to identification, a structural VMA representation corresponding to (3) is given as:

\[
z_t = \Gamma(L)e_t
\]

where \( \Gamma(L) = \Gamma_0 + \Gamma_1 L + \Gamma_2 L^2 + \cdots, \ \Gamma(1) = \sum_{i=0}^{\infty} \Gamma_i, \) and \( e_t = (\varepsilon_{1t}, \varepsilon_{2t})' \) is a vector of structural shocks. Following BQ, \( \varepsilon_{1t} \) and \( \varepsilon_{2t} \) are denoted as an aggregate supply (AS) shock and an aggregate demand (AD) shock, respectively. These are assumed to have a mean of zero and a covariance matrix of the form:

\[
E(\varepsilon_t \varepsilon_t') = \Sigma = \begin{pmatrix} 1 & \sigma_{12} \\ \sigma_{12} & 1 \end{pmatrix}
\]

where each structural shock is normalized to have unit variance without loss of generality. From (3) and (4), the relationships between the reduced-form and structural parameters are:

\[
\Gamma(L) = C(L)\Gamma_0
\]

and

\[
\varepsilon_t = \Gamma_0^{-1} e_t.
\]

\(^4\) The constant term is suppressed for the sake of illustration.
For exact identification of the structural shocks, the four parameters in $\Gamma_0$ or $\Gamma_0^{-1}$ must be uniquely determined. The two identifying assumptions come into action. Uncorrelatedness between AS and AD shocks leads to $\sigma_{12} = 0$ in (5), and then $\Gamma_0 \Gamma_0' = \Omega$ in (7) yields three restrictions on $\Gamma_0$. The long-run output neutrality assumption implies that the element (1,2) of the long-run impact matrix $\Gamma(1)$ in (4) is zero, i.e. $\Gamma(1)$ becomes lower triangular. This provides one remaining restriction on $\Gamma_0$ by setting the (1,2) element of $C(1)\Gamma_0$ in (6) to zero. Accordingly, solving $\Gamma_0 \Gamma_0' = \Omega$ under long-run output neutrality gives the four parameters in $\Gamma_0$.\(^5\) Once $\Gamma_0$ is estimated, the impulse responses and the forecast-error variances of the series can be computed from (6).

2-2. A cointegration representation of the Blanchard and Quah model

The BQ model can be cast in a cointegrated VAR framework. This is possible because the model assumes that real output is an $I(1)$ process and the unemployment rate is stationary in levels. An implication of this is that the two variables are cointegrated where the unemployment rate itself is the cointegration relation, i.e. $\beta = [0, 1]'$. The error correction term is given as:

$$\beta'z_t = u_t$$

\(^5\) An equivalent but simpler solution is to combine (6) and (7) as $C(1)\Omega C(1)' = \Gamma(1)\Gamma(1)'$. Taking the Choleski decomposition of $C(1)\Omega C(1)'$ provides all estimates in $\Gamma(1)$, and hence, $\Gamma_0$ from (6).
and a vector error correction (VEC) representation for (1) and (2) may be written as:

\[
\Delta y_t = \sum_{i=1}^{p} g_{yy,i} \Delta y_{t-i} + \sum_{i=1}^{p-1} g_{yu,i} \Delta u_{t-i} + \alpha_1 u_{t-p} + \epsilon_{1t} \tag{9}
\]

\[
\Delta u_t = \sum_{i=1}^{p} g_{uy,i} \Delta y_{t-i} + \sum_{i=1}^{p-1} g_{uu,i} \Delta u_{t-i} + \alpha_2 u_{t-p} + \epsilon_{2t} \tag{10}
\]

where \( \alpha = [\alpha_1, \alpha_2]' \) is a vector of error correction coefficients.

Let \( s_t \) be the vector \( (\Delta y_t, \Delta u_t)' \). The Granger Representation Theorem shows that the VEC model in (9) and (10) can be inverted to obtain the VMA representation (for details, see Engle and Granger, 1987):

\[
s_t = D(L)e_t \tag{11}
\]

where \( D(L) = D_0 + D_1 L + D_2 L^2 + \cdots, \ D_0 = I, \) and

\[
D(1) = \sum_{i=0}^{\infty} D_i = \beta_\perp \psi \alpha_\perp \tag{12}
\]

where \( \alpha_\perp = [\alpha_{1\perp}, \alpha_{2\perp}]' \) and \( \beta_\perp = [\beta_{1\perp}, \beta_{2\perp}]' \) are vectors orthogonal to \( \alpha \) and \( \beta \), respectively (i.e. \( \alpha' \alpha_\perp = 0 \) and \( \beta' \beta_\perp = 0 \)), \( \psi = (\alpha_\perp D(1) \beta_\perp)^{-1} \), and \( D(1) \) is the short-run impact matrix of reduced-form shocks from (9) and (10), given by:

\[
G(1) = \begin{bmatrix}
1 - \sum_{i=1}^{p} g_{yy,i} & -\sum_{i=1}^{p-1} g_{yu,i} \\
-\sum_{i=1}^{p} g_{uy,i} & 1 - \sum_{i=1}^{p-1} g_{uu,i}
\end{bmatrix}.
\]

A detailed derivation is found in Johansen (1991).

