Abstract

This study investigates duopolistic competition between hub ports for container transshipment. A container handling demand function which incorporates both gateway and transshipment traffic is developed. The study applies a non-cooperative two-stage game to a vertical-structure seaport market with ports as upstream players and shipping lines as downstream players. We explain the drivers behind port competition through the existence of a unique Nash equilibrium which incorporates the shipping lines’ supply decision and the ports’ pricing decision. We also analyze a port collusion model and a social optimum model, and compare it with the non-cooperative model for further insights.

Keywords: Container transshipment, Port competition, Non-cooperative game, Two-stage game, Nash equilibrium

1. Introduction

In 2000, Maersk Sealand relocated its major transshipment operations from the Port of Singapore (PSA) to the Port of Tanjung Pelepas (PTP) in Malaysia. The impact of this relocation on the regional transshipment market structure was significant. Maersk Sealand was then the largest shipping operator in Singapore. Its shift to PTP resulted in a decline of
approximately 11% in PSA’s overall business. In 2001, PSA’s total container throughput fell from 17 million TEUs to 15.52 million TEUs, marking a year-on-year drop of 8.9% (Tongzon, 2006). In the same period, PTP’s container throughput had increased nearly 5 folds, from 0.42 million to 2.05 million TEUs.\(^1\)

The shipping industry in Singapore and the region grew concerned about Maersk Sealand’s relocation and the potential ripple effect on other shipping lines’ decisions and related business activities.\(^2\) As shipping lines form strategic alliances to achieve economies of scale, the interdependency among alliance members and small- and medium-size shipping lines heightens. Consequently, Maersk Sealand’s decision on changing its transshipment port-of-call could well induce similar decisions among affiliating carriers. In 2002, Evergreen and its subsidiary Uniglory followed in Maersk Sealand’s footsteps and shifted most of their container operations, amounting to 1-1.2 million TEUs of annual throughput, from PSA to PTP. Since then, other shipping lines have also started to provide direct services to PTP. APL, for example, had chosen PTP for its West Asia Express service between Asia and the Middle East (Kleywegt et al., 2002).

This study aims to investigate a regional transshipment hub port competition problem within a duopolistic framework. In the case of competition between PTP and PSA, the acquisition of transshipment cargo is critical. Both ports are subjected to stringent growth limitations as gateway ports but possess excellent locations along the Strait of Malacca. Transshipment presents a good opportunity for these ports to expand beyond the demands of their respective catchment economies and more importantly, tap into the international cargo flows to enjoy superior profits. Beyond the potential spike in the number of cargo handling jobs and value-added activities, a transshipment port would also gain access to profitable feeder line networks which serve to transport containers to/from tributary ports. These networks give the transshipment port good connectivity, which in turn strengthens itself through the “ripple effect”. As the importance of achieving dominance in the market becomes apparent, it is foreseeable for regional ports to compete for transshipment container traffic.

In particular, we construct a demand function for a container port’s transshipment traffic. A review of the existing studies of port selection and competition (Lirn et al., 2003, 2004; Chou, 2007; Lin and Tseng, 2007; Chang et al., 2008; Yuen et al., 2011) have led us to

---


2 The issue was, for example, discussed in the article written by Allison of Asia Times Online on 2 September 2000.
identify port capacity, price, “transshipment coefficient” and port congestion as the primary factors of relevance to our study. While the port price and transshipment coefficient primarily determine the demand levels of a port, the resulting level of demand, together with the port capacity, determine the port’s congestion level, which in turn will influence the demands of the port and its rival port. This study attempts to uncover the relationships with greater clarity. In particular, we examine how different levels of port capacities, prices and transshipment coefficients affect the ports’ congestion levels and, more importantly, how a port can capture a greater transshipment demand with appropriate port pricing and capacity building.

We find that (i) at the non-cooperative model, the price difference between two ports is further accentuated when both port capacities are large, as in this case, congestion becomes less of an inhibiting factor. (ii) Shipping lines are inclined to make more port calls at the port that provides a higher transshipment coefficient, as long as the port capacity is sufficient to offset the accompanying congestion delay cost. (iii) The ports do take into consideration their capacity when setting prices. A bigger port can set a lower port price to attract more demand as it is more likely to have spare capacity and hence less congestion. (iv) The port collusion model yields a higher port price than that of non-cooperative model. (v) The profit margin of the social optimum model is higher than that of the non-cooperative model. The port price is internalized in the social optimum model therefore shipping lines can make more port calls than that of the non-cooperative model.

A number of studies have looked at port competition: Lam and Yap (2006, 2007) examined port competitiveness and the impact of competition in Southeast Asia while Yap and Lam (2006) considered the case of East Asia and Anderson, et al. (2008) considered the case of Northeast Asia. Saeed and Larsen (2010) considered intra-port competition and examined the possible combinations of coalitions among container terminals at the Karachi Port of Pakistan. Port competition is further investigated as part of rivalry between two alternative intermodal transportation chains; hence, recent studies have taken into account hinterland access and road congestion in order to observe their impact on ports and port

---

3 As global customers exert increasing pressure on shipping lines to lower their prices, the competition to reduce costs among shipping lines inevitably intensifies. Shipping lines are forced to explore the cheapest-cost options which in turn impose downward pressure on the port charges. For instance, the attractiveness of PTP’s port price, which is some 30% lower than that of PSA’s, becomes apparent. In fact, Evergreen had estimated that its shift to PTP would result in cost saving of between US$ 5.7 million and US$ 30 million per annum (Kleywegt et al., 2002). As such, port price will affect port demand. As for “transhipment coefficient,” it measures port connectivity and efficiency (the transhipment-coefficient definition is given below in section 2), which will affect the decision of shipping lines’ choice of port, thereby affecting port demand.
competition (Zhang, 2008; Yuen, et al., 2008; Wan and Zhang, 2011; Wan et al., 2011). Unlike our consideration of “transshipment container demand,” these papers have focused on “gateway container demand” – these are two different types of container demand at a transshipment port. The gateway demand represents the import and export container demands, whilst the transshipment demand is generated through the additional container handling jobs necessary for further seaborne transfers after unloading, including consolidation, deconsolidation and value-added activities of containers. Our transshipment focus can help advance the analysis on a port’s transshipment capabilities and enable the port to uncover and balance its demand and capacity levels so as to strategize for the long term. It can also provide better visibility on the shipping lines’ criteria when choosing a transshipment port.

Our study is closely related to the applications of game theory to port competition problem. Zan (1999) was one of the earliest authors who had attempted to use the game theory to investigate the behaviour of port users (carriers and shippers) in transshipment port management policy. He used a bi-level Stackelberg game to capture the flow of foreign trade containers. In more recent studies, Saeed and Larsen (2010) used the two-stage game to analyze possible coalitions: in the first stage, three container terminals at Karachi Port decide whether to act individually or to join a coalition; and in the second stage, the resulting coalition plays a non-cooperative game against non-members. In another study relevant to our work, De Borger, et al. (2008) used a two-stage game to analyze the interaction between the pricing behaviour of competing ports and the optimal investment policies in the ports and hinterland capacity. Beyond maritime-related research, studies have been performed on the duopolistic interactions between congestible facilities using a two-stage game (e.g. Van Dender, 2005; De Borger and Van Dender, 2006; Zhang and Zhang, 2006; Baake and Mitusch, 2007; Basso and Zhang, 2007). For example, Basso and Zhang (2007) developed a model for congestible facility rivalry in vertical structures which explains the relationship among the congestible facility and its intermediate user (airline) and final users (passengers). We will leverage on this approach to explain the drivers behind the container transshipment

---

4 The import container demand is further defined as containers destined for hinterland transportation out of the port after unloading from the vessels, and the export container demand as containers meant for seaborne transfer out of the port. The distinction between gateway and transshipment demands has been discussed in air transport research by Zhang (2003), where “gateway” traffic refers to “local and gateway” traffic, whereas “transshipment” traffic to “hub” (air to air) traffic.

