Tentative Syllabus for EC5880: Topics in Economics

Economics of Asymmetric Information
Instructor: Indranil Chakraborty

**Objective**
The module will cover some topics from the economics of asymmetric information that use theoretical models that are motivated by real life economic relationships to provide interesting insights. The aim is to prepare students for serious analysis using the tools of microeconomic theory. The target audience for this module consists of advanced undergraduate students with general interest in microeconomics. In particular, students who intend to write undergraduate thesis will find the preparation useful later on. Graduate students with interest in microeconomics may also find the module interesting.

Although the problems discussed in the module arise in real situations, this is a course in economic theory. Students are, therefore, asked to go over the detailed outline of the module given below and take a look at a sample lecture note attached with this syllabus to decide whether the module is appropriate for them. The sample lecture note will also provide some idea on the maximum level of mathematical difficulty that will be encountered by a student in this module. Needless to mention, I will walk the students through all the relevant details towards understanding the nature of applied theoretical research in microeconomics. However, a student will need to be willing to do the walking with me to find the course interesting.

**Material**
In order to encourage students to think independently the module will be conducted a bit loosely. We will base our material on class lectures, notes posted on IVLE, some classic research papers in economics, and interesting portions of books. However, all the theoretical results and derivations which will form the core of the module and will require help in understanding will be covered formally in the lectures. The students will then be expected to learn and build peripheral ideas around that material by working on problems and ideas that will be asked in regular assignments.

The books relevant for the module will be placed on RBR at the Central Library. The research papers can be downloaded via the NUS Library’s internet portal.

**Grading (Tentative)**
Students will be evaluated based primarily on their efforts, regular execution of the tasks assigned during the course of the semester, and the relative accomplishments of the students. The following activities will be taken into account (weights indicated in parenthesis) to determine the final grade:

*Problem sets (40%)*
Once every two weeks (and sometimes more frequently if necessary) there will be a problem set assignment. All students are required to solve the problem sets and make submissions within the due date.
Class participation and presentations (25%)
Students who come up with new ideas, interesting comments and observations, and a clear demonstration of staying on top of the readings (e.g. by answering questions asked in class), will receive credit for the effort. In addition, sometimes, students will be asked to solve certain portions of the problem sets or suggested readings in class using PowerPoint slides that they will prepare in advance. Those who volunteer to make such presentations will receive credit for the work.

Research Project (35%)
All students will be required to work on a research project that will culminate in a research paper. The work will involve thinking and discussing potential problems that could be modeled using the techniques that are used in this module (it is okay if some parts of the techniques are borrowed from other courses taken previously). The week after the midterm break students will need to be ready with ideas on the precise economic problem that the student will work on for the project. Between that time and the last day of classes each student will be developing the idea to ask the interesting questions for that situation and write down a tractable economic model that will help analyze and address the questions being asked by the student. Once some progress has been made in analyzing the model and some results obtained, the student will then put it together and write up as a research paper in the same format as the research papers covered in the module. There will be a short presentation of the research project by the student at the end of the semester.

There will be no in-class exam for this module if the number of students enrolled remains low. The percentages and nature of the above activities may have to be modified based on the enrollment size of the class. In the unlikely case that a large number of students enroll for this module, an in class exam with at least a 30% weight will be introduced to maintain grading objectivity.

A few words about the topics covered in this module
Asymmetric information deals with situations where the participants in the economic situation do not have the same kind of information. One or more of the economic agents have private information on some relevant aspect of the economic environment. The problem is to understand behavior of the participants in such situations and design the mechanisms (sometimes referred to as contracts) appropriately to serve the mechanism designer’s objective. The problem can arise in any of the following contexts:

Auction Theory
In economics a large number of monopoly type markets are modeled in auction theory. Essentially any situation where there is a single seller selling a limited amount of objects to multiple competing buyers, the bidders, (alternatively, a single buyer buying a limited amount of objects from multiple competing sellers) is modeled as an auction. A large number of markets fit this description in modern day economies. The objective of an analysis here is to predict the behavior of the bidders, and design auction mechanisms that are best for the auctioneer.
Moral Hazard
The models of moral hazard involve a principal (e.g. employer) who will pay an agent (e.g. an employee) to undertake an activity. The employer cannot observe the action undertaken by the agent (e.g., the amount of work put in by the employee) but observes the outcome (e.g., the amount sold by a salesman or the amount produced by a worker). The agent, of course, knows the action he has chosen. The problem is to design the incentives in terms of payments so that the agent puts the effort that is best for the principal.