The presence of one cointegrating relationship in the model implies that one shock
exhibits permanent effects, whereas the other shock demonstrates only transitory effects. The first ($\epsilon_1$) is designated as an AS shock and the second ($\epsilon_2$) as an AD shock. This interpretation is plausible in that the AS shock has permanent effects on real output but the AD shock has only transitory effects. To decompose the shocks into those with permanent effects and those with transitory effects, we employ the Beveridge and Nelson (1981) procedure of a type proposed for VEC models by Mellander et al. (1992), Englund et al. (1994), Levchenkova et al. (1996), and Fisher et al. (2000). The model posits for the relationship between the reduced-form and structural shocks of (7) that:

$$\Gamma_0^{-1} = \begin{bmatrix} \alpha'_{\perp} \\ \alpha' \Omega^{-1} \end{bmatrix}.$$  \hspace{2cm} (13)

Equation (13) is modified for each structural shock to have unit variance, which is given as:

$$\Gamma_0^{-1} = \begin{bmatrix} \Lambda_1 \alpha'_{\perp} \\ \Phi_1^{-1} \alpha' \Omega^{-1} \end{bmatrix}$$  \hspace{2cm} (14)

where $\Lambda_1^{-1} \Lambda_1^{-T} = \alpha'_{\perp} \Omega \alpha_{\perp}$ and $\Phi_1 \Phi_1 = \alpha' \Omega^{-1} \alpha$. Combining (7) and (14) reveals that $\Lambda_1 \alpha'_{\perp} e_t$ and $\Phi_1^{-1} \alpha' \Omega^{-1} e_t$ are permanent AS and transitory AD shocks, respectively.

It is straightforward to show that

$$E(\epsilon_t' \epsilon_t) = \Gamma_0^{-1} \Omega \Gamma_0^{-T} = \begin{bmatrix} \Lambda_1 \alpha'_{\perp} \\ \Phi_1^{-1} \alpha' \Omega^{-1} \end{bmatrix} \Omega \begin{bmatrix} \Lambda_1 \alpha'_{\perp} \\ \Phi_1^{-1} \alpha' \Omega^{-1} \end{bmatrix}' = 1.$$  \hspace{2cm} (15)

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6 See King et al. (1991), Mellander et al. (1992), and Levchenkova et al. (1996) for the implications of cointegration in the structural identification of VEC models.
The correlation between AS and AD shocks is zero. The inverse matrix of \( \Gamma_0^{-1} \) is calculated as:

\[
\Gamma_0 = \begin{bmatrix}
\Omega \alpha \Lambda_1' & \alpha \Phi_1^{-T}
\end{bmatrix}.
\] (16)

The application of (6), (12), and (16) demonstrates that

\[
\Gamma(1) = D(1)\Gamma_0 = \beta_\perp \psi \alpha_\perp' \begin{bmatrix}
\Omega \alpha_\perp \Lambda_1' & \alpha \Phi_1^{-T}
\end{bmatrix} = [\beta_\perp \psi \Lambda_1^{-1} , 0].
\] (17)

As the implied cointegrating vector of \( \beta = [0, 1]' \) gives \( \beta_\perp = [1, 0]' \), \( \Gamma(1) \) of dimension (2x2) becomes:

\[
\Gamma(1) = \begin{bmatrix}
\psi \Lambda_1^{-1} & 0 \\
0 & 0
\end{bmatrix}.
\] (18)

Equation (18) confirms that the AS shock has permanent effects on real output, while the AD shock has only transitory effects. Both shocks have transitory effects on the unemployment rate, which is consistent with the assumption that the rate is stationary in levels. This cointegration representation matches the BQ model exactly, and they generate identical results. A formal proof is provided in Appendix A.

2-3. Relaxation of the uncorrelatedness and long-run output neutrality assumptions

Now we present an extended model that can be used to jointly test the empirical relevance of the uncorrelatedness and long-run output neutrality assumptions. This model, denoted as BQ-E, shares the VEC representation in (9) and (10) but takes a different decomposition for the
identification of structural shocks. Specifically, it is assumed that
\[
\Gamma^{-1}_0 = \begin{bmatrix}
\Lambda_2 \beta'_\perp \\
\Phi_2^{-1} \alpha \Omega^{-1}
\end{bmatrix},
\]  
(19)
and its inverse is calculated as:
\[
\Gamma_0 = [\Omega \alpha_\perp (\Lambda_2 \beta'_\perp \Omega \alpha_\perp)^{-1} \beta (\Phi_2^{-1} \alpha' \Omega^{-1} \beta)^{-1}]
\]  
(20)
where \(\Lambda^{-1}_2 \Lambda_2^T = \beta'_\perp \Omega \beta_\perp\) and \(\Phi_2 \Phi_2' = \alpha' \Omega^{-1} \alpha\). From (7), \(\Lambda_2 \beta'_{1t} \epsilon_t\) and \(\Phi_2^{-1} \alpha' \Omega^{-1} \epsilon_t\) correspond to AS and AD shocks, respectively.