5 As defined by De Borger and Van Dender (2006), congestible facilities are facilities which are prone to congestion when the volume of simultaneous users increases amid constant capacity. Examples of such facilities include seaports, airports, Internet access providers and roads.
port competition through the existence of a unique Nash equilibrium for the shipping lines’ supply and the port pricing. We will also analyze a port collusion model and a social optimum model, and compare it with the non-cooperative model for further insights, where the port collusion model can behave like a monopoly, whereas the social optimum model reflects a maximization of the combined profits of all the players in the game.

The paper is organized as follows: Section 2 shows our model formulation with linear container handling demand functions. We then proceed to apply the non-cooperative two-stage game to our problem in section 3. In section 4, a port collusion model, and a social optimum model which may reflect the current business models of shipping lines, are analyzed and compared with the non-cooperative model. Results from our numerical simulations are then shown in section 5 to further explain the findings of this study. Finally, section 6 contains the concluding remarks.

2. The model

We assume there are two container transshipment ports, \( r = 1, 2 \), which provide homogenous container handling services to their customers within a stipulated period of time. Their customers are identical shipping lines, \( i = 1, \ldots, N \).

The notations used for developing our model are defined as follows:

- \( f \) loaded and unloaded gateway container demand
- \( g_r \) coefficient of loaded and unloaded transshipment container volume
- \( N \) number of ocean carriers
- \( q_{ir} \) a fraction of transshipment port calls made by carrier \( i \) at port \( r \)
- \( p_i \) price per container charged by carrier \( i \)
- \( c_i \) carrier \( i \)’s operating cost per container
- \( F_{ir} \) total loaded and unloaded containers of carrier \( i \) at port \( r \)
- \( \pi_i \) profit for carrier \( i \)
- \( \mu_r \) port price per container at port \( r \)
- \( K_r \) 85% capacity utilization of port \( r \)
- \( \Pi_r \) profit for port \( r \)
- \( Q_r = \sum_{i=1}^{N} q_{ir} \) total fraction of transshipment port calls at port \( r \)
- \( F_r = \sum_{i=1}^{N} F_{ir} \) total container demand at port \( r \)
- \( D_r (F_r, K_r) \) cost of congestion delays per container at port \( r \)
Our container handling demand function, $F_{ir}$, is the total number of containers which carrier $i$ loads and unloads at port $r$.

$$F_{ir} = f + g_r \cdot Q_r \quad \text{for } r = 1, 2$$

where $Q_r = \sum_{i=1}^{N} q_{ir}$, $0 < q_{ir} < 1$, $q_{is} = 1 - q_{ir}$ for $r \neq s$ and $g_r$ is nonnegative coefficient. $q_{ir}$ is a decision variable which indicates a fraction of transshipment port calls that carrier $i$ makes at port $r$, and is determined by the capacity of port $r$, its price and transshipment coefficient during the stipulated period of time.

$F_{ir}$ is made up of two components: The gateway container, i.e. import and export container demand, and the transshipment container that is generated through transshipment performance at port $r$. $f$ is the gateway container demand that the shipping line handles at port $r$, which is inclusive of both the unloading of import containers and loading of export containers. Since this study focuses more on the impact of transshipment demand, we assume that the gateway container demand is constant. This assumption helps simplifying analytical work. The coefficient $g_r$, on the other hand, refers to the transshipment container volume. At a conceptual level, $g_r$ can be used to represent port connectivity, which can be defined as the port’s network connection to other transport modes that extend to other destinations (e.g. feeder services, hinterland connection). In logistics, a transshipment port is akin to a transit facility. As such, shipping lines which adopt the hub-and-spoke transportation system are likely to prefer a transshipment port that has an extensive and strong network connection. To a certain extent, $g_r$ can also be used to represent port efficiency. An efficient port is one which can effectively and quickly perform the handling jobs arising from transshipment cargo, be it through an advanced IT system or an optimized scheduling algorithm. In general, transshipment containers require consolidation, deconsolidation and value-added activities such as assembly, calibration and customization (Frankel, 2002). The ability to handle these tasks efficiently therefore adds to the port’s overall attractiveness as a transshipment port. To take our argument further, a more efficient port would be able to get more capacity out of its fixed infrastructure, hence increasing the ‘real’ capacity of the port. It will also provide a faster turnover for vessels, hence improving the service quality of port. With the above characteristics in mind, it becomes comprehensible that a ‘stronger’ port would possess a
higher value for $g_r$ and hence would be more attractive to shipping lines calling that port for transshipment.

Meanwhile, it is noted that our container handling demand function generates an equal amount of transshipment volume to all shipping lines that calling at the same port, regardless of the number of port calls that each shipping line has made. This may not be practical; however this study aims to obtain the preliminary results of major factors, we therefore assume that the transshipment container demand depends only on ports’ handling capability and aggregate contribution of shipping lines’ port calls in the stipulated period of time. This may be regarded as two service providers – one service user problem since $N$- identical service user will show exactly the same pattern of characteristics of one service user.

From container handling demand function (1), each port’s demand function (2) can be derived.

$$F_1 = \sum_{i=1}^{N} F_{i1} = N (f + g_1Q_i); \quad F_2 = \sum_{i=1}^{N} F_{i2} = N (f + g_2Q_i)$$

where $Q_2 = \sum_{i=1}^{N} q_{i2} = \sum_{i=1}^{N} (1-q_{i1}) = N - Q_1$. Then we can derive the following properties by differentiating with respect to $q_{i1}$.

$$\frac{\partial F_1}{\partial q_{i1}} = g_1, \quad \frac{\partial F_2}{\partial q_{i1}} = -g_2$$

$$\frac{\partial F_1}{\partial q_{i1}} = Ng_1, \quad \frac{\partial F_2}{\partial q_{i1}} = -Ng_2$$

Properties (3) and (4) illustrate that an increase in the expected number of port calls to be made at expected port 1 would lead to a decrease in the expected number of port calls to be made at port 2. This is expected since we assume $q_{i2} = 1 - q_{i1}$. Furthermore, the gradient of increase or decrease in the expected number of port calls depends on transshipment coefficient of each port, $g_r$.