Adverse Selection
These models deal with situations where an agent has a privately known information the knowledge of which would allow the principal to maximize her profit. For instance, knowing an insured driver’s ability to avoid accidents would allow the insurer to charge the correct premium. Similarly, knowing an employee’s ability will allow an employer to assign him the right task and the appropriate wage. The problem here is to design the best mechanisms under this type of informational constraints. (Auctions are in fact models of adverse selection.)

Mixed Models
The mixed models involve economic situations that have elements of both moral hazard and adverse selection. The problem that we will discuss in this context will have elements of all the above three.

Details of topics and sources
We will cover four broad areas: Auction theory, moral hazard, adverse selection, and signaling or nonlinear pricing (if time permits):

Auction Theory
(ii) Lecture notes

Moral Hazard
(i) *Contract Theory*, Bolton and Dewatripont

Adverse Selection
(i) TBA
(ii) *Contract Theory*, Bolton and Dewatripont

Mixed Model of Moral Hazard and Adverse Selection
If time permits we will also cover one of the following two topics:

Signaling
(i) TBA
(ii) *Contract Theory*, Bolton and Dewatripont

Nonlinear pricing
(i) *The Theory of Industrial Organization*, Jean Tirole

The basic model for each topic will be followed up with a discussion of applications. The research papers and material that will be referred to in these discussions are not going to be covered in class. However, I will give introductions to some of the materials during the lectures and the students can follow their interests in reading up some of these materials for ideas on the research paper.

*The instructor reserves the right to make appropriate changes to this document.*
An introductory note on Auction Theory

What is an auction?

If you ask anyone what an “auction” is, the chances are very high that the person will think about the art or antique auctions at the Christies and the Sotheby’s. In today’s e-world people would also think about the eBay sales. An auction, to today’s economists, refers generally to the act of buying or selling of an item under a specific set of rules when there are either multiple buyers or multiple sellers are competing against each other to be successful in their act of buying or selling.

A very large quantity of transactions ranging from licenses for broadcasting using the wireless bands to Treasury bills, and from rare items to electricity are often done under a pre-specified set of rules through a competitive process. All these selling/buying processes are called auctions. In order to avoid confusion at this level, we will consider only auctions where an object is being sold by a seller and the buyers are competing to obtain the item.

Auction classifications

Auctions are classified in several ways for the purpose of developing a theory for each classification. One classification is done based on the buyers’ perception of the value of the object. If each buyer knows the precise value of the object to him and the value can possibly be different from that to another agent, it is called a private value auction. If all buyers have the same exact value for the object but the buyers do not know what it is, but they have formed their own estimates based on some privately held information then the auction is called a common value auction. In reality most auctions have elements of both private and common values.

Under another classification the auctions are classified based on the number of objects on sale. An auction where only one object is being sold is called a single-object auction. If there are multiple objects on sale in a single auction then it is called a multi-object auction. When the multiple objects are essentially identical units of the same object then it is called a multi-unit auction. Sometimes, the multiple objects are not sold in a single auction but in separate auctions. If the auctions are conducted simultaneously then they are called simultaneous auctions. If the auctions are conducted one after the other they are called sequential auctions. Auctions like those for radio spectrum licenses in the US use rules that are a mix of several types of auctions.
In the introductory session we will focus on the single-object auction for understanding the theory, and if there is time we will have a brief general discussion of the other types of auctions. We first consider the private value auctions. There are four standard auction rules that are studied at the theory level:

**First-price auction.** Under this auction rule each bidder submits a sealed bid. The highest bidder is awarded the object and the price he has to pay is equal to his bid amount.

**Second-price auction** (also called the Vickrey auction after the celebrated Nobel prize winning economist William Vickrey). This is also a sealed-bid auction where the highest bidder is awarded the object. In this case, however, the winning bidder is asked to pay a price equal to the second-highest bidders bid amount.

**English auction.** Under this rule the prices start at some low level, say zero, and is gradually increased. Initially, when the price is very low, almost all bidders will be interested in purchasing the item at the going price. However, as the price increases, bidders drop out of the race one after the other. When all but one bidder remains interested in making the purchase at the going price the price increase stops and the only remaining bidder is awarded the object at the going price. For theoretical analysis we often assume that the price increase happens smoothly rather than in jumps (as you would see at Chistie’s, for instance). In real life, the auction that is carried out in this way is called the Japanese clock auction, but in economics we often refer to it as the English auction.