The covariance matrix of the structural shocks is:
\[
E(\epsilon_1 \epsilon_1') = \Gamma^{-1}_0 \Omega \Gamma^{-T}_0 = \begin{bmatrix}
1 & \Lambda_2 \beta'_\perp \alpha \Phi_2^T \\
\Lambda_2 \beta'_\perp \alpha \Phi_2^T & 1
\end{bmatrix}.
\]  
(21)
The long-run impact matrix of the structural shocks is obtained from (12) and (20) as:
\[
\Gamma(1) = D(1) \Gamma_0 = [\beta'_\perp \psi \alpha'_\perp \Omega \alpha_\perp (\Lambda_2 \beta'_\perp \Omega \alpha_\perp)^{-1} \beta_\perp \psi \alpha'_\perp \beta (\Phi_2^{-1} \alpha' \Omega^{-1} \beta)^{-1}].
\]  
(22)
On using \(\beta_\perp = [1, 0]'\), (22) is summarized as:
\[
\Gamma(1) = D(1) \Gamma_0 = \begin{bmatrix}
\psi \alpha'_\perp \Omega \alpha_\perp (\Lambda_2 \beta'_\perp \Omega \alpha_\perp)^{-1} & \psi \alpha'_\perp \beta (\Phi_2^{-1} \alpha' \Omega^{-1} \beta)^{-1} \\
0 & 0
\end{bmatrix}.
\]  
(23)
Comparing (19) with (14) shows that the AD shock is identified as being the same across the BQ and BQ-E models. The key difference is that the BQ-E model in (21) and (23) allows this AD shock to correlate with the AS shock and to have long-run effects on real output, as determined by the data. The correlation between two structural shocks is not restricted to being zero, nor is
the long-run effect of the AD shock on real output. This offers one way of testing the empirical relevancy of the two assumptions underlying the BQ model. If results from the BQ model are different from those of the BQ-E model, this can be taken as evidence that the uncorrelatedness and long-run output neutrality assumptions, taken together, are not consistent with the actual data. If the results do not differ, the two assumptions are empirically supported, justifying the application of the BQ model. Further explanatory remarks will be found in the section to follow.

If \(\beta_\perp \alpha\) in (21) and \(\alpha_\perp \beta\) in (23) are zero, both the uncorrelatedness and the long-run output neutrality assumptions become valid for the BQ-E model.\(^7\) This occurs when real output is weakly exogenous to the cointegrating vector in the VEC model of (9) and (10), i.e. \(\alpha = [0, \alpha_2]'\) and hence \(\alpha_\perp = [\alpha_\perp, 0]'\). In fact, the BQ and BQ-E models coincide exactly. This can be easily demonstrated by noting that when \(\alpha = [0, \alpha_2]'\), \(\alpha_\perp\) can be reparameterized as \(\alpha_\perp = [1, 0]'\).\(^8\) Because \(\alpha_\perp = \beta_\perp\), all equations in the BQ-E model are identical to their counterparts in the BQ model. The equivalence between the BQ-E and BQ models offers stronger evidence in support of the joint assumptions of uncorrelatedness and long-run output neutrality. Note that the weak exogeneity of real output is a sufficient condition, but not a

\(^7\) Note that \(\Lambda_2, \Phi_2,\) and \(\psi\) cannot be zero by definition.

\(^8\) Consider the error correction coefficient matrix \(\alpha\) of dimension (nxr) in an n-variable model with r cointegration relationships present. Partition it as \(\alpha = [\alpha_m, \alpha_r]'\), where \(\alpha_m\) and \(\alpha_r\) are, respectively, the first \(m = (n-r)\) and last \(r\) rows of \(\alpha\). Using Gaussian elimination with complete pivoting gives \(\alpha_\perp = [I_m, -\alpha_m \alpha_r^{-1}]\). Since \(\alpha_m = 0\) in our case, \(\alpha_\perp = [1, 0]'\). For a full derivation, see Fisher and Huh (2007, p. 186).
necessary one, for accepting the two assumptions jointly. Even if real output is not weakly exogenous, it is possible for the uncorrelatedness and long-run output neutrality assumptions to be empirically supported, as determined by the data.

3. Tests on the results for the Blanchard and Quah model

The analysis outlined above was applied to quarterly observations of real GDP ($y_t$) and the unemployment rate ($u_t$) in three major economies: Germany, Japan, and the U.S. Data were taken from the IMF *International Financial Statistics*. The sample period was 1960:Q1 to 2006:Q4. The real GDP series was transformed using natural logarithms and differenced once to achieve stationarity. For estimation of the VEC models in (9) and (10), the lag length $p$ was chosen on the basis of the Sims likelihood ratio test. The chosen lengths are $p=8$ and $p=11$ for Germany and Japan, respectively. In the case of the U.S., $p=8$ is used to match the lag structure in the original BQ model. All lag lengths are consistent with the results of Breusch-Godfrey Lagrange Multiplier tests, which indicate the absence of serial correlation in both real output growth and unemployment rate equations at the 5% significance level.