---

6 The case of un-equal transshipment container demand would be studied in the next paper. In this case, the problem becomes more practical. The given conditions of each shipping lines are different and the port price may be varied to each shipping line in terms of demand that shipping lines bring into the port. As problem becomes complex, it may not be able to achieve the closed form of optimal port calls and port price, and there may exist multiple equilibrium solutions. Thus, we attempt to extend our model to asymmetric shipping lines and explore it with advanced numerical simulation skill in the next paper.
The congestion delay cost function, as shown in (5) below, possesses a quadratic form. Since this study only considers port capacity and port demand as a measurement of port congestion, this quadratic congestion function thus simply and efficiently captures the trend of congestion at the port; it also allows the analytical work feasible. To guarantee an interior solution, we further assume that \( F_r / K_r \leq 1 \). This is a practical range of studying \( F_r \) as we assume \( K_r \) is 85% utilization of port \( r \)'s maximum capacity \( K_r^{\text{max}} \).

\[
D_r \left( F_r, K_r \right) = a_r \left( \frac{F_r}{K_r} \right)^2 \quad \text{for } r = 1, 2
\]

(5)

where \( a_r \) is a positive parameter and \( K_r = 0.85 K_r^{\text{max}} \).

Keeping in mind that \( F_r \) is the container demand at port \( r \), \( F_r = \sum_{i=1}^{N} F_{ri} \), the congestion function \( D_r \) is designed to increase with the number of port calls made to port \( r \) and to decrease with the port’s capacity \( K_r \). The following properties are derived.

\[
\frac{\partial D_r}{\partial F_r} = 2a_r \frac{F_r}{K_r^2} \geq 0, \quad \frac{\partial^2 D_r}{\partial F_r^2} = 2a_r \frac{1}{K_r^2} \geq 0
\]

(6)

\[
\frac{\partial D_r}{\partial K_r} = -2a_r \frac{F_r^2}{K_r^3} \leq 0, \quad \frac{\partial^2 D_r}{\partial K_r^2} = 6a_r \frac{F_r^2}{K_r^4} \geq 0, \quad \frac{\partial^2 D_r}{\partial F_r \partial K_r} = -4a_r \frac{F_r}{K_r^3} \leq 0
\]

(7)

Property (6) depicts that the congestion externality is convex in the port’s demand. It is also intuitive that the congestion externality would decrease with an increase in the port handling capacity as shown in (7). Port congestion delay cost \( a_r \) in both ports is assumed to be analogous, hence notating it using \( a \) in further derivations.

3. A non-cooperative two-stage game

---

7 De Borger and Van Dender (2006) mentioned that strictly convex congestion functions prevent full capacity usage, so that interior solutions automatically result. For instance, the quadratic function or steady-state queuing theory (Zhang and Zhang, 2006) shows that the congestion costs approach infinity when demand approaches capacity.
We now study a non-cooperative two-stage game with two duopolistic transshipment ports and a continuum of identical shipping lines.

We first develop a non-cooperative profit function for shipping lines and ports. The sole objective of each shipping line is to maximize its own profits. The shipping lines’ profit function and constraints are given by

$$\max \pi_i = \sum_{r=1}^{2} (p_i - c_i - \mu_r - D_r) F_r$$

subject to

$$0 < q_{ir} < 1 ; i = 1, \ldots, N \quad r = 1, 2$$
$$\sum_{r=1}^{2} q_{ir} = 1 ; i = 1, \ldots, N$$
$$F_r \leq K_r$$

where $p_i$ is the shipping line’s container price (or revenue per container), $c_i$ is the operating cost and $\mu_r$ is the port price. Among the shipping lines, both pricing and cost incurrence are assumed to be identical, hence notating it using $p$ and $c$. In the shipping line’s profit function, the congestion function $D_r$ is captured as a cost component. This is to model the shipping line’s preference for a less congested port, considering that port congestion often leads to delays, which in turn translate to additional costs to the carriers. The first constraint shows that $q_{ir}$ is normalized between 0 and 1. The second constraint indicates that the total number of port call for all shipping lines is fixed to 1 so as to facilitate our observation of the shipping lines’ allocation decision to two ports in response to the ports’ capacities, prices and transshipment coefficients. The third constraint is a capacity constraint. This indicates that the total demand at each port cannot exceed its capacity $K_r$, and this constraint is enforced to ensure an interior solution to our model.

The port’s objective is also to maximize its own profit. The port’s profit function $\Pi_r$ is given by

$$\max \Pi_r = (\mu_r - O_r) F_r - m_r K_r \quad \text{for } r = 1, 2$$

where $O_r$ is the ports’ operation cost and $m_r$ is the capacity marginal cost. We assume that the ports’ operation cost and capacity cost are separable and that the marginal cost is constant.

Based on above non-cooperative profit functions, we now apply our model to two-stage game which would be solved using backward induction. We first investigated, in the second
stage, the shipping lines’ supply decision based upon the respective port’s capacity, price and transshipment coefficient. Thereafter, we advance to the first stage to derive, in relation to the shipping lines’ supply behaviour, a competitive port price. In addition, we apply comparative static analysis on equilibrium shipping lines’ supply decision with respect to the concerned parameters.

3.1 Stage two: Shipping line’s supply decision

Given the port capacity $K = (K_1, K_2)$, port price $\mu = (\mu_1, \mu_2)$, and transshipment coefficient $g = (g_1, g_2)$, shipping lines simultaneously assign their port calls at each port. We assume Cournot behaviour in shipping line competition\(^8\), thus maximizing the shipping lines’ profit function in (8) and, as a result, deriving their respective best response function of shipping lines’ supply decision. For this, the first-order condition of (8) can be written as

$$
\frac{\partial \pi_i}{\partial q_{i1}} = \left( p - c - \mu_1 - D_1 \right) \frac{\partial F_{i1}}{\partial q_{i1}} - \frac{\partial D_1}{\partial F_{i1}} \frac{\partial F_{i1}}{\partial q_{i1}} F_{i1} + \left( p - c - \mu_2 - D_2 \right) \frac{\partial F_{i2}}{\partial q_{i1}} - \frac{\partial D_2}{\partial F_{i2}} \frac{\partial F_{i2}}{\partial q_{i1}} F_{i2} = 0 \quad (10)
$$

The assumption of symmetry among the shipping lines’ price $p$ and cost $c$, and partial derivatives from (3) and (4) imply that the best response function of shipping line $i$, for all $i$, is identical, i.e., symmetric equilibrium $q_{i1} = ... = q_{iN}$. Hence, it implies that $Q_r = \sum_{i=1}^{N} q_{i1} = N q_{i1}$. Applying our earlier analysis on the shipping lines’ profit function, we obtain

**Lemma 1.** Shipping line $i$’s profit $\pi_i$ is concave in $q_{i1}$.

**Proof.** See the Appendix 1.

Lemma 1 shows that $\pi_i$ has a maximum in $q_{i1}$. Hence, there exists a unique Nash equilibrium in shipping lines’ supply decision.

We solve (10) in order to achieve the best response function of port call supply decision made by shipping line $i$ at port 1.