**Dutch auction.** This auction derives its name from the Dutch flower auctions where the rule is used. The price starts at a very high level and is gradually decreased. The first bidder who expressed interest to purchase is awarded the item at a price equal to the price at which the bidder expresses his interest. (The price actually stops the moment the person says he wants to purchase. So the price is called the stop-out price.)

**Predicting bidder behavior**

Let us now try to do the theoretical analysis of these auction rules. The first step is to predict the behavior of the bidders. We take a game theoretic approach and use the concept of equilibrium to predict bidder behavior. The basic auction theory considers the situation where each of \( n \) bidders has privately known value for the object. From the point of view of the other bidders and the seller the value \( V_i \) that bidder \( i \) has for the object is unknown and can really be anything and in that sense it is a random variable.
The random variable $V_i$ has a probability distribution $F(v_i)$ on, say, $[0, b]$. For instance if the value is uniformly distributed on $[0, b]$ then the probability that the value is less than or equal to $v$ is $F(v) \equiv P(V_i \leq v) = \frac{v}{b}$.

**First-price auction rule.** The analysis starts with finding the equilibrium bidding strategy of this auction game. The equilibrium strategy that we will find is called the Bayes-Nash equilibrium strategy since it is chosen by a bidder to maximize his expected payoff given the other bidders’ strategies. A strategy in this case is a function $\beta(v)$ that assigns a number (bid) to each possible private value of the bidder. We will consider a symmetric Bayes-Nash equilibrium, i.e., an equilibrium where all bidders behave similarly and use the same equilibrium strategy $\beta(v)$. The challenge is to find a strategy $\beta(v)$ for bidder $i$ such that if all the other $n-1$ bidders use this strategy then it is optimal (in the sense that it maximizes his expected utility) for the bidder $i$ to use it, too.

To easily understand how this is done let us consider the example where the values are uniformly distributed on $[0, 1]$, i.e., $F(v) = v$. It can be shown that the equilibrium bidding strategy must be strictly increasing in this case. We are not going to show it here, but take this knowledge as given throughout our analysis. So suppose the others are using the equilibrium bidding strategy $\beta(v)$ that we need to find. If bidder $i$ who has value $v_i$ bids $b$ then his expected payoff is given by

$$
(v_i - b) \left[ P(\beta(V_j) \leq b) \right]^{n-1} = (v_i - b) \left[ P(V_j \leq \beta^{-1}(b)) \right]^{n-1} = (v_i - b) \left[ F(\beta^{-1}(b)) \right]^{n-1}.
$$

Since we are considering the case of the uniform distribution the expected payoff to consider is

$$(v_i - b) \left[ \beta^{-1}(b) \right]^{n-1}
$$

If $\beta(v)$ is indeed an equilibrium bidding strategy then bidder $i$’s expected payoff must be maximum when he chooses $b = \beta(v_i)$. The first order condition for this is

$$
\left[ - \left[ \beta^{-1}(b) \right]^{n-1} + (v_i - b)(n - 1) \left[ \beta^{-1}(b) \right]^{n-2} \frac{1}{\beta'(\beta^{-1}(b))} \right]_{b=\beta(v_i)} = 0
$$

i.e.,

$$
-v_i^{n-1} + (v_i - \beta(v_i))(n - 1)v_i^{n-2} \frac{1}{\beta'(v_i)} = 0
$$

3
or,
\[-v_i^{n-1} \beta'(v_i) + (v_i - \beta(v_i))(n-1)v_i^{n-2} = 0\]
or,
\[-v_i \beta'(v) + (v_i - \beta(v))(n-1) = 0\]
This is a differential equation with a solution \(\beta(v_i) = \frac{n-1}{n}v_i\). In other words, the Bayes-Nash equilibrium bidding strategy for the first-price auction in this case is given by
\[\beta(v_i) = \frac{n-1}{n}v_i\]
for all bidders \(i = 1, ..., n\).

You can now check that in this equilibrium the expected revenue that is generated by the auction is \(\frac{n-1}{n+1}\).

**Second-price auction.** In this case it is a weakly dominant strategy for a bidder to bid his true value!!! In equilibrium, therefore, each bidder bids his true value and the price in the auction is equal to the second-highest value in the auction. That is the starting point of the celebrated Revenue Equivalence Theorem.

**English auction.** It is a weakly dominant strategy for a bidder to not quit the auction until the price in the auction reaches his true value. The price, again, is equal to the second-highest value in the auction.

**Dutch auction.** The Dutch auction is also strategically similar to the first-price auction, and it turns out to be equivalent to the first-price auction.