The estimated VEC models are expanded to models in the levels of the series. They are then inverted numerically to generate estimates of the reduced-form shocks. After these estimates have been obtained, the structural shocks are identified and their impacts on the series are
calculated by utilizing the procedures outlined in Section 2. Figure 1 depicts the responses of the levels of the series to one-standard-error structural shocks. The BQ and BQ-E models produce responses that are consistent with standard economic theory. A favorable AS shock causes real output to increase across the horizons. This shock lowers the unemployment rate at short horizons. In response to a favorable AD shock, real output increases and the unemployment rate declines. Real output in the BQ model eventually returns to its pre-shock levels as a consequence of the long-run output neutrality assumption.

The issue is how significantly the responses in the BQ model differ from those of the BQ-E model. To assist in assessing this, depicted together in Figure 1 are 95% confidence bands generated by 500 bootstrap replications of the BQ model. The Japan and U.S. cases show that the BQ and BQ-E models produce impulses of a quite similar shape and magnitude. Without doubt, all responses from the BQ-E model are located well within the 95% confidence bands of the responses in the BQ model. The BQ-E model clearly exhibits the long-run output neutrality too, while the AD shock was allowed to have long-run effects on real output. For the two countries, the evidence is that the uncorrelatedness and long-run output neutrality assumptions in the BQ model are jointly accepted by the data.

Germany produces different results, however. The BQ-E model indicates that the AD shock has positive effects on real output in the long run. In fact, the responses after 10 quarters
are all above the 95% upper bound of the responses from the BQ model. The AD shock also leads to a larger fall in the unemployment rate than the BQ model implies. The effect is pronounced for short horizons at which the responses are significantly below the 95% lower bound of the responses from the BQ model. As the BQ model is statistically different from the BQ-E model, the joint test of uncorrelatedness and long-run output neutrality is rejected by the data. It is thus likely that the BQ model when applied to Germany provides misleading inferences on the dynamic responses of the series.

As a consistency check, Table 1 reports estimates for the error correction coefficients $\alpha_1$ and $\alpha_2$ in (9) and (10). Based on the t-test, the coefficient $\alpha_1$ is not statistically different from zero in Japan and the U.S., implying that real output is weakly exogenous to the cointegration relationship. The null hypothesis of $\alpha_2 = 0$ is rejected, as expected from the Granger Representation Theorem. A consequence is that the BQ and BQ-E models produce identical results for these countries (see Section 2).\(^9\) This offers strong evidence supporting the use of the uncorrelatedness and long-run output neutrality assumptions in the BQ model. For the case of Germany, the coefficient $\alpha_1$ is statistically different from zero, while the coefficient $\alpha_2$ is not. The unemployment rate, not real output, is the variable that is weakly exogenous to the cointegrating relationship. The BQ and BQ-E models do not coincide. In fact, Figure 1 revealed

\(^9\) To save space, we have not reported the responses of the model imposing weakly exogeneity of real output.
that the two models were statistically different and the joint test of uncorrelatedness and long-run neutrality was rejected by the data.

4. Uncorrelatedness of the shocks *versus* long-run output neutrality

This section examines the question of which assumption, uncorrelatedness of the shocks or long-run output neutrality, has greater impact leading to rejection of the BQ model for Germany. Two companion models are developed along the lines of the BQ-E model of Section 2. One allows for correlation between AD and AS shocks under the long-run output neutrality assumption. The other admits that an AD shock can have potentially long-run effects on real output under the uncorrelatedness assumption between AD and AS shocks. These two models, denoted as BQ-C and BQ-L, respectively, may be regarded as restricted cases of the BQ-E model. Detailed derivations of the BQ-C and BQ-L models are presented in Appendix B for the sake of compactness.

Figure 2 presents the responses of the levels of the series generated from the BQ-C and BQ-L models for Germany. Those from the BQ model and their 95% confidence bands are reproduced for comparison. The BQ-C model shows responses that are of a very similar shape and magnitude to those from the BQ model. The only possible exceptions are the responses of unemployment rate to the AS shock, but these are within the 95% confidence bands of the
responses in the BQ model. In contrast, the BQ-L model produces different implications. Again, the AD shock has positive long-run effects on real output above the 95% upper bound of the responses from the BQ model. Both AS and AD shocks also show larger effects on the unemployment rate than the BQ model suggests at short to medium horizons. On taking the results together, the uncorrelatedness assumption is consistent with the data, but the long-run output neutrality assumption is not. The imposition of long-run output neutrality turns out to be too restrictive, and this appears to have contributed to the rejection of the BQ model for Germany.