---

\(^8\) Cournot behavior in congestible facility users, such as airlines and shipping lines, has been assumed in Zhang and Zhang (2006) and Lam and Yap (2006).
\[
q_{Ki}^r = \frac{\left\{ f\left( g_1^2 + g_2^2 \right) + g_1^2 N \right\}}{N \left( g_2^3 - g_1^3 \right)} \left( g_2^3 - g_1^3 \right) \left( p - c \right) \left( g_2 - g_1 \right) + g_1 \mu_1 - g_2 \mu_2 \\
- \frac{\left\{ f\left( g_1 + g_2 \right) + g_1 g_2 N \right\}}{N \left( g_2^3 - g_1^3 \right)} \sqrt{g_1 g_2 + \frac{\left( g_2^3 - g_1^3 \right)^2}{3aN^2 \left( f\left( g_1 + g_2 \right) + g_1 g_2 N \right)^2}}
\]

(11)

where superscript \( K \) represents the standardized port capacities, \( K = K_1 = K_2 \). Then we relax our assumption on port capacities considering the case that two ports have different capacities. The port call supply decision made by shipping line \( i \) at port 1 is:

\[
q_{Ki}^{K_1, K_2} = \frac{\left\{ f\left( g_1^2 K_1^2 + g_2^2 K_2^2 \right) + g_1^2 N K_1^2 \right\}}{N \left( g_2^3 K_1^2 - g_1^3 K_2^2 \right)} \left( g_2^3 K_1^2 - g_1^3 K_2^2 \right) \left( p - c \right) \left( g_2 - g_1 \right) + g_1 \mu_1 - g_2 \mu_2 \\
- \frac{K_1 K_2 \left\{ f\left( g_1 + g_2 \right) + g_1 g_2 N \right\}}{N \left( g_2^3 K_1^2 - g_1^3 K_2^2 \right)} \sqrt{g_1 g_2 + \frac{\left( g_2^3 K_1^2 - g_1^3 K_2^2 \right)^2}{3aN^2 \left( f\left( g_1 + g_2 \right) + g_1 g_2 N \right)^2}}
\]

(12)

where superscript \( K_1 \neq K_2 \) stands for the different capacity levels of ports.

Accompanying above derivation works, we check on the consistency between (11) and (12) by assuming that in the latter, \( K_1 > K_2 \) and the difference between them, denoted using \( \sigma \), is insignificantly small. By substituting \( K_2 = K_1 - \sigma \) to (12), we examine \( \lim_{\sigma \to 0} q_{Ki}^{K_1, K_2} \). The result shows a convergence to \( \lim_{\sigma \to \infty} q_{Ki}^{K_1, K_2} = q_{Ki}^K \) (proof shown in Appendix 2). \( q_{Ki}^{K_1, K_2} \) led similar results.

3.2 Stage one: Port pricing strategies

The port pricing strategy is analyzed in this stage. Ports are maximizing their profit (9) by differentiating with respect to port price \( \mu_r \). The first order condition is shown in (13).

\[
\frac{\partial \Pi_r}{\partial \mu_r} = F_r + (\mu_r - O_r) \frac{\partial F_r}{\partial \mu_r} = 0
\]

(13)

Based on the results obtained from stage two and (13), we have following propositions.

**Proposition 1.** Port \( r \)'s profit function \( \Pi_r \) is concave in \( \mu_r \).
**Proof.** See the Appendix 4.

**Proposition 2.** Port competition has a unique equilibrium in strategic pricing, \( \mu^* = (\mu_1^*, \mu_2^*) \).

**Proof.** See the Appendix 5.

Proposition 1 assures that \( \Pi \) has a maximum in \( \mu_r \), and proposition 2 gives that there exists a unique Nash equilibrium in port pricing. With (11) and (12), we have (14) and (15), respectively.

\[
\frac{\partial \Pi_1}{\partial \mu^K_1} = N_f + N_g_1 \left( N q_{i_1}^K + \left( \mu_i - O_i \right) \frac{\partial q_{i_1}^K}{\partial \mu_i^K} \right) = 0
\]  

(14)

\[
\frac{\partial \Pi_1}{\partial \mu^K_{i_1,i_2}} = N_f + N_1 \left( N q_{i_1}^{K_1,K_2} + \left( \mu_i - O_i \right) \frac{\partial q_{i_1}^{K_1,K_2}}{\partial \mu_i} \right) = 0
\]

(15)

where superscript \( K \) stands for the identical port capacities and \( K_1 \neq K_2 \) stands for different port capacities of two ports. Solving these best response functions yield the Nash equilibrium in port prices. However, the best response function cannot be given on a closed form in this model, so we will explore this through numerical experiments.

### 3.3 Comparative statics analysis

We apply to comparative statics analysis in order to exam the changes in equilibrium port call supply decision with respect to the changes in the important parameters, such as the capacity, price and transshipment coefficient. As mentioned in our earlier analysis, the best response of shipping lines’ supply decision is identical to all shipping lines and the shipping lines’ best response function depends only on the aggregate strategies of shipping lines at each port. The characteristics of aggregate shipping lines’ supply decision, \( Q_r \), possess the characteristics of individual shipping line’ supply decision, \( q_{ir} \). Thus, we further investigate the comparative static analysis concerning the shipping lines’ aggregate output at port \( r \) with respect to parameter set \( X = \{ \mu, \mu_2, K_1, K_2, g_1, g_2 \} \). The results are shown below and the details of derivation works are given in Appendix 3.

\[
\frac{\partial Q_r}{\partial \mu_r} < 0, \quad \frac{\partial Q_r}{\partial \mu_i} > 0, \quad \frac{\partial Q_r}{\partial K_1} > 0, \quad \frac{\partial Q_r}{\partial K_2} < 0
\]

(16)
As shown in (16), shipping lines’ aggregate output at port $r$ decreases with own port’s price and increases with own port’s capacity. We obtain the below (17) and (18), the response of transshipment coefficient to aggregate shipping lines’ output.

\[
\frac{\partial Q_r}{\partial g_r} = \frac{\left(p - c + \mu_r - D_r\right) - 2D_r}{\left(\frac{\partial^3 D_r}{\partial F_r^2} g_r^2 F_r + \frac{\partial^3 D_r}{\partial F_s^2} g_s^2 F_s \right) + 2 \left(\frac{\partial D_r}{\partial F_r} g_r^2 + \frac{\partial D_s}{\partial F_s} g_s^2 \right)}
\]

(17)

\[
\frac{\partial Q_s}{\partial g_s} = \frac{\left(-p - c + \mu_s - D_s\right) + 2D_s}{\left(\frac{\partial^3 D_r}{\partial F_r^2} g_r^2 F_r + \frac{\partial^3 D_r}{\partial F_s^2} g_s^2 F_s \right) + 2 \left(\frac{\partial D_r}{\partial F_r} g_r^2 + \frac{\partial D_s}{\partial F_s} g_s^2 \right)}
\]

(18)

It can be implied that if the shipping lines’ profit margin at port $r$ is bigger than the double summation of port $r$’s total congestion delay cost, aggregate shipping lines’ output increases with the transshipment coefficient, which means higher transshipment coefficient leads to a more port calls from shipping lines. If not, total congestion delay cost overtakes profit margin. On the other hand, as given in (18), if the competitor port has higher profit margin than their overall congestion delay cost, shipping lines may assign more port call at competitor port.

4. Ports collusion and social optimum

Thus far in this study, we had focused mainly on a non-cooperative game where every port and shipping line makes independent decisions to maximize own profit. We now consider the case of two ports cooperating on prices and capacities, and of all market players, two ports and N-shipping line, cooperating to maximize the market profit to compare against the non-cooperative model.