**Expected Equilibrium Payoff to the Seller**

The same approach is used for calculating the equilibrium bidding strategies under the different auction rules for more general value distribution and when a reserve price is used. The next step is to calculate the expected revenue to the seller:

**Second-Price Auction/English Auction**

The expected payment of a bidder with value \(v \geq r\) is given by
\[rF(r)^{n-1} + n(n-1) \int_r^v xF(x)^{n-2}(1 - F(x))f(x)dx\]
The expected payment conditional on winning the object is given by

\[ \frac{1}{F(v)^{n-1}} \left( rF(r)^{n-1} + (n - 1) \int_r^v xF(x)^{n-2} f(x) dx \right) \]

**First-Price Auction**

The equilibrium bidding strategy in the first price auction is

\[ b(v) = \frac{1}{F(v)^{n-1}} \left( rF(r)^{n-1} + (n - 1) \int_r^v xF(x)^{n-2} f(x) dx \right) \]

Note that this is the expected payment conditional on winning in the second price auction. The expected payoff to the seller with value \( x_0 \) for the object from the first price (also the second-price) auction is

\[ x_0 F(r)^n + \int_r^w b(v) n F(v)^{n-1} f(v) dv = x_0 F(r)^n + \int_r^w \frac{1}{F(v)^{n-1}} \left( rF(r)^{n-1} + (n - 1) \int_r^v xF(x)^{n-2} f(x) dx \right) nF(v)^{n-1} f(v) dv \]

\[ = x_0 F(r)^n + \int_r^w rF(r)^{n-1} n f(v) dv + n(n - 1) \int_r^w \int_r^v xF(x)^{n-2} f(x) dx f(v) dv \]

\[ = x_0 F(r)^n + nrF(r)^{n-1} \int_r^w f(v) dv + n(n - 1) \int_r^w \int_x^w f(v) dv x F(x)^{n-2} f(x) dx \]

\[ = x_0 F(r)^n + nr(1 - F(r)) F(r)^{n-1} + n(n - 1) \int_r^w x(1 - F(x)) F(x)^{n-2} f(x) dx \]

To calculate the optimal reserve we take the derivative of this expected payoff to the seller and set it equal to zero:

\[ x_0 n F(r)^{n-1} f(r) + n(1 - F(r)) F(r)^{n-1} + n(n - 1)r(1 - F(r)) F(r)^{n-2} f(r) - nr F(r)^{n-1} f(r) - n(n - 1)r(1 - F(r)) F(r)^{n-2} f(r) = 0 \]

or

\[ x_0 f(r) + (1 - F(r)) - rf(r) = 0 \]
or,

\[ x_0 + \frac{1 - F(r)}{f(r)} - r = 0 \]

Therefore the optimal reserve \( r^* \) satisfies the condition

\[ r^* - \frac{1}{\lambda(r^*)} = x_0 \]

**Reserve and Entry Fee**

The same screening level induced by a reserve price could also be induced by an *entry fee*. It is easily checked that the expected payoff to a type \( r \) bidder is given by

\[ \int_0^r F(x)^{n-1} dx. \]

This entry fee will keep all bidders with values less than \( r \) out of the auction. Therefore the expected payoff to the seller from charging this entry fee is given by

\[ x_0 F(r)^n + n F(r) \int_0^r F(x)^{n-1} dx + n(n-1) \int_r^w x F(x)^{n-2} (1 - F(x)) f(x) dx \]

The expected payoff from having a reserve price \( r \) is

\[ x_0 F(r)^n + nr(1 - F(r))F(r)^{n-1} + n(n-1) \int_r^w x(1 - F(x)F(x)^{n-2} f(x) dx. \]

Suppose

\[ n F(r) \int_0^r F(x)^{n-1} dx \geq nr(1 - F(r))F(r)^{n-1} \]

Then we have

\[ \int_0^r F(x)^{n-1} dx \geq r(1 - F(r))F(r)^{n-2} \]

or, using integration by parts,

\[ r F(r)^{n-1} - \int_0^r x(n - 1)F(x)^{n-2} f(x) dx \geq r(1 - F(r))F(r)^{n-2} \]

i.e.,

\[ F(r) - (1 - F(r)) \geq \frac{1}{r F(r)^{n-2}} \int_0^r x(n - 1)F(x)^{n-2} f(x) dx \]

or,

\[ 2F(r) - 1 \geq \frac{n - 1}{r F(r)^{n-2}} \int_0^r x F(x)^{n-2} f(x) dx \]

This is a contradiction. Therefore, the expected revenue from using a reserve price is strictly higher than that using an entry fee that imposes a similar screening level.