To ascertain how the results change with the long-run output neutrality assumption, Table 2 reports the forecast error variance of the series attributable to each structural shock in the BQ and BQ-L models for Germany. Both models show that the AS shock is the major determinant of the movement in real output across the horizons. One particular difference arises, however. In the BQ model, the AD shock accounts for some fraction of the forecast error variance at short horizons, but the contribution eventually converges to zero as a result of the long-run output neutrality assumption. The BQ-L model indicates that this shock affects real output in the long run. It accounts for around a one-third of the forecast error variance at long horizons. The results differ more considerably for the unemployment rate. The BQ model suggests that the AS shock explains the vast majority of the variation in the rate. The contribution of the AD shock is limited to accounting for no more than 15% of the forecast error variance. In the BQ-L model, the AD
shock is more important for explaining the variation in the unemployment rate. It accounts for more than 74% of the forecast error variance across the horizons. The AS shock plays only a small role in influencing the rate.

The BQ-C and BQ-L models were not applied for Japan and the U.S. because the uncorrelatedness and long-run neutrality assumptions were jointly accepted by the data in Section 3. An obvious prediction is that both BQ-C and BQ-L would not be statistically different from the BQ model. This can easily be checked by recalling that real output is weakly exogenous to the cointegrating vector in Japan and the U.S. Since $\alpha_\perp\beta = 0$, the BQ-C model has correlation between AD and AS shocks of zero, and the AD shock has no long-run effects on real output in the BQ-L model. See (B-3), (B-9), and (B-10) in the Appendix B. A consequence of this is that the BQ-C and BQ-L models preserve both the uncorrelatedness and the long-run output neutrality. In fact, the BQ, BQ-C, and BQ-L models become identical. When $\alpha_\perp = \beta_\perp$ and hence $\alpha = \beta$, the BQ-C and BQ-L models yield exactly the same equations as the BQ model. The equivalence between these three models confirms our earlier finding that the joint test of uncorrelatedness and long-run output neutrality is accepted by the data for Japan and the U.S.

It is interesting to see why Germany produces different results from the other two countries. One possibility is suggested in Table 1, which reports the error correction coefficient estimates. Unlike Japan and the U.S., the unemployment rate in Germany is weakly exogenous to
the cointegrating relationship and real output is not. An implication of this is that the unemployment rate contains a unit root. To check this statistically, we applied the Augmented Dickey-Fuller (ADF) and Kwiatkowski, Phillips, Schmidt, and Shin (KPSS, 1992) tests for a unit root. The German unemployment rate is indeed characterized as an I(1) process. For the other two countries, the ADF and KPSS tests confirm that the U.S. unemployment rate is stationary, while the rate for Japan is stationary around a deterministic trend.

In principle, the unemployment rate cannot have a unit root since it is a bounded variable. The fact that the variance of a unit root process grows indefinitely over time rules out the possibility of a unit root for any bounded variable. The unemployment rate has a unit root, however, if it exhibits hysteresis. This scenario seems sensible for Germany, as Figure 3 demonstrates that the unemployment rate has risen persistently from its low bound. In fact, a number of studies report that hysteresis has played a key role in explaining persistently high unemployment in Germany (Blanchard and Summers, 1986; Fitoussi et al., 2000; Ball and Mankiw, 2002; Franz, 2005; Blanchard, 2006; Ball, 2009). When there is hysteresis, a rise in the unemployment rate following a negative AD shock is extended into the long run, causing the natural rate of unemployment to rise. Jaeger and Parkinson (1994), Logeay and Tober (2006), and Horn et al. (2007) find this mechanism accountable for permanent shifts that the German

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10 See Johansen and Juselius (1990) for the implications of weak exogeneity in cointegrated models.
11 The testing results are available upon request.
natural rate has undergone. As the natural rate of unemployment is affected, so is real output in the long run. A further implication is that an AD shock can have long-run effects on real output, in contrast to long-run output neutrality.

With this in view, we look at possible effects of hysteresis when using the BQ model for Germany. Several studies, including Blanchard (2006) and Ball (2009), assert that hysteresis has played a major part in the unemployment story since 1980, whilst the effects were only secondary in the 1970s. Following this, the data were split into the two subsamples: 1960:Q1 – 1979:Q4 (period 1) and 1980:Q1 – 2006:Q4 (period 2). Figure 4 reports the responses generated from the BQ and BQ-L models for each period, together with 95% confidence bands generated by 500 bootstrap replications of the BQ model. It is apparent that period 1 displays no significant difference between the BQ and BQ-L models. Neither does the AD shock have a long-run effect on real output in the BQ-L model, consistent with the long-run output neutrality assumption. In period 2, however, the BQ-L model indicates that the AD shock now affects real output in the long run. This result can be explained in association with hysteresis effects. A negative AD shock raises the unemployment rate beyond the short run. The natural unemployment rate rises and real output declines in the long run, as Figure 4 shows. It can thus

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12 Higher unemployment during the 1970s was mainly attributed to the two oil price crises and a marked slowdown in productivity.
13 The lag length p was set at 4 and 8, respectively, for periods 1 and 2, on the basis of the Sims likelihood ratio test. 
14 The responses are symmetric by definition.
be concluded that the long-run output neutrality assumption in the BQ model is rejected by the data for Germany since 1980.