4.1 Ports collusion model

Considering the case in which the two ports decide their prices and capacities concurrently. The profit function of ports collusion model is shown in below (19).

\[
\max_{\mu_r, \mu_s} \Pi_1 + \Pi_2 = (\mu_r - O_r) F_1 - m_1 K_1 + (\mu_s - O_s) F_2 - m_2 K_2
\]

(19)

The first-order conditions for each port pricing from (19) is

\footnote{This result is consistent with the comparative statics in Zhang and Zhang(2006) and Basso and Zhang(2007).}
\[
\frac{\partial (\Pi_1 + \Pi_2)}{\partial \mu_i} = F_i + (\mu_i - O_i) \frac{\partial F_1}{\partial \mu_i} + (\mu_2 - O_2) \frac{\partial F_2}{\partial \mu_i} = 0
\]
\[
\frac{\partial (\Pi_1 + \Pi_2)}{\partial \mu_2} = F_2 + (\mu_i - O_i) \frac{\partial F_1}{\partial \mu_2} + (\mu_2 - O_2) \frac{\partial F_2}{\partial \mu_2} = 0
\]

where \( \frac{\partial F_i}{\partial \mu_i} = -\frac{\partial F_2}{\partial \mu_2} \). Therefore, \( \frac{\partial (\Pi_1 + \Pi_2)}{\partial \mu_i} > 0 \) which means if there is no boundary for maximum port price, the port price would reach infinity. This is due to our assumption that shipping lines must call at both port (first constraint in (8)). Hence, compared with the non-cooperative case, we can easily observe that the port collusion model yields higher port price than the non-cooperative model. The result is intuitive that this case becomes a monopoly market. Therefore, in this case of monopolistic cooperation, the ports can have a free hand in escalating their prices (infinity) to maximize their profits. This analysis would help better explain the competitive landscape and strategies among regionally bounded terminal operators.

4.2 Social optimum model

We now consider a social optimum model that reflects cooperation among all players in the game to maximize their combined profits. This is captured in the social-welfare function (SW) given below:

\[
SW = \max_{\mu_i, \delta_i} \sum_{i=1}^{2} \Pi_i + \sum_{i=1}^{N} \pi_i = \sum_{i=1}^{2} \left( (\mu_i - O_i) \cdot F_i - m_i K_i \right) + \sum_{i=1}^{N} \sum_{r=1}^{2} (p - c - \mu_r - D_r) F_{ir}
\]

(21)

A reduced form of (21) is presented by

\[
SW = (p - c)(F_1 + F_2) - (O_i + D_i) \cdot F_i - m_i K_i - (O_2 + D_2) \cdot F_2 - m_2 K_2
\]

(22)

It is important to note that the port price disappears from (21) to give (22). This is because of the internalization of the port price due to the equality between total port revenue and the sum of shipping lines’ port cost. Therefore, there is no pricing stage in this model, but the supply decision game among shipping lines is considered. The shipping lines’ supply decision in social optimum model is characterized by first order condition below.
\[
\frac{\partial SW}{\partial q_{i1}} = -(O_i + D_i) \frac{\partial F_1}{\partial q_{i1}} - \left( \frac{\partial D_1}{\partial F_1} \frac{\partial F_1}{\partial q_{i1}} \right) F_1 - (O_1 + D_2) \frac{\partial F_2}{\partial q_{i1}} - \left( \frac{\partial D_2}{\partial F_2} \frac{\partial F_2}{\partial q_{i1}} \right) F_2 + (p-c) \left( \frac{\partial F_1}{\partial q_{i1}} + \frac{\partial F_2}{\partial q_{i1}} \right) = 0
\]

(23)

The standardized port capacities, \( K=K_1=K_2 \), give an explanation only to a port operation cost which is the only factor to affect on supply decision compared to non-cooperative model, hence relaxing this assumption to consider the different capacities of two ports, i.e. \( K_1 \neq K_2 \).

Solving (23) we can achieve the derived shipping lines’ supply decision below.

\[
q_{i1}^{SW} = \frac{\left\{ f \left( g_i^2 + g_2^2 \right) + g_2^3 N \right\}}{N \left( g_2^3 - g_i^3 \right)} \sqrt{g_1 g_2 + \left( g_2^3 - g_i^3 \right) K^2 \left\{ \left( p-c \right) \left( g_2 - g_i \right) + g_i O_i - g_2 O_2 \right\}}
\]

\[
q_{i1}^{SW} = \frac{\left\{ f \left( g_i^2 K_1^2 + g_2^2 K_2^2 \right) + g_2^3 N K_i^2 \right\}}{N \left( g_i^3 K_1^2 - g_i^3 K_2^2 \right)} \sqrt{g_1 g_2 + \left( g_i^3 K_1^2 - g_i^3 K_2^2 \right) \left\{ \left( p-c \right) \left( g_2 - g_i \right) + g_i O_i - g_2 O_2 \right\}}
\]

(24)

where superscript SW stands for social welfare maximization. It depicts that in social optimum model, the shipping lines’ decision in number of port calls seeks to minimize congestion delay cost and port operation cost so as to maximize overall profits.

To better appreciate the model, we can perceive the stipulated conditions as a vertical integration of shipping lines, where they extend their core businesses from liner shipping to terminal operation, such as A.P.Moller-Maersk whose business scope is in container shipping services and terminal operations. In this case, social optimum model explains the structure of these carriers’ profit, where ports’ prices are internalized and ports’ demands are determined by port operation cost, capacity and transshipment volume.
5. Numerical results

In addition to the derived functions for the shipping lines’ supply decision and the port pricing, we further explore the comparative statics through a set of numerical experiments. Firstly, we explain the effect on shipping lines’ supply decision conditional on differing port prices between two ports while applying various levels of port capacities. Secondly, we show the effect of transshipment coefficient on shipping lines’ supply decision while applying various levels of port capacities. Thirdly, as it is not able to achieve the closed form of port pricing stage, exploring the changes in equilibrium port prices with the changes in the parameters. Lastly, we show the comparison results of social optimum model and non-cooperative model in terms of shipping lines’ supply decision and market profits.

Figure 1 shows the shipping lines’ supply decision subjected to differing prices between two equally-sized ports. As expected, similar port prices will yield an equal portion of port calls to both ports. The portion of port calls to port 1 decreases as port 1’s price increases. This implies that shipping lines are easily attracted by a cheaper port price. Another important finding from Figure 1 is the combined effect of capacity and price level. Different slope gradients were obtained when different port capacities were subjected to similar price differentials (see \(K=4\) and \(K=8.29\)). While capacity \(K=4\) carried a gentle slope, the capacity \(K=8.29\) resulted in the steep slope. These results showed that when the both ports’ capacities are large, a marginal difference between two port prices is sufficient to drive significant demand to the cheaper port. In contrast, when both ports’ capacities are small, the shift of demand becomes inelastic to the difference in port prices. Therefore, it is reasonable to assume that the congestion effect associated with a small port capacity offsets the price difference between ports, and that a large port capacity offsets the congestion effect and hence amplifies the effect of price difference.
Since \( g_r \) is a factor that determines the transshipment demand of shipping lines, shipping lines are inclined to make more port calls at the port that provides a higher \( g_r \). However, this only holds true when the port capacity is sufficient to handle the additional transshipment demand and offset the congestion effect, which is shown at the intersection point between the curves in Figure 2. In other words, when the port capacity is small, the congestion effect takes precedence over the transshipment effect, and hence result in the lower portion of port calls despite a higher \( g_r \).