5. Concluding remarks

One of the most well-known structural VAR models is that of Blanchard and Quah (BQ, 1989). They developed a model that identifies the effects of aggregate supply and aggregate demand shocks on real output and the unemployment rate. Since then, numerous applications and extensions have followed. The BQ model employs as identifying assumptions uncorrelatedness between aggregate supply and aggregate demand shocks and the long-run output neutrality condition. Presumably, the model implications would be dependent on the adequacy of the assumptions employed in the model. Nevertheless, the uncorrelatedness and long-run output neutrality assumptions have rarely been tested for their empirical relevance. Furthermore, some studies have provided evidence against these assumptions.

The present paper examines the extent to which the uncorrelatedness and long-run output neutrality assumptions of the BQ model are consistent with actual data for Germany, Japan, and the U.S. To derive a testable form, the BQ model is transformed into a cointegration representation that produces identical results. This alternative setup is extended to allow for the possibility that the structural shocks are mutually correlated and an aggregate demand shock has
long-run effects on real output. Comparison of the results with those from the BQ model has shown no significant difference in Japan and the U.S. The two models differ for Germany, however. The joint test of uncorrelatedness and long-run output neutrality is rejected by the data, thus questioning the appropriateness of BQ application to Germany. Further analysis suggests that the imposition of long-run output neutrality is responsible for such rejection. The hysteresis hypothesis is shown to offer one explanation for the inconsistency between the actual data and long-run output neutrality.
Appendix A

The BQ VAR model of (1) and (2) can be expressed in matrix form as:

\[ F(L)z_t = e_t. \]  \hspace{1cm} (A-1)

Similarly, the VEC model of (9) and (10) can be presented in matrix form as:

\[ G(L)s_t - \alpha u_{t-p} = \Pi(L)s_t = e_t \]  \hspace{1cm} (A-2)

where

\[
G(L) = \begin{bmatrix} G_{11}(L) & G_{12}(L) \\ G_{21}(L) & G_{22}(L) \end{bmatrix} \quad \text{and} \quad \Pi(L) = \begin{bmatrix} G_{11}(L) & G_{12}(L) - \alpha_1 L^p(1-L)^{-1} \\ G_{21}(L) & G_{22}(L) - \alpha_2 L^p(1-L)^{-1} \end{bmatrix}.
\]

To derive the relationship between the two models, consider the following transformation on (A-2):

\[
\Pi(L) \begin{bmatrix} 1 \\ 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & (1-L)^{-1} \end{bmatrix}^{-1} \begin{bmatrix} 1 & 0 \\ 0 & (1-L)^{-1} \end{bmatrix} s_t = e_t. \hspace{1cm} (A-3)
\]

This yields

\[
\begin{bmatrix} G_{11}(L) & G_{12}(L)(1-L) - \alpha_1 L^p \\ G_{21}(L) & G_{22}(L)(1-L) - \alpha_2 L^p \end{bmatrix} \begin{bmatrix} \Delta y_t \\ u_t \end{bmatrix} = e_t. \hspace{1cm} (A-4)
\]

Comparison with (A-1) shows that

\[
F(L) = \begin{bmatrix} F_{11}(L) & F_{12}(L) \\ F_{21}(L) & F_{22}(L) \end{bmatrix} = \begin{bmatrix} G_{11}(L) & G_{12}(L)(1-L) - \alpha_1 L^p \\ G_{21}(L) & G_{22}(L)(1-L) - \alpha_2 L^p \end{bmatrix}. \hspace{1cm} (A-5)
\]

Equations (3), (A-1), and (A-5) imply that

\[15\text{ Campbell and Shiller (1988) and Mellander et al. (1992) provide useful formulae for transforming VEC models to VAR counterparts in a general setup.}\]
\( C(1)^{-1} = F(l) = \begin{bmatrix} G_{11}(l) & -\alpha_1 \\ G_{21}(l) & -\alpha_2 \end{bmatrix}. \)  \hspace{1cm} (A-6)

By using (6), (A-6), and (14) with \( \alpha' \alpha = 0 \), it is proven that

\[
\Gamma(l)^{-1} = \Gamma_0^{-1} C(l)^{-1} = \begin{bmatrix} \Lambda_1\alpha_{11} & \Lambda_1\alpha_{12} \\ \Xi(\alpha_1\Omega_{22} - \alpha_2\Omega_{12}) & \Xi(\alpha_2\Omega_{11} - \alpha_1\Omega_{12}) \end{bmatrix} \begin{bmatrix} G_{11}(l) & -\alpha_1 \\ G_{21}(l) & -\alpha_2 \end{bmatrix} = \begin{bmatrix} \times & 0 \\ \times & \times \end{bmatrix} \]  \hspace{1cm} (A-7)

where \( \Xi = (1/|\Omega|) \Phi_1^{-1} \), \( \Omega_{11} \), \( \Omega_{22} \), and \( \Omega_{12} \) are the variances of \( e_{1t} \) and \( e_{2t} \) and their covariance in \( \Omega \), respectively, and \( \times \) denotes a non-zero number. Because \( \Gamma(l)^{-1} \) is lower triangular, so is \( \Gamma(l) \). The long-run output neutrality condition is evident. The lower triangularity of \( \Gamma(l) \) together with uncorrelatedness between AS and AD shocks in (15) satisfy the assumptions of the BQ model. The BQ model and its cointegrated representation thus become equivalent and they generate identical empirical results.
Appendix B