We examine the equilibrium port price of port 1 and port 2 while varying difference between two ports capacities which can be regarded as port expansion in practice. As shown in Figure 3, the capacity expansion in either port decreases the equilibrium prices, i.e.
This finding can be interpreted as such: a larger port can aggressively lower its price to attract more demand as they more likely to have spare capacity and hence less congestion. The other port has no choice but to lower the price since there is no other predominant factor to compete with. As a result, capacity expansion increases its own demand and decrease the demand of the competitor port.

As explained in Figure 2, a higher transshipment coefficient assures a higher transshipment volume for shipping lines and this works when the port capacity is sufficient to hold total demand. Figure 4 shows the equilibrium port price subjected to differing transshipment coefficient between two equally-sized ports. We can observe the combined effect of capacity and transshipment coefficient level. The port price of \( K = 7.68 \) increases as carrying a higher transshipment coefficient than the other port, but the port price is relatively lower than the port price of \( K = 4.8 \) in terms of low congestion level. Port 2’s price increases as port 1’s transshipment level increases and own transshipment level decreases. It is implied that the port has to raise the price in order to achieve the profit margin while their transshipment level is lower than competitor port.
In previous analysis on the social optimum model, we found that the shipping lines’ supply decision depends on port capacity, transshipment coefficient and port operation cost when the port price is internalized in the objective function. We now take our analysis further by comparing the non-cooperative model and the social optimum model. In the following analysis, we give both models identical parameters and compare the resulting shipping lines’ supply decisions and market profits. We fix the port operation cost of port 1 to be lower than that of port 2, while subjecting both models to a varying difference between the two port capacities. The transshipment coefficient is set to be identical for two ports in order to reduce
the complexities of results. Therefore, the given conditions for this experiment are summarized as: \( O_1 < O_2, K_1 > K_2 \) and \( g_1 = g_2 \).

As shown in Figure 5, in the social optimum model, the shipping lines have a higher propensity to call at port 1 under all capacity scenarios of port 2 because of port 1’s lower operation cost. In fact, when the capacity of port 2 is 40% or below the capacity of port 1, maximum port call is diverted to port 1. In the non-cooperative model, the propensity to make a port call at port 1 is comparatively lower than the social optimum model because of port prices. However, when the capacity of port 2 is 40% or lower than that of port 1, the percentage of port call diverted to port 1 increases rapidly. This is because the capacity difference between two ports is large enough to disregard the effect of port prices.

![Figure 5. Supply decision of non-cooperative and social optimum model](image)

Figure 6 gives that the market profit of the social optimum model is shown to be higher than that of the non-cooperative model. As examined in Figure 5, the port call diverted to port 1 in social optimum model is comparatively higher, hence the profit result it is. In addition, the decline tendency of profits of both models according to a varying difference between the two port capacities is because we set port 1’s capacity to be constant and varied the port 2’s capacity with decreasing every 20% of port 1’s. Therefore, the congestion delay cost increased as a varying difference between the two port capacities became larger.
In this study, we investigated the competition between container transshipment hub ports based on a duopolistic game model. We developed a unique container handling demand function which considers gateway and transshipment demand from the shipping lines’ perspective. These container handling demand functions were then combined to form the port demand function. Other important functions such as the congestion delay cost function and the profit functions for both carriers and ports were subsequently derived to set us up for the game analysis. We first conducted a non-cooperative two-stage game with duopolistic transshipment ports and a continuum of identical shipping lines, and solved it via backward induction. The shipping lines’ supply decision was first investigated with respect to the respective port’s capacity, price and transshipment coefficient in the second stage of the game. We had shown that the shipping lines’ supply decision follows a behavior of assigning more port calls to the port that provides a lower price and a larger capacity. We then advanced to the first stage to derive, in relation to the shipping lines’ supply behaviour, a competitive port price. We considered a port collusion and social optimum model that reflects cooperation between two ports and cooperation among all players in the game to maximize their combined profits. In the port collusion model, we found a monopolistic port market which resulted in infinite port price. In the social optimum model, we obtained the derived shipping lines’ supply best

![Figure 6. Total profits of non-cooperative and social welfare model](image-url)
response function that showed that the shipping lines’ decision in number of port calls seeks to minimize the ports’ operation costs and congestion delay costs so as to maximize overall profits.

To further our observation of the Nash equilibrium point, numerical experiments are conducted. We first found that price difference between two ports is further accentuated when both port capacities are large, as in this case, congestion becomes less of an inhibiting factor. Next, shipping lines are inclined to make more port calls at the port that provides a higher $g_r$, as long as the port capacity is sufficient to offset the accompanying congestion delay cost. In other words, when the port capacity is small, the congestion delay cost takes precedence over the transshipment volume, and hence result in a lower number of port calls despite a higher $g_r$. We had shown that ports do take into consideration their capacity when setting prices. A bigger port can set a lower port price to attract more demand as they more likely to have spare capacity and hence less congestion. And finally, the result of comparing non-cooperative model against social optimum model showed that the internalized port price paved the way for social optimum model focusing on minimizing overall cost so as to maximize the market profit. The non-cooperative model, on the other hand, is restricted by port price, hence resulting less profit margin.

In the main, this study may provide further insights to the shipping lines’ behaviour and the characteristics of transshipment demand. Such insights can serve as useful information to the port operators when optimizing their port pricing strategies. Furthermore, this study can be improved with nonlinear demand function to be shown more obvious behaviour of shipping lines’ preference on their supply decision. The tradeoffs of the congestion cost and transshipment benefits can be shown clearly in nonlinear demand function model. Asymmetric shipping lines’, unlike this study only considered the symmetric conditions of shipping lines’, would also be taken into account in the future study. The shift of port call supply decision can be determined by shipping lines’ own operating cost and price to the consignors which can affect the marginal profit of shipping lines’.
Appendix

1. Proof of Lemma 1

From (3),(4),(6),(7), we obtain

$$\frac{\partial^2 \pi_i}{\partial q_{i1}^2} = -2 \frac{\partial D_i}{\partial F_1} \frac{\partial F_1}{\partial q_{i1}} - \frac{\partial^2 D_i}{\partial F_2^2} \left( \frac{\partial F_1}{\partial q_{i1}} \right)^2 F_{i1} - 2 \frac{\partial D_i}{\partial F_2} \frac{\partial F_i}{\partial q_{i1}} - \frac{\partial^2 D_i}{\partial F_2^2} \left( \frac{\partial F_2}{\partial q_{i1}} \right)^2 F_{i2}$$

$$= -2 \frac{\partial D_i}{\partial F_1} Ng_i^2 - \frac{\partial^2 D_i}{\partial F_2^2} (Ng_i)^2 F_{i1} - 2 \frac{\partial D_i}{\partial F_2} Ng_i^2 - \frac{\partial^2 D_i}{\partial F_2^2} (Ng_i)^2 F_{i2}$$

$$= -g_i^2 \left( 2 \frac{\partial D_i}{\partial F_1} N + \frac{\partial^2 D_i}{\partial F_2^2} N^2 F_{i1} \right) - g_i^2 \left( 2 \frac{\partial D_i}{\partial F_2} N + \frac{\partial^2 D_i}{\partial F_2^2} N^2 F_{i2} \right) < 0$$

(A.1)

Therefore, $\pi_i$ is concave in $q_{i1}$.

2. Proof of $\lim_{\sigma \to 0} q_{i1}^{K_i K_2} = q_{i1}^K$

Let $C = \frac{\{(p - c)(g_2 - g_1) + g_i \mu_i - g_2 \mu_i\}}{3aN^2 \{f(g_1 + g_2) + g_1 g_2 N\}^2}$

where $C$ is constant.

$$q_{i1}^{K_i K_2} = \frac{\left\{ f \left( g_2 K_1^2 + g_1^2 K_2^2 \right) + g_1^2 N K_2^2 \right\} - K_1 K_2 \left\{ f \left( g_1 + g_2 \right) + g_1 g_2 N \right\} \sqrt{g_1 g_2 + C \left( g_1^2 K_1^2 - g_2^3 K_2^2 \right)}}{N \left( g_2^2 K_1^2 - g_1^2 K_2^2 \right)}$$

Assume $K_1 > K_2$ and then substitute $K_2 = K_1 - \sigma$.

$$q_{i1}^{K_i K_2} (\sigma) = \frac{\left\{ f \left( g_2 K_1^2 + g_1^2 (K_1 - \sigma)^2 \right) + g_1^2 N K_2^2 \right\} - K_1 (K_1 - \sigma) \left\{ f \left( g_1 + g_2 \right) + g_1 g_2 N \right\} \sqrt{g_1 g_2 + C \left( g_1^3 K_1^2 - g_2^3 (K_1 - \sigma)^2 \right)}}{N \left( g_2^2 K_1^2 - g_1^2 (K_1 - \sigma)^2 \right)}$$

Study the limit of $q_{i1}^{K_i K_2}$ as $\sigma$ approaches 0.
\[
\lim_{\sigma \to 0} q_{ii}^{K_i K_j} (\sigma) \\
= \left\{ f\left(g_2^2 + g_1^2\right) + g_2^2 N - f\left(g_1 + g_2\right) + g_1 g_2 N\right\} / \sqrt{g_1 g_2 + K_1^2 C\left(g_1^3 - g_1\right)} / \sqrt{g_2^3 - g_1^3} \\
\]

As shown above, the value of limit \(\sigma\) approaches 0 exactly converges to (11).
Therefore, \(\lim_{\sigma \to 0} q_{ii}^{K_i K_j} = q_{ii}^K\).

3. Derivations of comparative statics for the equilibrium shipping lines’ supply decision

Let \(X = (\mu, \mu_s, K_r, K_s, g_r, g_s)\),

\(q_{\nu} = q_{\nu}^* (X)\)

Using (10), we can express \(\frac{\partial \pi_i}{\partial q_{\nu}} (q_{\nu}^*; X) = 0\)

Applying the total differential implicit function and chain rule, differentiate (10) with respect to \(X\). Then, we obtain

\[
\frac{\partial^2 \pi_i}{\partial q_{\nu} \partial q_{\nu}} + \sum_{i \neq j} \left( \frac{\partial^2 \pi_i}{\partial q_{\nu} \partial q_{\nu}} \frac{\partial q_{\nu}}{\partial X} \right) + \frac{\partial^2 \pi_i}{\partial q_{\nu} \partial X} = 0
\]

(A.2)

\[
\frac{\partial \pi_i}{\partial q_{\nu} \partial q_{\nu}} = \frac{\partial^2 D_r}{\partial F_r \partial F_r} \frac{\partial F_r}{\partial q_{\nu}} F_{ir} - \frac{\partial D_r}{\partial F_r} \frac{\partial F_r}{\partial q_{\nu}} \frac{\partial F_r}{\partial q_{\nu}} - \frac{\partial D_r}{\partial F_r} \frac{\partial F_r}{\partial q_{\nu}} \frac{\partial F_r}{\partial q_{\nu}} \\
- \frac{\partial^2 D_s}{\partial F_s \partial F_s} \frac{\partial F_s}{\partial q_{\nu}} F_{is} - \frac{\partial D_s}{\partial F_s} \frac{\partial F_s}{\partial q_{\nu}} \frac{\partial F_s}{\partial q_{\nu}} - \frac{\partial D_s}{\partial F_s} \frac{\partial F_s}{\partial q_{\nu}} \frac{\partial F_s}{\partial q_{\nu}} < 0
\]

(A.3)

Substituting (A.1) and (A.3) into (A.2),

\[
\left\{ - \frac{\partial^2 D_r}{\partial F_r^2} \left( \frac{\partial F_r}{\partial q_{\nu}} \right)^2 F_{ir} - 2 \frac{\partial D_r}{\partial F_r} \frac{\partial F_r}{\partial q_{\nu}} \frac{\partial F_r}{\partial q_{\nu}} - \frac{\partial^2 D_s}{\partial F_s^2} \left( \frac{\partial F_s}{\partial q_{\nu}} \right)^2 F_{is} - 2 \frac{\partial D_s}{\partial F_s} \frac{\partial F_s}{\partial q_{\nu}} \frac{\partial F_s}{\partial q_{\nu}} \right\} \frac{\partial q_{\nu}}{\partial X} \\
+ \sum_{i \neq j} \left\{ \left( \frac{\partial^2 D_r}{\partial F_r \partial q_{\nu}} \frac{\partial F_r}{\partial q_{\nu}} F_{ir} - \frac{\partial D_r}{\partial F_r} \frac{\partial F_r}{\partial q_{\nu}} \frac{\partial F_r}{\partial q_{\nu}} - \frac{\partial D_r}{\partial F_r} \frac{\partial F_r}{\partial q_{\nu}} \frac{\partial F_r}{\partial q_{\nu}} \\
- \frac{\partial^2 D_s}{\partial F_s \partial q_{\nu}} \frac{\partial F_s}{\partial q_{\nu}} F_{is} - \frac{\partial D_s}{\partial F_s} \frac{\partial F_s}{\partial q_{\nu}} \frac{\partial F_s}{\partial q_{\nu}} - \frac{\partial D_s}{\partial F_s} \frac{\partial F_s}{\partial q_{\nu}} \frac{\partial F_s}{\partial q_{\nu}} \right\} \frac{\partial q_{\nu}}{\partial X} \right\} + \frac{\partial^2 \pi_i}{\partial q_{\nu} \partial X} = 0
\]

(A.4)
From the properties of demand and congestion delay cost function in (3) and (4), the (A.4) can be expressed as:

\[
\begin{pmatrix}
\frac{\partial^2 D_r}{\partial F_r^2} \left( N g_s^2 \right) F_r - 2 \frac{\partial D_r}{\partial F_r} \left( N g_s^2 \right) \\
- \frac{\partial^2 D_s}{\partial F_s^2} \left( -N g_s \right) F_s - 2 \frac{\partial D_s}{\partial F_s} \left( -N g_s \right) (-g_s)
\end{pmatrix}
\frac{\partial q_{ir}}{\partial X} + \frac{\partial^2 \pi_i}{\partial q_{ir} \partial X} = 0
\]

\[
+ \sum_{i \neq j}
\begin{pmatrix}
- \frac{\partial^2 D_r}{\partial F_r^2} \left( N^2 g_r^2 \right) F_r - 2 \frac{\partial D_r}{\partial F_r} \left( N g_r \right) (g_r) \\
- \frac{\partial^2 D_s}{\partial F_s^2} \left( N^2 g_s^2 \right) F_s - 2 \frac{\partial D_s}{\partial F_s} \left( -N g_s \right) (-g_s)
\end{pmatrix}
\frac{\partial q_{ir}}{\partial X} + \frac{\partial^2 \pi_i}{\partial q_{ir} \partial X} = 0
\]