The BQ-C model assumes, for the relationship between reduced-form and structural shocks of (7), that

\[
\begin{bmatrix}
\Lambda_3 \alpha_3 \\
\Phi_3^{-1} \beta \Omega^{-1}
\end{bmatrix}
\]

and its inverse is calculated as:

\[
\Gamma_0 = \left[ \Omega \beta \left( \Lambda_3 \alpha_3 \Omega \beta \right)^{-1} \alpha \left( \Phi_3^{-1} \beta \Omega^{-1} \alpha \right)^{-1} \right]
\]

where \( \Lambda_3^{-1} \Lambda_3^{-T} = \alpha_3' \Omega \alpha_3 \) and \( \Phi_3 \Phi_3' = \beta' \Omega^{-1} \beta \). From (7) and (B-1), \( \Lambda_3 \alpha_3 \epsilon_t \) and \( \Phi_3^{-1} \beta \Omega^{-1} \epsilon_t \) correspond to permanent AS and transitory AD shocks, respectively. Gonzalo and Ng (2001) introduced a similar expression to (B-1) but adopted a different normalization. In the BQ-C model, the covariance matrix of structural shocks is given as:

\[
\begin{bmatrix}
1 \\
\Lambda_3 \alpha_3' \beta \Phi_3^{-T}
\end{bmatrix}
\]

The correlation between AS and AD shocks is not restricted to being zero but is determined by the data. The long-run impact matrix of structural shocks is obtained from (12) and (B-2) as:

\[
\Gamma(1) = D(1) \Gamma_0 = [\beta', \psi \Lambda_3^{-1} 0].
\]

Imposition of \( \beta' = [1, 0]' \) gives:

\[
\Gamma(1) = D(1) \Gamma_0 = \begin{bmatrix}
\psi \Lambda_3^{-1} & 0 \\
0 & 0
\end{bmatrix}.
\]
5)

The AD shock has no long-run effect on real output. In fact, the BQ-C and BQ models share the same long-run impact matrix \( \Gamma(1) \) as \( \Lambda_1 = \Lambda_3 \) from \( \Lambda_1^{-1}A_1^{-\top} = \Lambda_3^{-1}A_3^{-\top} = \alpha_1'\Omega\alpha_1 \).

Proceeding to the BQ-L model, it is assumed that

\[
\begin{align*}
\Gamma_0^{-1} &= \begin{bmatrix}
\Lambda_4\beta_4' \\
\Phi_4^{-1}\beta_4'\Omega^{-1}
\end{bmatrix}, \\
\Gamma(1) &= D(1)\Gamma_0 = \begin{bmatrix}
\beta_4'\Omega\beta_4' & \beta_4'\Phi_4'T
\end{bmatrix},
\end{align*}
\]  

where \( \Lambda_4^{-1}A_4^{-\top} = \beta_4'\Omega\beta_4' \) and \( \Phi_4^{-1}\beta_4'\Omega^{-1}\beta_4' \). Escribano and Peña (1994) proposed a similar procedure for the decomposition of trend and cycle. In (B-6), \( \Lambda_4\beta_4'\epsilon_t \) and \( \Phi_4^{-1}\beta_4'\Omega^{-1}\epsilon_t \) correspond to AS and AD shocks, respectively. The remaining formulae can be calculated similarly, as before, and are therefore listed without detailed derivations. They are:

\[
\begin{align*}
\Gamma_0 &= [\Omega\beta_4\Lambda_4' & \beta_4\Phi_4'T], \\
E(\epsilon_1\epsilon_1') &= \Gamma_0^{-1}\Omega\Gamma_0^{-\top} = I, \\
\Gamma(1) &= D(1)\Gamma_0 = \begin{bmatrix}
\beta_4'\psi\alpha_4'\Omega\beta_4' & \beta_4'\psi\alpha_4'\beta_4'T
\end{bmatrix},
\end{align*}
\]  

and, with imposition of \( \beta_4 = [1, 0]' \),

\[
\Gamma(1) = D(1)\Gamma_0 = \begin{bmatrix}
\psi\alpha_4'\Omega\beta_4' & \psi\alpha_4'\beta_4'T \\
0 & 0
\end{bmatrix}.
\]  

The BQ-L model admits uncorrelatedness between AD and AS shocks in (B-8). Equations (B-9) and (B-10) show that the long-run output neutrality condition is not imposed a priori and the AD shock is allowed to have long-run effects on real output.
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Table 1: Tests for error-correction coefficients