Finally,

\[
\begin{pmatrix}
\frac{\partial^2 D_r}{\partial F_r^2} \left( N g_s^2 \right) F_r + 2 \frac{\partial D_r}{\partial F_r} \left( N g_s \right) (g_s) \\
+ \frac{\partial^2 D_s}{\partial F_s^2} \left( N^2 g_s^2 \right) F_s + 2 \frac{\partial D_s}{\partial F_s} \left( -N g_s \right) (-g_s)
\end{pmatrix}
\frac{\partial Q_r}{\partial X} + \frac{\partial^2 \pi_i}{\partial q_{ir} \partial X} = 0
\]

(A.5)
Summing up $i$ in (A.5), then we obtain

$$\frac{\partial Q_r}{\partial X} = \frac{1}{N} \sum_{i=1}^{\infty} \frac{\partial^2 \pi_i}{\partial q_i \partial X}$$

$$= \left( N \left( \frac{\partial^2 D_r}{\partial F_r^2} g_r^2 F_r + \frac{\partial^2 D_s}{\partial F_s^2} g_s^2 F_s \right) + 2 \left( \frac{\partial D_r}{\partial F_r} g_r^2 + \frac{\partial D_s}{\partial F_s} g_s^2 \right) \right)$$

Using (10) to calculate $\frac{\partial^2 \pi_i}{\partial q_i \partial X}$, we obtain:

$$\frac{\partial Q_r}{\partial \mu_r} = \frac{-g_r}{N} \left( N \left( \frac{\partial^2 D_r}{\partial F_r^2} g_r^2 F_r + \frac{\partial^2 D_s}{\partial F_s^2} g_s^2 F_s \right) + 2 \left( \frac{\partial D_r}{\partial F_r} g_r^2 + \frac{\partial D_s}{\partial F_s} g_s^2 \right) \right) < 0$$

$$\frac{\partial Q_r}{\partial \mu_s} = \frac{g_s}{N} \left( N \left( \frac{\partial^2 D_r}{\partial F_r^2} g_r^2 F_r + \frac{\partial^2 D_s}{\partial F_s^2} g_s^2 F_s \right) + 2 \left( \frac{\partial D_r}{\partial F_r} g_r^2 + \frac{\partial D_s}{\partial F_s} g_s^2 \right) \right) > 0$$

$$\frac{\partial Q_r}{\partial K_r} = \frac{6g_r D_r}{K_r} \left( N \left( \frac{\partial^2 D_r}{\partial F_r^2} g_r^2 F_r + \frac{\partial^2 D_s}{\partial F_s^2} g_s^2 F_s \right) + 2 \left( \frac{\partial D_r}{\partial F_r} g_r^2 + \frac{\partial D_s}{\partial F_s} g_s^2 \right) \right) > 0$$

$$\frac{\partial Q_r}{\partial K_s} = \frac{-6g_r D_r}{K_r} \left( N \left( \frac{\partial^2 D_r}{\partial F_r^2} g_r^2 F_r + \frac{\partial^2 D_s}{\partial F_s^2} g_s^2 F_s \right) + 2 \left( \frac{\partial D_r}{\partial F_r} g_r^2 + \frac{\partial D_s}{\partial F_s} g_s^2 \right) \right) < 0$$

$$\frac{\partial Q_r}{\partial g_r} = \frac{(p - c - \mu_r - D_r) - 2D_r}{N} \left( N \left( \frac{\partial^2 D_r}{\partial F_r^2} g_r^2 F_r + \frac{\partial^2 D_s}{\partial F_s^2} g_s^2 F_s \right) + 2 \left( \frac{\partial D_r}{\partial F_r} g_r^2 + \frac{\partial D_s}{\partial F_s} g_s^2 \right) \right)$$

$$\frac{\partial Q_r}{\partial g_s} = \frac{-(p - c - \mu_s - D_s) + 2D_s}{N} \left( N \left( \frac{\partial^2 D_r}{\partial F_r^2} g_r^2 F_r + \frac{\partial^2 D_s}{\partial F_s^2} g_s^2 F_s \right) + 2 \left( \frac{\partial D_r}{\partial F_r} g_r^2 + \frac{\partial D_s}{\partial F_s} g_s^2 \right) \right)$$

4. Proof of Proposition 1
Referring to a result of Appendix 3, $\frac{\partial Q_r}{\partial \mu_r} < 0$, and the second order conditions of $\Pi_r$ with respect to own port price $\mu_r$, shows that it is strictly concave since,

$$\frac{\partial^2 \Pi_r}{\partial \mu_r^2} = 2 \frac{\partial F_r}{\partial Q_r} \frac{\partial Q_r}{\partial \mu_r} + (\mu_r - O_r) \frac{\partial F_r}{\partial Q_r} \frac{\partial^2 Q_r}{\partial \mu_r^2} = 2 \frac{\partial F_r}{\partial Q_r} \left( \frac{\partial Q_r}{\partial \mu_r} \right) = 2 N g_r \left( \frac{\partial Q_r}{\partial \mu_r} \right) < 0$$

(A.6)

5. **Proof of Proposition 2**

To prove uniqueness of port price, we show the diagonal dominant condition. From the port profit function (9) and above (A.6), we know that

$$\frac{\partial^2 \Pi_r}{\partial \mu_r \partial \mu_r} = 2 N g_r \frac{\partial Q_r}{\partial \mu_r} < 0$$

$$\left| \frac{\partial^2 \Pi_r}{\partial \mu_r \partial \mu_r} \right| = \left| \frac{\partial F_r}{\partial Q_r} \frac{\partial Q_r}{\partial \mu_r} + \frac{\partial F_r}{\partial Q_r} \frac{\partial Q_r}{\partial \mu_r} + (\mu_r - O_r) \left( \frac{\partial^2 F_r}{\partial Q_r^2} \frac{\partial Q_r}{\partial \mu_r} \frac{\partial Q_r}{\partial \mu_r} + \frac{\partial F_r}{\partial Q_r} \frac{\partial^2 Q_r}{\partial \mu_r^2} \right) \right| = \left| \frac{\partial F_r}{\partial Q_r} \left( \frac{\partial Q_r}{\partial \mu_r} + \frac{\partial Q_r}{\partial \mu_r} \right) \right|$$

Since $\frac{\partial Q_r}{\partial \mu_r} = - \frac{\partial Q_r}{\partial \mu_r}$, we obtain

$$\frac{\partial^2 \Pi_r}{\partial \mu_r^2} + \sum_{r \neq s} \left| \frac{\partial^2 \Pi_r}{\partial \mu_r \partial \mu_s} \right| < 0$$

which implies that the Nash equilibrium is unique.
References