<table>
<thead>
<tr>
<th>Country</th>
<th>Real output</th>
<th>Unemployment</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\alpha_1$</td>
<td>t-test</td>
</tr>
<tr>
<td>Germany</td>
<td>−0.062 (0.03)</td>
<td>0.04</td>
</tr>
<tr>
<td>Japan</td>
<td>−0.050 (0.14)</td>
<td>0.72</td>
</tr>
<tr>
<td>U.S.</td>
<td>0.048 (0.05)</td>
<td>0.32</td>
</tr>
</tbody>
</table>

The second column reports the estimate for the error correction coefficient $\alpha_1$ in the output equation (9). Figures in parentheses are the standard errors. The third column reports the marginal significance level (p-value) of t-test statistics for the null hypothesis that $\alpha_1$ is equal to zero. The fourth and fifth columns do the same for the equation of the unemployment rate in (10).
Table 2: Forecast error variance decompositions for Germany

| Quarter | BQ model | | | BQ-L model | | |
|---------|----------|------------|------------|----------|------------|------------|------------|
|         | AS       | AD         | AS         | AD       | AS         | AD         | AS         | AD         |
| Real output | Unemployment | | Real output | Unemployment | | |
| 1       | 54.2     | 45.8       | 87.0       | 13.0     | 100.0      | 0.0        | 19.6       | 80.4       |
| 2       | 62.0     | 38.0       | 87.5       | 12.5     | 98.4       | 1.6        | 20.2       | 79.8       |
| 4       | 70.4     | 29.6       | 89.8       | 10.2     | 95.8       | 4.2        | 23.5       | 76.5       |
| 8       | 78.4     | 21.6       | 91.4       | 8.6      | 91.5       | 8.5        | 25.8       | 74.2       |
| 10      | 82.2     | 17.8       | 90.5       | 9.5      | 87.7       | 12.3       | 24.5       | 75.5       |
| 20      | 90.5     | 9.5        | 86.5       | 13.5     | 76.8       | 23.2       | 19.7       | 80.3       |
| 30      | 93.3     | 6.7        | 86.0       | 14.0     | 73.4       | 26.6       | 18.9       | 81.1       |
| 40      | 94.8     | 5.2        | 85.6       | 14.4     | 71.1       | 28.9       | 18.5       | 81.5       |
| 50      | 95.8     | 4.2        | 85.5       | 14.5     | 69.2       | 30.8       | 18.3       | 81.7       |
| 60      | 96.5     | 3.5        | 85.4       | 14.6     | 67.7       | 32.3       | 18.2       | 81.8       |
| 70      | 97.1     | 2.9        | 85.3       | 14.7     | 66.5       | 33.5       | 18.1       | 81.9       |
| 80      | 97.4     | 2.6        | 85.3       | 14.7     | 65.4       | 34.6       | 18.0       | 82.0       |
| 90      | 97.7     | 2.3        | 85.2       | 14.8     | 64.5       | 35.5       | 18.0       | 82.0       |
| 100     | 98.0     | 2.0        | 85.2       | 14.8     | 63.7       | 36.3       | 18.0       | 82.0       |

The figures are the fractions of the forecast error variance of the series attributable to aggregate supply (AS) and aggregate demand (AD) shocks.
Figure 1: (a) Responses of the series from the BQ and BQ-E models for Germany

AS shock $\rightarrow$ Real output

AD shock $\rightarrow$ Real output

AS shock $\rightarrow$ Unemployment rate

AD shock $\rightarrow$ Unemployment rate

BQ $\quad$ 95% Confidence bands of BQ $\quad$ BQ-E
Figure 1: (b) Responses of the series from the BQ and BQ-E models for Japan

AS shock $\rightarrow$ Real output

AD shock $\rightarrow$ Real output

AS shock $\rightarrow$ Unemployment rate

AD shock $\rightarrow$ Unemployment rate

BQ  \hspace{1cm} 95\% Confidence bands of BQ  \hspace{1cm} BQ-E
Figure 1: (c) Responses of the series from the BQ and BQ-E models for the U.S.
Figure 2: Responses of the series from the BQ, BQ-C, and BQ-L models for Germany

AS shock $\rightarrow$ Real output

AD shock $\rightarrow$ Real output

AS shock $\rightarrow$ Unemployment rate

AD shock $\rightarrow$ Unemployment rate

BQ  
$95\%$ Confidence bands of BQ  
BQ-C  
BQ-L
Figure 3: Plot of the unemployment rate (percent)
Figure 4: (a) Responses of the series for Period 1 (1960:Q1 – 1979:Q4)

AS shock → Real output

AD shock → Real output

AS shock → Unemployment rate

AD shock → Unemployment rate

(b) Responses of the series for Period 2 (1980:Q1 – 2006:Q4)

AS shock → Real output

AD shock → Real output

AS shock → Unemployment rate

AD shock → Unemployment rate

BQ  95% Confidence bands of BQ  BQ-L